UTHEP-388
UTCCP-P-48

One-loop renormalization factors and mixing coeffecients of bilinear quark operators for improved gluon and quark actions *

Sinya Aoki [a], Kei-ichi Nagai [b], Yusuke Taniguchi [a] and Akira Ukawa [a b]

[a]Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki-305, Japan

[b]Center for Computational Physics, University of Tsukuba, Tsukuba, Ibaraki-305, Japan

We calculate one-loop renormalization factors and mixing coefficients of bilinear quark operators for a class of gluon actions with six-link loops and O(a)-improved quark action. The calculation is carried out by evaluating on-shell Green's functions of quarks and gluons in the standard perturbation theory. We find a general trend that finite parts of one-loop coefficients are reduced approximately by a factor two for the renormalization-group improved gluon actions compared with the case of the standard plaquette gluon action.

1. Introduction

In numerical studies of field theories through computer simulations finite lattice spacing effects poses an obstacle in extracting physical predictions for observable quantities. In an effort to reduce this problem, a recent full QCD simulation by the CP-PACS Collaboration[1] employs a combination of the $O(a)$-improved clover quark action[2] and a renormalization-group-improved gluon action with 6-link loops [3], based on previous results of a comparative study of various action combinations[4]. With this development it has become necessary to calculate renormalization factors and mixing coefficients of quark operators for improved gluon and quark actions. We have carried out such a calculation to one-loop order of perturbation theory for bilinear quark operators [5,6]. In this article we summarize the results.

Our calculations are performed within standard perturbation theory for on-shell Green's functions of quarks and gluons. For the plaquette gluon action, perturbative calculation of the mixing coefficients of bilinear quark operators has been previously carried out using the Schrödinger functional technique[7,8]. A recent work with the same technique[9] extends the results to the same set of improved gluon actions as examined by us. We find a good agreement between results by us and those reported in these references.

*Talk presented by Y. Taniguchi

2. Choice of actions

We adopt the clover action for quark, which is given by adding the clover term

$$S_{\rm C} = -c_{\rm SW} a^3 \sum_n \sum_{\mu,\nu} ig \frac{r}{4} \overline{\psi}_n \sigma_{\mu\nu} P_{\mu\nu}(n) \psi_n$$

to the ordinary Wilson fermion action. The gluon action we consider is defined by

$$S_{\rm gluon} = \frac{1}{g^2} {\rm Tr} \left\{ c_0 U_{pl} + c_1 U_{rtg} + c_2 U_{chr} + c_3 U_{plg} \right\},$$

where the first term represents the standard plaquette term, and the remaining terms are six-link loops formed by a 1×2 rectangle, a bent 1×2 rectangle (chair) and a 3-dimensional parallelogram. The coefficients $c_0, \cdots, c_3$ satisfy $c_0 + 8c_1 + 16c_2 + 8c_3 = 1$. At the one-loop level, the choice of the gluon action is specified by the pair of numbers c_1 and $c_{23} = c_2 + c_3$. We adopt the following five cases in our calculation: (i) the standard plaquette action $c_1 = 0, c_{23} = 0$, (ii) the tree-level improved action in the Symanzik approach $c_1 = -1/12, c_{23} = 0$[10], and (iii) three choices suggested by an approximate renormalization-group analysis, $c_1 = -0.331, c_{23} = 0$ and $c_1 = -0.27, c_{23} = -0.04$ by Iwasaki[3], and $c_1 = -0.252, c_{23} = -0.17$ by Wilson[11].

Table 1
Finite part z_Γ of renormalization factor for bilinear operator of form $(1+ma)\overline{\psi}\Gamma\psi$ (*i.e.,* $z=0$ in (1)) with m the subtracted quark mass incorporating one-loop self-energy correction. Coefficients of the term $c_{SW}^n (n=0,1,2)$ in an expansion $z_\Gamma = z_\Gamma^{(0)} + c_{SW} z_\Gamma^{(1)} + c_{SW}^2 z_\Gamma^{(2)}$ are given in the column labeled (n). Errors are at most in the last digit given.

gauge action		V			A			S			P		
c_1	c_{23}	(0)	(1)	(2)	(0)	(1)	(2)	(0)	(1)	(2)	(0)	(1)	(2)
0	0	−20.618	4.745	0.543	−15.797	−0.248	2.251	−12.953	−7.738	1.380	−22.596	2.249	−2.036
−1/12	0	−16.603	4.228	0.464	−12.540	−0.198	2.021	−9.607	−6.836	1.367	−17.734	2.015	−1.745
−0.331	0	−11.099	3.326	0.336	−8.192	−0.125	1.610	−4.858	−5.301	1.266	−10.673	1.601	−1.281
−0.27	−0.04	−11.540	3.418	0.353	−8.523	−0.131	1.657	−5.260	−5.454	1.292	−11.292	1.644	−1.316
−0.252	−0.17	−10.525	3.248	0.338	−7.707	−0.117	1.587	−4.366	−5.166	1.287	−10.001	1.565	−1.212

3. Bilinear operators and improvement

We consider bilinear quark operators of form,

$$\mathcal{O}^\Gamma = \overline{\psi}\Gamma\psi, \qquad \Gamma = 1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}. \tag{1}$$

At the tree-level, there exists a one-parameter family of $\mathcal{O}(a)$-improved operators given by[12]

$$\begin{aligned}\mathcal{O}_0^\Gamma &= [1 + ar\,(1-z)\,m_0]\,\overline{\psi}\Gamma\psi \\ &\quad + z\overline{\psi}\Gamma^\otimes\psi - z^2\overline{\psi}\Gamma'\psi,\end{aligned} \tag{2}$$

where $\Gamma^\otimes$ and Γ' are $\mathcal{O}(a)$ and $\mathcal{O}(a^2)$ vertices, and m_0 is the bare quark mass.

At the one-loop level, on-shell Green's functions of these operators do not have terms of $\mathcal{O}(g^2 a\log a)$ when we set the clover term coefficient $c_{\rm SW}=1$ [12]. The $\mathcal{O}(g^2a)$ terms still remain, however. In order to extract the renormalized operators without $\mathcal{O}(g^2a)$ errors we need to calculate mixing coefficients B_Γ and C_Γ as well as the renormalization factor Z_Γ defined by

$$\begin{aligned}\mathcal{O}_0^\Gamma = Z_\Gamma^{-1}\mathcal{O}_{\rm R}^\Gamma &- g^2 C_F a m_R\, B_\Gamma\, \mathcal{O}_{\rm R}^\Gamma \\ &- g^2 C_F a\, C_\Gamma\, \widetilde{\mathcal{O}}_{\rm R}^\Gamma,\end{aligned} \tag{3}$$

where C_F denotes the second-order Casimir eigenvalue for the quark field, and the last two terms are needed to remove $\mathcal{O}(g^2a)$ errors from on-shell matrix elements, with $\widetilde{\mathcal{O}}_{\rm R}^\Gamma$ a dimension 4 operator with derivative, *e.g.,* $\widetilde{\mathcal{O}}^A = \partial_\mu \mathcal{O}^P$ and $\widetilde{\mathcal{O}}^V = \partial_\mu \mathcal{O}^T$.

4. Strategy of one-loop calculation

We apply standard perturbation theory to the Green's functions of the operator $\mathcal{O}_0^\Gamma$ and two on-shell external quarks to calculate renormalization factors and mixing coefficients. In order to isolate terms of $\mathcal{O}(g^2a)$, one-loop amplitudes are expanded in powers of a, or equivalently in the external momenta p and quark mass m. Infrared divergences that generally appear in on-shell amplitudes are regularized by a gluon mass λ.

For this procedure to work, infrared divergences that contribute to the mixing coefficients B_Γ and C_Γ have to cancel out among diagrams. We have explicitly checked that this is in fact the case; infrared divergences appear only in the ordinary renormalization factor Z_Γ, which, however, cancel against those in the renormalization factor for the continuum operator. As a result, the renormalization factor in the $\overline{\rm MS}$ scheme takes the form,

$$Z_\Gamma^{-1} = 1 + \frac{g^2 C_F}{16\pi^2}\left(\left(\frac{h_2(\Gamma)}{4} - 1\right)\log(\mu a)^2 + z_\Gamma\right),$$

where $h_2(\Gamma)$ is an integer given by $h_2(\Gamma) = 4(A), 4(V), 16(P), 16(S), 0(T)$.

We note that the bare quark mass m_0 in (2) may be replaced by a subtracted mass $m = m_0 - g^2 C_F \Sigma_0/a$ incorporating the one-loop self-energy correction by a redefinition of z_Γ, B_Γ and C_Γ. We adopt this definition for numerical results presented below.

4.1. Results

The calculational procedure described above yields z_Γ, B_Γ and C_Γ as sums of one-loop integral constants. While straightforward in principle, the actual algebra to evaluate the integrands

Table 2
Mixing coefficients for axial vector, vector, pseudo scalar and scalar density at $z = 0$.

gauge action		C_A	C_V	B_A	B_V	B_P	B_S
c_1	c_{23}						
0	0	−0.005680(2)	−0.01226(3)	0.1141(1)	0.1150(2)	0.1148(1)	0.1444(2)
−1/12	0	−0.00451(1)	−0.01030(4)	0.0881(1)	0.0886(2)	0.0890(1)	0.1144(2)
−0.331	0	−0.00285(1)	−0.00729(4)	0.0547(1)	0.0550(2)	0.0561(1)	0.0747(2)
−0.27	−0.04	−0.00302(1)	−0.00757(4)	0.0572(1)	0.0575(2)	0.0586(1)	0.0777(2)
−0.252	−0.17	−0.00281(1)	−0.00705(4)	0.0512(1)	0.0514(2)	0.0527(5)	0.0706(2)

is extremely tedious. We employ *Mathematica* to carry out this task, and also to generate the FORTRAN code from the results. Infrared divergences that can appear in individual integrals are subtracted by working out the leading term of integrands for small loop momenta. Integrals are then evaluated by the Monte Carlo routine VEGAS in double precision. We generally employ 20 sets of 10^5 points for integration, except for C_A for the plaquette action for which 20 sets of 10^6 points are used. Errors are estimated from variation of integrated values over the sets.

We present our results for z_Γ, C_Γ and B_Γ for the case of $z = 0$ in Tables 1 and 2. Results for $z \neq 0$ can be found in Refs. [5,6]. The Wilson parameter is taken to be $r = 1$. For z_Γ, the coefficients of the expansion $z_\Gamma = z_\Gamma^{(0)} + c_{SW} z_\Gamma^{(1)} + c_{SW}^2 z_\Gamma^{(2)}$ are given (Table 1), while the constants C_Γ and B_Γ are evaluated for $c_{\rm SW} = 1$ (Table 2).

We observe in the results that the renormalization factors and mixing coefficients are reduced by about a factor two for renormalization-group improved gluon actions compared to those for the plaquette action. More generally we find that z_Γ's monotonically decrease as c_1 and c_{23} become large and negative[5].

Numerical calculations have been carried out at Center for Computational Physics, University of Tsukuba and at Research Institute for Fundamental Physics, Kyoto University. This work is supported in part by the Grants-in-Aid of the Ministry of Education (Nos. 2373, 09304029,10640246). KN and YT are JSPS Research Fellows.

REFERENCES

1. R. Burkhalter for the CP-PACS Collaboration, in these proceedings; CP-PACS Collaboration (presented by K. Kanaya, T. Kaneko and H. Shanahan), in these proceedings.
2. B. Sheikholeslami and R. Wohlert, Nucl. Phys. B259 (1985) 572.
3. Y. Iwasaki, preprint, UTHEP-118 (Dec. 1983), unpublished.
4. CP-PACS Collaboration, S. Aoki *et al.*, Nucl. Phys. B(Proc. Suppl.)60A (1998) 335; Nucl. Phys. B(Porc. Suppl.)63 (1998) 221.
5. S. Aoki, K. Nagai, Y. Taniguchi and A. Ukawa, hep-lat/9802034 (to appear in Phys. Rev. D).
6. Y. Taniguchi and A. Ukawa, hep-lat/9806015 (to appear in Phys. Rev. D).
7. M. Lüscher and P. Weisz, Nucl. Phys. B479 (1996) 429.
8. S. Sint and P. Weisz, Nucl. Phys. B502 (1997) 251; Nucl. Phys. B (Proc. Suppl.) 63A-C (1998) 856.
9. S. Aoki, in these proceedings; S. Aoki, R. Frezzotti and P. Weisz, hep-lat/9808007.
10. P. Weisz, Nucl. Phys. B212 (1983) 1; P. Weisz and R. Wohlert, Nucl. Phys. B236 (1984) 397; erratum, *ibid.* B247 (1984) 544.
11. K. G. Wilson, in *Recent Development of Gauge Theories*, eds. G. 'tHooft *et al.* (Plenum, New York, 1980).
12. G. Heatlie *et al.,* Nucl. Phys. B352 (1991) 266