PORTFOLIO EQUILIBRIUM, INVESTMENT AND CURRENT ACCOUNT ADJUSTMENT

BY

Kiyoshi OTANI*

The Faculty of Social Sciences
The University of Tsukuba

ABSTRACT

The typical mechanism of current account adjustment envisioned by the portfolio approach no longer works if the unrealistic assumptions of non-tradable domestic assets and the absence of home investment are abandoned. This paper explores a mechanism that still works without these assumptions. It shows that, the current account deficit being associated with excessive home investment, the supply of domestic financial assets increases under the deficit to finance such investment; and that, given imperfect substitutability of domestic and foreign financial assets, the rate of return on domestic assets has to rise, thereby depressing excessive home investment and hence eliminating the current account deficit.

I Introduction

As in Kouri (1976), the mechanism of current account adjustment in the portfolio approach is typically based on two assumptions, both of which allow us to identify a current account surplus with increasing home real wealth. The first assumption is that home investment is non-existent. This renders a current account surplus as equivalent to positive saving, and hence an increase in home real wealth. Second, it is assumed that home assets are nontradable, while foreign ones tradable. This assumption means a current account surplus constitutes an increase in the supply of foreign assets (money) to home residents. This increase calls for depreciation of the foreign currency which is necessary to place the increase in the portfolio of home residents. The depreciation also increases the foreign (real) value of home wealth. Because of the wealth effect, an increase in home wealth, brought about in the two channels, depresses home saving, and hence reduces a current account surplus given the first assumption. Needless to say, the assumptions of non-existence of home investment and non-tradable home assets are unrealistic, but the above current account adjustment does not work without them. For, then, a current account surplus is no longer equivalent to either positive saving or an increase in the supply of foreign money to home residents; it follows that the surplus does not necessarily increase home wealth, and hence create the wealth effect to reduce home saving.

This paper advances an alternative process of current account adjustment that works even when domestic assets as well as foreign assets are tradable, and home investment is present. Many theories of open economy macroeconomics attribute to changes in wealth and saving the key role in current account adjustment[1]. In contrast, this paper stresses the importance of home investment in the adjustment. Specifically, it demonstrates the following adjustment mechanism. The current account deficit is equivalent to an excess of home investment over home saving. From this follows that the deficit is accompanied by an increase in the supply of home securities

relative to home saving as home firms have to raise funds necessary for the investment. To keep the home financial market in equilibrium this requires that, given less than perfect substitutability between home and foreign securities, the rate of return to domestic securities must rise to provoke substitution in the portfolios of both home residents and foreigners in favor of home securities. A rise in the rate of return reduces home investment, and hence reduces the current account deficit.

There are, of course, works such as Dornbusch (1975, 1980, Ch, 14.), Frenkel and Rodriguez (1975), Lipton and Sachs (1983), and Niehans (1984, Ch, 6) that also put the stress on investment and the financial market in relation to the current account adjustment problem.

Dornbusch (1975, 1980, Ch. 14) describes the following current account adjustment mechanism that involves fluctuation of home investment. A large investment that is partly responsible for a current account deficit calls forth a higher yield on capital to get new equities created by the large investment to be absorbed into the portfolios of home residents. A high yield on capital implies a low relative price of capital goods, assuming a labor intensive capital goods industry. Then, it reduces production of investment goods, and hence home investment under the assumption of non-tradable newly produced capital goods. Accordingly, a high yield on capital works to reduce the current account deficit, though this effect on the deficit could be offset by the wealth effect.

The above process crucially depends upon the unrealistic assumption that even newly produced capital goods are non-tradable[2]. When they are tradable realistically, their relative price in terms of the consumption good is exogenously determined under the small country assumption, so that the level of production of new capital goods is determined independently of the

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1   Though current account adjustments in the specie-flow doctrine and the elasticity approach appear to emphasize substitution between domestic and foreign products to be invoked by changes in relative price of the products, they, when put in a general equilibrium framework, rely upon the real balance effect and associated changes in saving as the model in Negishi (1972, Ch. 15) shows. The Keynesian model asserts that a change in saving induced by a change in income which in turn was induced by a current account surplus partially eliminates the initial current account surplus.

magnitude of the current account imbalance. Further, when newly produced capital goods are tradable, nothing in the Dornbusch model determines the volumes of newly produced capital goods to be exported abroad and to be installed at home. This also means the current account imbalance is un-related to home investment in the model. Accordingly, the model does not involve the current account adjustment through fluctuations of home invest-ment under the realistic assumption of tradable newly produced capital goods.

Investment is present in the model of Frenkel and Rodriguez (1975), but it plays no role in current account adjustment; as home investment depends only upon capital stock existing in the home country and the world interest rate (see equation (9′) in their paper), it takes place independently of disturbances such as monetary shocks that create current account imbal-ances.

Lipton and Sachs (1983) develop an international trade model that involves the intertemopral optimization of both firms and households, in particular, the investment behavior of firms. Their model appears more general than ours, and hence to include the adjustment mechanism this paper explores. However, it does not. To see it concisely, suppose in their model as well as in the model of this paper that the home country is small, and that the production function is linear in capital and labot. One can then find that home investment and the current account imbalance move indepen-dently in their model. This means that, the current account imbalance is not eliminated through repercussions in the asset market and changes in home investment in the model of Lipton and Sachs[3].

Niehans (1984, Ch. 6) explores a process of current account adjustment through changes in the marginal productivity of capital and home invest-ment. His mechanism is relevant to the long-run movement of the current account, while ours is concerned with the short-run movement in which capital stock and hence the marginal productivity of capital do not change

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2   Assuming newly produced capital goods are non-tradable is entirely different from the natural assumption that old, already installed capital goods are not tradable.

substantially. We will compare the two mechanisms in the end of this paper.

This paper is organized as follows. Section II introduces a model that incorporates a current account adjustment mechanism through investment and financial markets. Section III provides a stability analysis of the model and shows the mechanism that eliminates current account imbalances. Section IV briefly discusses other mechanisms of current account adjustment, which work over a long period in contrast to our mechanism which is of the short run character.

II   The Model

The home country is specialized in the production of a single commodity, and is a price taker in the international market of the commodity. These assumptions allow one to identify the exchange rate with the price level of the home country. We therefore denote both variables by P.

Firm's Behavior

Given the number of labor force, N, and real capital, K, in the sense of Uzawa (1969), the level of production Q at any moment is determined as:

(1) $Q = aK + bN$

with $a > 0$ and $b > 0$. This admittedly special specification of production technology is used to single out an autonomous mechanism of current

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3   Under the two suppositions, r* and Π in the Lipton-Sachs model are constant, and hence (1, 29) through (1, 33) in Lipton and Sachs (1983) reduce to

$$\dot{q} = (r^* + d)q - \gamma - (q - 1)^2 2b.$$

Since q and K are the determinants of home investment, this equation means that capital accumulation at home does not depend upon other state variables, namely, human wealth H and the amount of home-held foreign securities Z. The current account imbalance on the other hand depends upon Z in various ways. Accordingly, different levels of the initial value of Z produce different movements of the current account imbalance over time, but capital accumulation proceeds in the same manner whichever levels the initial value of Z is set at. This implies the conclusion in the text.

account adjustment that this paper explores. Given the usual strictly quasi
-concave and linearly homogeneous production technology, the marginal
productivity of K falls as capital accumulates. This fall tends to choke off
investment, thereby eliminating a current account deficit that arises from
excessive investment. We recognize the importance of this mechanism. But,
the use of (1) puts our mechanism in a traceable form by isolating it from
the other mechanism through changes in the marginal productivity of
capital. In Section IV, comparison will be made between the two mecha-
nisms.

Due to (1), the real wage is always equal to b under the competetive
labor market.

We follow Uzawa's investment theory (Uzawa (1969)). The represen-
tative firm maximizes

$$\int_0^\infty [(aK+bN)-\omega N-\Phi(\dot{K}, K)]\exp[-\int_0^t \rho_s ds]dt.$$

Here, ω is the real wage rate, ρ_s the instantaneous real rate of interest at
time s, and $\Phi(\dot{K}, K)$ the amount of the commodity necessary to increase
capital K in the rate of $\dot{K}$. Φ is homogeneous of degree one, and strictly quai
-concave. Let $\alpha = \dot{K}/K$ and $\phi(\alpha) = \Phi(\alpha, 1)$. We assume $\phi(0) = 0$; that is, the
zero rate of capital accumulation requires no commodity. By $b = \omega$, the
firm's optimal plan of capital accumulation must satisfy

(2) $$\dot{\alpha} = \frac{a - \phi(\alpha) + \phi'(\alpha)(\alpha - \rho_t)}{-\phi''(\alpha)}.$$

Further, since $\phi(\alpha)$ is convex,

(3) $$\lim_{t \to \infty} K_t \phi'(\alpha_t) \exp[-\int_0^t \rho_s ds] = 0$$

must hold (see Benveniste and Scheinkman (1982)).

The firm issues shares to finance its investment. Let S and P_s be the
number of home representative firm's shares and their price, respectively.
The number of shares increases according to:

$$\dot{P}_s S = P \phi (\alpha) K.$$

When the firm is disinvesting (selling its previously installed equipment), the proceeds from the sale are used to redeem its shares. Let $x = P_s/P$ and $v = xS$. These variables reduce the above equation to:

(4) $\quad \dot{v} = \phi (\alpha) K + \dfrac{\dot{x}}{x} v.$

Portfolio Equilibrium

We assume for simplicity that there are only two kinds of domestic financial assets; money and shares of the home representative firm. The home real interest rate, ρ, in the present model is then the real rate of return on domestic shares. Let η and π be the expected rates of share price appreciation and home inflation (the home currency depreciation). In our production technology, profit is given as PaK, Then, we have

(5) $\quad \rho = \dfrac{PaK}{P_s S} + \eta - \pi = \dfrac{aK}{v} + \eta - \pi,$

supposing all profit is paid out as dividends when earned.

Domestic and foreign shares are less than perfect substitutes to each other. Put another way, generally, $\rho \neq \rho^*$ where ρ^* represents the real rate of return on foreign shares. The assumption of perfect substitutability of domestic and foreign assets, in effect, takes that there is only one kind of internationally tradable asset, and allows us to identify current account imbalances with changes in the amount of the particular asset held by home residents as we can do so in the absence of tradable domestic assets. This oversimplifies the current account adjustment.

In addition to home money and shares of the domestic firm, there are foreign money and shares of the foreign representative firm in the model. For simplicity, it is assumed that residents hold foreign shares, but that they, living in the home country and not trading goods and services in the foreign country, do not hold foreign money.

Let Y be the value of domestic production PQ, and W the wealth of

home residents. Let $L(\rho, \pi, Y, W)$ and $D(\rho, \pi, Y, W)$ denote demand (in terms of value) functions of home residents for domestic money and domestic firm's shares, respectively. Without their demond for foreign money, their value demand for foreign firm's shares is given by $F(\rho, \pi, Y, W) = W - L(\rho, \pi, Y, W) - D(\rho, \pi, Y, W)$ on account of the budget constraint on assets transactions. The expected real rate of return on domestic money, $-\pi$, is relevant to the portfolio decisions of home households along with ρ and ρ^*, and hence is assumed as an arguments of L and D functions as well as ρ. (ρ^* is, however, suppressed in the functions because it is an exogenous variable under the small country assumption).

D and L functions have the following properties; $\partial D/\partial \rho > 0$, $\partial L/\partial \rho < 0$, and $\partial F/\partial \rho = -\partial(D+L)/\partial \rho < 0$. In other words, the higher the expected real rate of return on domestic firm's shares, the more demand for them, but the less demand for domestic money and foreign firm's shares. It is also assumed $\partial D/\partial \pi > 0$, $\partial L/\partial \pi < 0$, and $\partial F/\partial \pi = -\partial(D+L)/\partial \pi > 0$. That is, the more rapid the home inflation (the lower the expected rate of return on domestic money), the less demand for domestic money, but, the more demand for domestic and foreign firm's shares.

We make a usual assumption of $\partial L/\partial Y > 0$—on account of the transactions motive of demand for money, a rise in the value of domestic production Y increases demand for money. Because of the budget constraint on asset transactions, an increase in demand for money in response to an increase in Y must be accompained by a decrease in demand for domestic and/or foreign shares. Put the other way, given wealth, one can increase money in one's portfolio only by selling other assets in the portfolio. Since there is no specific reason to suppose that individuals try to obtain the necessary money balance by selling only one kind of asset, we assume home residents sell both foreign and domestic shares; $\partial D/\partial Y < 0$ and $\partial F/\partial Y < 0$. Naturally, demand of home residents for all assets rises when their wealth increases; $\partial L/\partial W > 0$, $\partial D/\partial W > 0$, and $\partial F/\partial W > 0$.

It is natural to assume that proportional increases in both the value of domestic production and wealth increase value demands for all assets in the same proportion. We assume D, F, and L are homogeneous of degree one

with respect to Y and W.

Foreigners' demand for domestic assets (demand for domestic money when there is only one kind of domestic asset) is frequently assumed absent; that is, domestic assets are assumed non-tradable. In this paper, however, foreigners as well as home residents demand domestic representative firm's shares. This means that, unlike in such models as Kouri (1976), the supply of foreign assets (foreign assets available) to home residents is not predetermined each time, and that home residents can obtain foreign assets instantaneously by selling domestic assets to foreigners. As noted in the introduction, the restrictive assumption of non-existence of foreigners' demand for domestic assets has played a critical role in the portfolio approach. We relax this assumption. Let $D^*(\rho)$ with $dD^*/d\rho > 0$ designate foreigners' value (in terms of foreign currency) demand for shares of the domestic firm. Foreigners, who principally transact goods and services in the foreign market, do not hold domestic money. Therefore its rate of return, $-\pi$, is irrelevant to foreigners' portfolio decisions. As an exogenous variable, the expected rael rate of return on the foreign firm's share, ρ^*, is again suppressed in the D^* function.

The markets for domestic firm's shares and domestic money are in equilibrium when

(6) $\quad D(\rho, \pi, Y, W) + D^*(\rho)P = P_sS,$

(7) $\quad L(\rho, \pi, Y, W) = M$

hold. Here, M is the stock of domestic money which expands at the rate of μ over time. Because of homogeneity with respect to Y and W, dividing (6) and (7) by P yields:

(6)' $\quad D(\rho, \pi, aK+b, w) + D^*(\rho) = v,$

(7)' $\quad L(\rho, \pi, aK+b, w) = m$

when $w = W/P$ and $m = M/P$.

Solving equations (6)' and (7)' for ρ and π yields functions ρ (v, K, m,

w) and π (v, K, m, w). Let ρ_v, ρ_K, ρ_m, π_v, π_K, and π_m be partial derivatives of ρ and π with respect to v, K and m.

One can show

(16)
$$\begin{vmatrix} \rho_K, & \rho_m \\ \\ \pi_K, & \pi_m \end{vmatrix} = -\frac{1}{|B|}\frac{\partial D}{\partial K} \quad \text{and} \quad \begin{vmatrix} \rho_K, & \rho_v \\ \\ \pi_K, & \pi_v \end{vmatrix} = \frac{a}{|B|}\frac{\partial L}{\partial K}$$

where B denotes the matrix of

$$\begin{pmatrix} \dfrac{\partial D}{\partial \rho}+\dfrac{dD^*}{d\rho}, & \dfrac{\partial D}{\partial \pi} \\ \\ \dfrac{\partial L}{\partial \rho} & , & \dfrac{\partial L}{\partial \pi} \end{pmatrix}$$

Hence, $\rho_v>0$, $\pi_v<0$, $\rho_m>0$, $\pi_m<0$, and $\pi_K>0$.

We make the following assumption:

ASSUMPTION 1 $\qquad \dfrac{1}{v}\dfrac{\partial(D+D^*)}{\partial\rho}+\dfrac{\partial D}{\partial K}>0.$

Since the assumption is equivalent to $\dfrac{d}{daK}\ [D(\dfrac{aK}{v}+\eta-\pi,\ \pi,\ aK+b,\ w)+$ $D^*(\dfrac{aK}{v}+\eta-\pi)]>0$, it means that, given expectations, a rise in production capacity aK increases the value demand for domestic shares. It follows from Assumption 1 and equation (6)' that, given expectations and wealth, the ·market evaluation of the representative firm, v, must increase as its production capacity increases.

Note that, as it increases profit, an increases in production capacity of a particular firm raises its market value, but that the same thing does not necessarily hold for the representative firm. For, an increase in production capacity of the representative firm increases not only its profit, but also total domestic production. The former raises the value as in the case of a particular firm, but the latter decreases the value due to $\partial D/\partial y<0$. Assumption 1 ensures that the first effect outweights the second.

The Foreign Exchange Market

There is another way of looking at portfolio equilibrium. Let $\bar{S}$ and $\bar{S}^*$

denote the quantities of domestic shares held by home residents and for-
eigners respectively before transactions in the asset market. Let $\bar{S}_F$ and $\bar{M}$
denote quantities of foreign shares and domestic money held by home
residents before transactions in the asset market. Let P_F signify the foreign
share price in terms of the home currency. With these notations, net foreign
indebtedness of the home country is given as $P_s\bar{S}^* - P_F S_{\cdot F}$. Since transac-
tions of stocks between foreigners and home residents do not alter net
foreign indebtedness of the home country, the net indebtedness before
transactions must be equal to that after transactions; that is,

(8) $P_s\bar{S}^* - P_F\bar{S}_F = PD^*(\rho) - F(\rho, \pi, Y, W)$

must hold.

One can interpret (8) as the equilibrium condition of the foreign
exchange market. Home residents plan to change their holding of foreign
shares from $\bar{S}_F$ to F/P_F through transactions in the asset market. Their
plans give rise to demand for foreign exchanges in the amount of $[F - P_F$
$\bar{S}_F]/P$. On the other hand, foreigners plan to change their holding of
domestic shares from $\bar{S}^*$ to PD^*/P_s. Their plans give rise to the supply of
foreign exchanges in the amount of $P_s\bar{S}^*/P - D^*$. Hence, equilibrium of the
foreign exchange market is given as

$$[F - P_F\bar{S}_F]/P = P_s\bar{S}^*/P - D^*$$

which is equivalent to (8).

The above alternative formulation of portfolio equilibrium (8) along
with either (6) or (7) is equivalent to (6) and (7) together. This is seen as
follows. The budget constraint on asset transactions for home residents is

$$W = P_s\bar{S} + P_F\bar{S}_F + \bar{M} = D + F + L.$$

By the assumption that foreigners do not hold domestic money, $M = \bar{M}$.
(This is not essential to the following argument.) Rearranging the budget
constraint with the use of $S = \bar{S} + \bar{S}^*$ yields:

$$(F - P_F\bar{S}_F) + (P_s\bar{S}^* - PD^*) = (M - L) + (P_sS - D - PD^*).$$

It follows from this identity that (6) and (7) imply (8), that (6) and (8) imply (7), and that (7) and (8) imply (6).

Saving Behavior

This paper assumes that the saving behavior of home households is such that real wealth remains constant; that is,

(9) $\qquad \dfrac{d}{dt} \dfrac{W}{P} = 0.$

This assumption concerning saving behavior is admittedly special, and more elaborate specifications of saving behavior are possible, and might appear desirabe. With its more elaborate specification, in particular, if real wealth is allowed to change, current account adjustment mechanisms through changes in wealth also operate as well as the mechanism through investment this paper attempts to explore. We will then have a more realistic model, but such a model blends our mechanism of current account adjustment with the other adjustment mechanisms through changes in wealth. Our mechanism would then become obscure and less traceable. For this reason, the model, though more general than ours, does not serve the purpose of this paper of bringing to focus an adjustment mechanism through home investment. Let $\bar{w}$ denote the constant value of W/P implied by (9).

The Current Account

Home residents receive not only their income, but also transfer of money from the government. They spend them on consumption and purchase of assets. Therefore, one has:

$$\text{Income} + \mu M$$

$$= PQ - (PaK) \frac{PD^*}{P_s S} + r^* F + \mu M$$

$$= \text{Consumption} + \frac{dL}{dt} + P \frac{d}{Sdt} \frac{D}{P_s} + P \frac{d}{dt} \frac{F}{P}$$

holds where r* is the given yield on the foreign share. (PaK) PD*/P$_s$S in the above equation represents the payment of devidents to foreign show holders, and r*F home receipt of the foreign firms devidents. From the above identity, saving in the sense of national accounting becomes

$$\text{Saving} = \text{Income} - \text{Consumption}$$

$$= (\frac{dL}{dt} - \mu M) + P_s \frac{d}{dt} \frac{D}{P_s} + P \frac{d}{dt} \frac{F}{P}$$

$$= (\frac{dL}{dt} - \mu M) - \frac{\dot{P}_s}{P_s} D + \frac{d}{dt} D + P \frac{d}{dt} \frac{F}{P}.$$

On the other hand, by (9),

$$0 = \frac{d}{dt} \frac{W}{P} = \frac{d}{dt} [\frac{D}{P} + \frac{F}{P} + \frac{L}{P}]$$

$$= -\frac{\dot{P}}{P} \frac{D}{P} + \frac{1}{P} \frac{d}{dt} D + \frac{d}{dt} \frac{F}{P} - \frac{\dot{P}}{P} \frac{L}{P} + \frac{1}{P} \frac{dL}{dt}.$$

Due to (7) and the above two equations,

$$\text{Saving} = (\frac{\dot{P}}{P} - \mu) L + (\frac{\dot{P}}{P} - \frac{\dot{P}_s}{P_s}) D.$$

Accordingly, we obtain the formula for the current account in terms of the foreign currency, C;

$$C = \frac{\text{Saving} - \text{Investment}}{P} = (\frac{\dot{P}}{P} - \frac{\dot{P}_s}{P_s}) \frac{D}{P} - (\mu - \frac{\dot{P}}{P}) \frac{M}{P} - \phi(\alpha) K$$

$$= -\frac{\dot{x}}{x} \frac{D}{P} - \dot{m} - \phi(\alpha) K.$$

On account of (4) and (6)′, this means

(10) $$C = -\dot{v} - \dot{m} + \frac{x}{x} D^*(\rho).$$

Expectations

It remains to specify how π and η are determined before completing our model. Following the recent prevalence of the rational expectations hypothesis and its related concepts, this paper assumes

(11) $\qquad \pi = \dfrac{\dot{P}}{P}$ and $\eta = \dot{P}_s/P_s$.

Under the assumption of rational expectations, (2) and (3) give not only the plan of the firm's future investment behavior, but also its actual investment over time. (11) reduces (5) to;

(5)″ $\qquad \rho = \dfrac{aK}{v} + \dfrac{\dot{x}}{x}.$

Then, (4) becomes

(4)′ $\qquad \dot{v} = (\phi(\alpha) - a)K + \rho v.$

The Equilibrium of the Dynamic Model

By the definitionscs of α and m, and by (11), we have

(12) $\qquad \dot{K} = \alpha K,$

(13) $\qquad \dot{m} = (\mu - \pi)m.$

These two differential equations along with (2), (4)′, and $w = \bar{w}$ constitute the dynamic model of the current account adjustment in this paper.

Let us signify the values of the variables in the dynamic model at its equilibrium by putting a bar over them; For instance, $\bar{\alpha}$ denotes the equilibrium value of α. The equilibrium values are unique as is easily seen: By (12), $\bar{\alpha} = 0$. Hence, $a = \phi'(0)\bar{\rho}$ because of $\phi(0) = 0$ and (2). $\dfrac{\dot{x}}{x} = 0$ at equilibrium by (4)′ and $\phi(0) = 0$. Then, by (5)′, $\phi'(0)\bar{K} = \bar{v}$. By (13), $\bar{\pi} = \mu$. Accordingly, at equilibrium, (6)′ and (7)′ become

$\qquad D(a/\phi'(0), \mu, a\bar{K}+b, \bar{w}) + D^*(a/\phi'(0)) = \phi'(0)\bar{K},$

$$L(a/\phi'(0),\ \mu,\ a\bar{K}+b,\ \bar{w}) = \bar{m}.$$

By $\partial D/\partial Y > 0$, $\bar{K}$ and $\bar{m}$ are uniquely determined by these equations. P increases at the rate of μ at equilibrium.

III   A Mechanism of Current Account Adjustment

We now analyze, in a neighborhood of equilibrium, the dynamic behavior of the model specified in the last section, and will see how the current account imbalance is to be eliminated through the asset market and changes in home investment.

Let A denote the matrix of

$$\begin{pmatrix} \bar{p} & , & \rho_K\phi'(0)/\phi''(0), & \rho_v\phi'(0)/\phi''(0), & \rho_m\phi'(0)/\phi''(0) \\ \bar{K} & , & 0 & , & 0 & , & 0 \\ \phi'(0)\bar{K}, & -\phi'(0)\bar{p}+\rho_K\bar{v}, & \bar{p}+\rho_v\bar{v} & , & \rho_m\bar{v} \\ 0 & , & -\bar{m}\pi_K & , & -\bar{m}\pi_v & , & -\bar{m}\pi_m \end{pmatrix}$$

where, ρ_K, ρ_v, ρ_m, π_K, π_v and π_m are evaluated at $\bar{K}$, $\bar{v}$, $\bar{m}$ and $\bar{w}$. When linearized around their equilibrium, (2), (4)′,(12) and (13) reduce to:

(14)
$$\begin{pmatrix} \dot{\alpha} \\ \dot{K} \\ \dot{v} \\ \dot{m} \end{pmatrix} = A \begin{pmatrix} \alpha \\ K-\bar{K} \\ v-\bar{v} \\ m-\bar{m} \end{pmatrix} + \varepsilon.$$

Here and hereafter, ε signifies a (scalar or vector) term of the first order smallness. Appendix 1 shows that A has only one negative eigen value, with other three being either positive or complex with positive real parts. Let ξ denote the negative eigenvalue of A. Let $(h_1,\ h_2,\ h_3,\ h_4)'$ be the column eigenvector of A associated with ξ. It is easy to see $h_1 = \xi/\bar{K}$, $h_2 = 1$, $h_3 = \phi'$

$(0) + \xi\phi''(0)$. Appendix 2 shows $h_3 > 0$ and $h_3 + h_4 > 0$ under Assumption 1.

The rational expectations hypothesis takes that initial values of prices (P and P_s, and so v and m in the present model.) are determined at such levels that paths starting from the initial values are stable. Thus, that matrix A has only one negative eigen value with three others being either positive or complex with positive real parts means that the movement of the dynamic system consisting of (2), (4)′, (12) and (13) over time is unique under the rational expectations hypothesis. The unique paths of α, K, and m are expressed as:

(15)
$$
\begin{cases}
\alpha(t) = \xi(K(0) - \bar{K})e^{\xi t}/\bar{K} + \varepsilon, \\[2mm]
K(t) - \bar{K} = (K(0) - \bar{K})e^{\xi t} + \varepsilon, \\[2mm]
v(t) - \bar{v} = (\phi'(0) + \xi\phi''(0))(K(0) - \bar{K})e^{\xi t} + \varepsilon, \\[2mm]
\dot{m}(t) - \bar{m} = h_4(K(0) - \bar{K})e^{\xi t} + \varepsilon.
\end{cases}
$$

Note (15) satisfies (3). Further, ρ, π, $\dot{x}/x$ and C take the following paths:

(16)
$$
\begin{cases}
\dot{\rho} = (\xi - \bar{\rho})\xi^2(K(0) - \bar{K})e^{\xi t}\,\phi''/\phi'\bar{K} + \varepsilon, \\[2mm]
\dot{\rho} = -\xi^2 h_4(K(0) - \bar{K})e^{\xi t}/\bar{m} + \varepsilon, \\[2mm]
\dfrac{\dot{x}}{x} = \xi\phi''(0)\dot{K}/\bar{v} + \varepsilon, \\[2mm]
C = [-(h_3 + h_4) + \xi\phi''(0)D^*(\rho)/\bar{v}]\,\dot{K} + \varepsilon.
\end{cases}
$$

The formula for the current account in (16) tells us how the current account imbalance comes about in the present model; It does so when the (long run) equilibrium amount of capital, which is to be determined by portfolio decisions of foreigners and home residents changes in face of exogenous disturbances. Let us see some of such disturbances.

By Otani (1982),

$$\lim_{K(0) \to \bar{K}} \frac{dC}{d\mu} \bigg|_{t=0} = -\ [-(h_3+h_4) + \xi\phi''(0)D^*/\phi'(0)\bar{K}]\ \xi\frac{d\bar{K}}{d\mu}.$$

This is negative because $h_3+h_4 > 0$ under Assumption 1, and because one can easily see $d\bar{K}/d\mu > 0$. Accordingly, an increase in the rate of monetary expansion produces a current account deficit. Intuitively, a sustained increase in the rate of monetary expansion raises the rate of inflation to make real balances less attractive as an asset; it thus induces home residents to have more domestic and foreign shares in their portfolios. The real rate of return on domestic shares must thus decline. This has the effect of stimulating home investment and creating a current account deficit.

The preceding argument means that money is non-neutral in our model. This is because the government transfer of money is made to each home resident irrespectively of the amount of money he has, though the total stock of money increases in the constant rate of μ. If money transfer is made to each home resident proportionally to the amount of money he has, the real rate of return on real balances is $\mu-\pi$ instead $-\mu$, and is independent of μ by (6)' and (7)'. By (15), $\bar{m}$ also becomes independent of μ. It follows that, under this alternative money supply rule, the rate of monetary expansion has no effect upon the steady state, the path approaching it and, in particular, the current account.

A fall of the rate of return on foreign shares also produces a current account deficit of the home country. We have:

$$\lim_{K(0) \to \bar{K}} \frac{dC}{d\rho^*} \bigg|_{t=0} = -\ [-(h_3+h_4) + \xi\phi''(0)D^*/\phi'\bar{K}]\ \xi\frac{d\bar{K}}{d\rho^*}.$$

This formula is negative under Assumption 1 by $h_3+h_4 > 0$ and $\frac{d\bar{K}}{d\rho^*} < 0$; a rise in the rate of return on foreign shares brings about a home current account deficit. The reason for this is that a low real rate of return on foreign shares induces both home and foreign residents to demand for more domestic shares and less foreign shares. The real rate of return on domestic shares then falls, which has the effect of stimulating home investment, and so brings the current account into dificit.

Acconding to (16), which describes movements of the current account and related variables out of the long run equilibrium, one can say:

The current account deficit is associated with positive home investment, and tends to be eliminated automatically; under the current account deficit the real rate of return on domestic firm's shares (the real rate of interest at home) is low, but rising so that home investment is eventually choked off; the real share price is high, but declining, and the real value of the representative domestic firm is low, but rising under the current account deficit.

In (16), the current account imbalance is self-correcting. Intuition behind the automatic self-correction is the following. The mechanism is a process through the home financial market and home investment. To be specific, suppose a rise in the rate of monetary expansion had provoked positive investment at home in the way expounded ealier, and hence that a home current account deficit is created. With positive home investment taking place at home, the number of shares of the representative domestic firm increases as the firm issue shares to finance their investment. At the sametime, as the level of production at home rises on account of increasing production capacity that results from the investment, demand for real balances increases because of the transactions motive. The increased demand for real balance is accompanied by home residents' sales of domestic shares. Positive investment at home thus creates an excess supply of domestic shares in the two ways. Since domestic and foreign shares are not perfect substitutes for each other, the real rate of return on domestic shares must rise, and the domestic share price must fall to induce substitution away from foreign shares to domestic shares in the protfolios of both home residents and foreigners. The substitution restores portfolio equilibrium of the asset market.

A higher real rate of return on domestic shares (equivalently, their lower price) means a higher real home interest rate, tending to reduce home investment and so the current account deficit of the home country. A current account imbalance is thus automatically eliminated through the asset market and changes in home investment.

This mechanism of the current account adjustment may be interpreted as an aspect of the self-adjustment mechanism in the financial market. When shares of the domestic firm is scarce in relation to the rate of return on foreign shares, the home financial market is easy, and the real rate of return on domestic shares is low (equivalenty, the price of the shares is high). This condition stimulates home investment. The domestic firm issues shares to finance the increased investment. This turns the home financial market to be less easy, and lowers the share price to reduce home invest-ment. A current account dificit appears in the initial stage of the self-adjusting process of the financial market when the market is easy, being accompoined by large investment. The current account is to be brought into balance when the market turns less easy, causing a smaller investment.

It must be noted that our mechanism of current account adjustment does not work if domestic shares are perfect substitutes for foreign shares. Under the condition, the rate of return on domestic firm's shares is set equal to that of foreign firm's shares which is exogenous under the small country assumption. Then, investment and capital accumulation at home increase the number of domestic shares, but have no effect upon the rate of return on domestic firm's shares. This means investment at home does not produce the force that works to eventually choke itself off. It follows that the current account imbalance cannot be eliminated in the manner we explored.

It must be also noted that, though Assumption 1 is critical for a current account imbalance to be eliminated in the way expounded above, the imbalance is still self-correcting even without the assumption. Matrix A in (14) still has only one negative eigenvalue. Others are either positive or complex with positive real parts even without the assumption. Our system then converges to its equilibrium, and the current account imbalance still tends to disappear automatically under the rational expectations hypothesis even without the assumption. However, if Assumption 1 does not hold, $h_3 + h_4$ could be negative. It is then possible that positive home investment is exceeded by positive saving that is undertaken to make up for capital losses and to maintain real wealth at the constant level, Positive home investment could be accompanied by a current account surplus. Yet, the current

account adjustment is accomplished, but in a different manner. A fall of the foreign rate of return, for instance, stimulates home investment. Unless Assumption 1 holds, a smaller invectment could reduce the value of shares of the domestic firm. This means a capital loss, and forces home recidents to save more. Thus, increases in home saving, despite the increased home investment, may imply, a current account surplus. The smaller value (hence a lower price) of domestic shares also works to reduce home investment eventually. The above mechanism is then to be reversed; a smaller home investment shrinks the current account surplus.

In the portfolio approach, typically as in Kouri (1976), domestic money is assumed non-tradable, but foreign money tradable. Because of this assumption, when running a current account deficit, the home country must pay foreign money to foreigners. Therefore, a current account deficit means a decrease in the supply of foreign money to home residents. The price of foreign money then rises to maintain portfolio equilibrium of the home asset market. A rise in the value of foreign money implies a fall in the value of domestic money stock that is entirely held by home residents. In addition, a current account deficit is identical with dissaving at home on account of the assumed absence of home investment. For these two reasons, the value of home wealth in terms of foreign currency, that is, in terms of commodities under the small country assumption is decreasing when the home country runs a current account deficit. By the wealth effect, the decrease in the value of home wealth brings about an increase in saving, thereby eliminating the current account deficit under the assumed absence of home investment.

This mechanism of current account adjustment no longer operates if domestic money is also tradable, and investment is undertaken at home. If foreigners receive domestic money in return for their current account surplus, the current account deficit on the part of the home country does not necessarily decrease the amount of foreign money held by home residents, and does not call forth a rise in the price of foreign money. If investment is present at home, the current account deficit is due to home investment in excess of saving, and is not identical with dissaving. Therefore, a current

account deficit does not necessarily mean decumulation of home wealth. It follows that, if the unrealistic assumptions of non-tradable domestic assets and the absence of home investment are abandoned, the current account adjustment mechanism typically considered in the portfolio approach no longer operates. By contrast, the current account adjustment mechanism we have demonstrated in this paper operates under the presence of home investment and tradable domestic assets.

IV   A Long-Run Mechanism of the Current Account Adjustment

By assuming that the marginal productivity of capital and the real value of wealth are constant, we neglected two important current account adjustment mechanisms. First, a large investment that produces a current account deficit eventually increases capital stock; an increase in capital stock makes the marginal productivity of capital decline on account of the decreasing marginal productivity of capital; then, home investment becomes less profitable, and so decreases; accordingly, the current account deficit shrinks. Niehans (1983, Ch, 6) gives a simple, but very clear account of this mechanism. Second, relatively scarce home saving compared with a high level of home investment can be a cause of the current account deficit; but, increasing domestic income due to capital accumulation allows home residents to increase saving; an increase in saving reduces the current account deficit.

These two neglected mechanisms are important, especially for the long -run movement of the current account imbalance. It is sometimes observed (for instance, Samuelson (1980, p 617) that countries, such as the U. S. before the Civil War, and the N. I. C's at present, in early stages of economic development are capital-importing countries and run a current account deficit. As economic development proceeds, current accounts of such countries become balanced and then turn into a surplus eventually, so that they become capital-exporting countries such as the U. S. after World War I. This long-run movement of the current account well fits with the picture of

the above mentioned mechanisms through changes in the marginal productivity of capital and saving. In countries with abundant potential for economic development but with low income, the marginal productivity of capital is high with large home investment taking place, though saving is small. Such countries run current account deficits. As capital accumulation and economic development proceed, the marginal productivity of capital declines and home investments decrease. At the same time, saving increases on account of increased domestic income. Accordingly, as capital accumulation proceeds in former developing countries, their current accounts become balanced, even turn into surplus as other yet underdeveloped countries now begin their economic development and undertake large investment with the high marginal productivity of capital, but with scarce savings.

The short-run fluctuations of the current account are imposed upon the above long-run movement of the current account, causing even a country with a current account surplus on trend to occasionally run a current account deficit. The mechanisms we neglected in this paper concern the long-run movement of the current account imbalance. On the other hand, our mechanism in this paper concerns short-run fluctuations of the current account to be imposed upon its long-run movements. Our model presents an adjustment mechanism that works to eliminate a current account imbalance created by short-run factors such as changes in the rate of monetary expansion, the rate of return on foreign shares, evaluation of risks, fiscal policy, and so forth.

APPENDIX 1

In this appendix, matrix A introduced in Section III has only negative eigen value with others being positive is complex with positive real parts.
One can see

$$| A - \lambda I | = (\lambda - \bar{\rho}) \begin{vmatrix} -\lambda + \bar{\rho}, & \rho_K \phi'/\phi'', & \rho_v \phi'/\phi'', & \rho_m \phi'/\phi'' \\ \bar{K} , & -\lambda , & 0 , & 0 \\ \phi'' \bar{K} , & \phi' , & -1 , & 0 \\ 0 , & -\bar{m} \pi_K , & -\bar{m} \pi_v , & -m \pi_m - \lambda \end{vmatrix}$$

Let $h(\lambda)$ denote the determinant on the right side of the above equation. Let solutions of $h(\lambda) = 0$ be λ_1, λ_2 and λ_3. One has:

$$-(\lambda_1 + \lambda_2 + \lambda_3) = \text{the coefficient of } \lambda^2 \text{ of } h(\lambda) = -\bar{v} \rho_v - \bar{\rho} + \bar{m} \pi_m < 0,$$

$$-\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = h(0) = -\frac{\phi' \bar{K} \bar{m}}{\phi''} \left[\phi' \begin{vmatrix} \rho_v, & \rho_m \\ \pi_v, & \pi_m \end{vmatrix} + \begin{vmatrix} \rho_K, & \rho_m \\ \pi_K, & \pi_m \end{vmatrix} \right] > 0.$$

It accordingly follows that A has only one negative eigenvalue, and that its other three eigenvalues are $\bar{\rho}$ and either positive numbers or conjugate complex numbers with positive real parts.

APPENDIX 2

This appendix shows $h_3 > 0$ and $h_3 + h_4 > 0$ under Assumption 1.

Let $g(z)$ be the determinant of $\begin{vmatrix} z + \bar{\rho}, & \rho_K , & \rho_m \\ \bar{K} , & 1 , & 0 \\ 0 , & -\bar{m} \pi_K, & -\bar{m} \pi_m + z \end{vmatrix}$

One can see $h(-\phi'(0)/\phi''(0)) = -g(\phi'/\phi'') \phi'/\phi''$. Consider a quadratic equation $g(z) = 0$.

The product of the two roots of this equation $=g(0)$

$$= -\bar{m} \left[\bar{\rho}\pi_m - \bar{K} \begin{vmatrix} \rho_K, & \rho_m \\ \\ \pi_K, & \pi_m \end{vmatrix} \right] = -\frac{\bar{m}a\bar{K}}{|B|} \left[\frac{1}{\bar{v}} \frac{\partial\theta_1+\theta_2}{\partial\rho} + \frac{\partial\theta_1}{\partial aK} \right] > 0.$$

by Assumption 1. Accordingly, under the assumption,

$$\pi_m(\bar{\rho} - \bar{K}\rho_K) + \bar{K}\pi_K\rho_m < 0.$$

Therefore, $\bar{\rho} - \bar{K}\rho_K > 0$. Then,

the sum of the two roots of the equation $= -g'(0)$

$$= \pi_m - \bar{\rho} + \bar{K}\rho_K < 0.$$

It follows that both roots of $g(z) = 0$ are negative, and so that $g(\phi'/\phi'') > 0$. Therefore, $h(-\phi'/\phi'') = -g(\phi'/\phi'')\phi'/\phi'' < 0$. Obviously, $-\phi'/\phi'' \neq \xi$. Let $-\phi'/\phi'' > \xi$. By $h(0) > 0$, then, there is a negative solution of $h(\lambda) = 0$ between $-\phi'/\phi''$ and 0. This contradicts the fact that ξ is the only negative solution of $h(\lambda) = 0$. Hence, $\xi > -\phi'/\phi''$ and so $h_3 = \phi'(0) + \xi\phi''(0) > 0$ under the assumption.

Further, $\bar{m}\pi_K + \bar{m}\pi_v h_3 + \bar{m}\pi_m h_4 = -\xi h_4,$

$$\bar{m}_K + \bar{m}(\pi_v - \pi_m)h_3 - \xi h_3 = -(\xi + \bar{m}\pi_m)(h_3 + h_4).$$

By (14), $\pi_v - \pi_m = -[\partial(L+D+D^*)/\partial\rho]/|B| > 0$. Then, by $h_3 = \phi'(0) + \xi\phi''(0) > 0$, $h_3 + h_4 > 0$.

REFERENCES

Benveniste, L. M. and J. A. Scheinkman, "Duality Theory for Dynamic Optimization Models of Economics: The Continuous Time Case," *Journal of Economic Theory* 27, (June 1982) 1-19.

Dornbusch, R., "A Portfolio Balance Model of the Open Economy" *Journal of Monetary Economics* 1, (January, 1975) pp. 3-20.

————, "Expectation and Exchange Rate Dynamics" *Journal of Political Economy* 84 (December, 1976) pp. 1161-1176.

————, and S. Fischer, "Exchange Rates and the Current Account" *American Economic Review* 70 (December, 1980) pp. 960-971.

————, *Open Economy Macroeconomics* Basic Books, 1980

Frenkel, J. A. and C. A. Rodriguez, "Portfolio Equilibrium and the Balance of Payments: A Monetary Approach," *American Economic Review* 65 (September, 1975) pp. 674-688.

Kouri, P. J., "The Exchange Rate and the Balance of Payments in the Short Run and the Long Run: A Monetary Approach", *Scandinavian Journal of Economics* 76 (May, 1976) pp. 280-304.

Lipton, D. and J. Sachs, "Accumulation and Growth in a Two-Country Model, A Simulation Approach," *Journal of International Economics* 15 (August, 1983), pp. 135-159.

McKinnon, R. I., "The Exchange Rate and Macroeconomic Policy: Changing Postwar Perceptions" *Journal of Economic Literature* 19 (January, 1981) pp. 531-557.

Negishi, T, *General Equilibrium Theory and International Trade* North Holland/American Elservier, Amsterdom London, 1972.

Niehans, J., "Exchange Rate Dynamics with Stock/Flow Interaction" *Journal of Political Economy* 85, (December, 1977) pp. 1245-1257.

————, *International Monetary Economics*, Baltimore; Johns Hopkins U. P., 1984.

Oniki, H. and H. Uzawa, "Patterns of Trade and Investment in a Dynamic Model of International Trade" *Review of Economic Studies* 32 (January, 1965) pp. 15-38.

Otani, K. "Explicit Formulae of Comparative Dynamics," *International Economic Review* 23, (June, 1982) 411-419.

Rodriguez, C. A., "The Rate of Trade Flows in Exchange Rate Determination: A Rational Expectation Approach", *Journal of Political Economy* 88, (December 1980) pp. 1148-1158.

Samuelson, P. A., *Economics* the 11th ed. MacGrow-Hill 1980.

Uzawa, H., "Time Preference and the Penrose Effect in a Two Class Model of Economic Growth," *Journal of Political Economy* 77 (July/August, 1969) pp. 628-652.