Quenched Light Hadron Spectrum with the Wilson Quark Action: Final Results from CP-PACS *

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We report the final results of the CP-PACS calculation for the quenched light hadron spectrum with the Wilson quark action. Our data support the presence of quenched chiral singularities, and this motivates us to use mass formulae based on quenched chiral perturbation theory in order to extrapolate hadron masses to the physical point. Hadron masses and decay constants in the continuum limit show unambiguous systematic deviations from experiment. We also report the results for light quark masses.

1. Introduction

At Lattice'97 we presented first results from the CP-PACS calculation of the quenched light hadron spectrum with the Wilson quark action on large lattices ($La \gtrsim 3$ fm) at small quark masses ($m_\pi/m_\rho = 0.75$ down to 0.4) with high statistics (800, 600, 420 and 91 configurations at $\beta = 5.9, 6.1, 6.25$ and 6.47) [1]. We have since increased the statistics at $\beta = 6.47$ to 150, and have completed the analysis. In this article, we report the final spectrum results and the main points of analyses behind them.

2. Quenched chiral singularities

Chiral extrapolation is a basic element of spectrum calculations, for which a choice has to be made of the functional form of hadron masses in terms of quark masses. An important issue in considering the choice is the validity of quenched chiral perturbation theory(QχPT) [2–5], which predicts characteristic singularities in hadron masses in the chiral limit. We have therefore made a detailed examination of this issue.

2.1. pseudo-scalar mesons

For pseudo-scalar (PS) mesons made of quarks of mass m and m_s, QχPT formula reads[3]

$$
\begin{aligned}
m_{PS}^2 &= A(m_s + m)\{1 - \delta[\ln(2mA/\Lambda_\chi^2) \\
&+ m_s/(m_s - m)\ln(m_s/m)]\} \\
&+ B(m_s + m)^2 + C(m_s - m)^2 + \cdots. \quad (1)
\end{aligned}
$$

To test the presence of the logarithm term, we combine our results to form two quantities

$$
y = \frac{2m}{m_s + m}\frac{m_K^2}{m_\pi^2} \times \frac{2m_s}{m_s + m}\frac{m_K^2}{m_\eta^2}, \quad (2)
$$

$$
x = 2 - \frac{m_s + m}{m_s - m}\log(\frac{m_s}{m}), \quad (3)
$$

where π (η) is the degenerate PS meson with quark mass m (m_s) at $m_\pi/m_\rho = 0.6, 0.5, 0.4$ (0.75, 0.7) and K is the non-degenerate one with m and m_s. The two quantities are related by $y = 1 + \delta \cdot x$, where the leading correction depends only on the $O((m_s - m)^2)$ term in (1).

In Fig. 1 we plot the two quantities calculated with quark masses determined from an extended axial current Ward identity (m_q^{AWI}) as they have no ambiguity associated with the determination of the critical hopping parameter. The data fall within a narrow wedge spanned by the lines $y \approx$
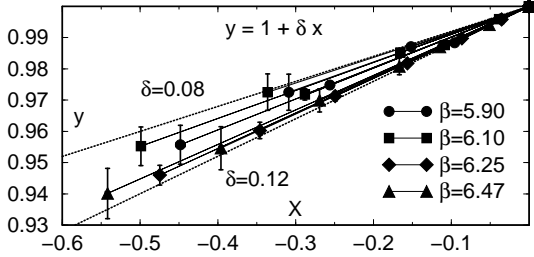
Figure 1. Results of a test for the presence of the quenched chiral logarithms in PS meson mass data. See text for details.

$1 + (0.08 - 0.12)x$, implying the value $\delta \approx 0.10(2)$. We note that the $O((m_s + m)^2)$ term can not be ignored for the range of our quark masses; results for the original ratio m_K^2/m_π^2[3], which receive corrections both from $O((m_s + m)^2)$ and $O((m_s - m)^2)$ terms, do not fall on a common line.

A different test using a ratio of decay constants $y = f_K^2/(f_\pi f_\eta)$[3] leads to a similar result; our data fall within the lines $y = 1 - \delta/2 \cdot x$ with $\delta = 0.08 - 0.16$.

Finally, making full correlated fits to m_{PS} using (1) but imposing $C = 0$, independently for degenerate and non-degenerate data, we find $\delta \approx 0.06 - 0.12$ for the range $\Lambda_\chi \approx 0.6 - 1.4$ GeV.

These results lead us to conclude that our PS data show evidence for QχPT logarithms.

2.2. vector meson and baryon masses

For vector mesons and baryons, we perform uncorrelated simultaneous fits to degenerate and non-degenerate data together as a function of m_{PS}, assuming QχPT mass formulae[4,5] with $\delta = 0.1$. For vector mesons and decuplet baryons, all $O(m_{PS})$ terms of QχPT are included as well as $O(m_{PS}^2)$ terms. For octet baryons, we include $O(m_{PS}^3)$ terms in addition to $O(m_{PS})$ and $O(m_{PS}^2)$ terms since the nucleon mass shows a negative curvature which is opposite to that of the $O(m_{PS})$ term. We omit decuplet-octet coupling (C in the notation of Ref. [5]) and coupling to η' (γ), and set $\alpha_\Phi = 0$.

These mass formulae fit our data well. The values for the coefficient $C_{1/2}$ of $O(m_{PS})$ terms, however, are small. We obtain $C_{1/2} = -0.071(8)$ for ρ, $-0.118(4)$ for nucleon, and $-0.14(1)$ for Δ, to be compared with phenomenological estimates:
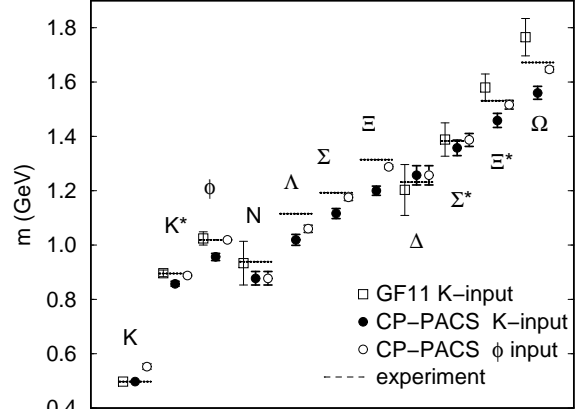

Figure 2. Final spectrum results compared to GF11's[6] and experiment.

$C_{1/2} = -4\pi g_2^2 \delta \approx -0.71$ for ρ if $\delta = 0.1$ and $g_2 = 0.75$, and $C_{1/2} = -(3\pi/2)(D - 3F)^2 \delta \approx -0.27$ for nucleon if $\delta = 0.1$ and $(F, D) = (0.5, 0.75)$.

We conclude that vector and baryon masses are compatible with the presence of $O(m_{PS})$ terms, but that their magnitudes are smaller than expected. These terms yield an effect of at most $0.2 \times m_\pi \approx 25$ MeV, which is about 3% of light hadron masses.

3. Final spectrum results

The results of analyses above motivate us to adopt the QχPT fits to calculate masses at each value of β. The physical point for degenerate u and d quarks and the lattice scale are determined from the experimental values of m_π and m_ρ, and the strange quark mass by that of m_K or m_ϕ. We then extrapolate the results linearly in a. The final result for the spectrum in the continuum limit is shown in Fig.2.

In order to examine how results differ if we do not employ QχPT mass formulae, we repeat the analysis employing a quadratic polynomial in $1/K$ (cubic for N) for chiral extrapolations. While masses at each value of β differ, by about 3% in the largest case, the differences in the continuum limit do not exceed 1.5% of the results of QχPT fits.

Compared to the results presented at Lattice'97[1] where we employed a linear chiral extrapolation in $1/K$ (cubic for N and quadratic
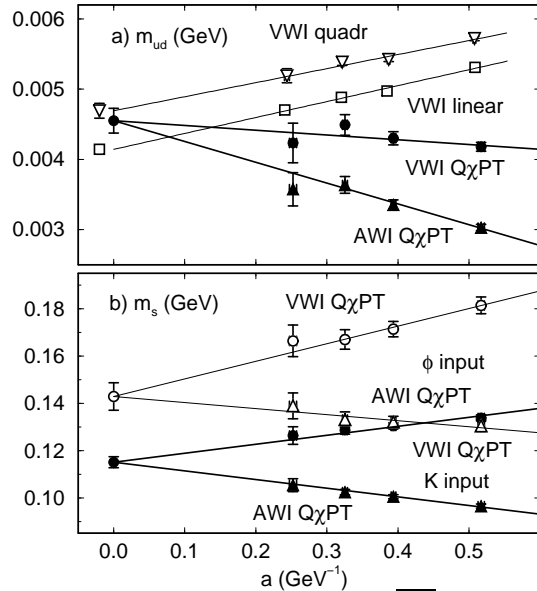
Figure 3. Light quark masses in $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV.

for Λ), the nucleon and Δ masses have decreased by 4.5% and 3.5%, respectively. Strange baryon masses with m_K used as input have also decreased. The shift, however, is within 1.5σ for all particles, with either m_K or m_ϕ as input.

In summary, we find that differences in chiral extrapolations and an increase of statistics at $\beta = 6.47$ do not alter the conclusions we drew at the time of Lattice'97: With m_π, m_ρ and m_K used as input, the meson hyperfine splitting and decuplet baryon mass splitting are too small compared to experiment, and so are the octet baryon masses. When we use m_ϕ instead of m_K as input, the discrepancies for baryon masses are reduced, but the meson hyperfine splitting remains smaller.

4. Light quark masses

The QχPT fit to pseudo-scalar meson masses has a significant effect on light quark masses at finite β. Due to a negative curvature of the QχPT formula, values of the averaged u and d quark mass defined with vector Ward identity m_{ud}^{VWI} become smaller than those from polynomial chiral extrapolations as shown in Fig. 3. The results extrapolated to the continuum limit, however, are

consistent among various definitions. We adopt a combined fit to m_q^{VWI} and m_q^{AWI}, both estimated with QχPT fits, to calculate our final result. We obtain $m_{u,d}$=4.6(2) MeV, and m_s=115(2) MeV (m_K input) or 143(6) MeV (m_ϕ input) in the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV.

5. PS meson decay constants

We determine f_π and f_K from the local axial current, employing a quadratic polynomial and linear chiral extrapolation, respectively. We obtain $f_\pi = 120(6)$ MeV and $f_K = 139(4)$ MeV, which are 10 and 15% smaller than experiment, respectively. The ratio $f_K/f_\pi - 1 = 0.156(29)$ is also smaller than experiment.

6. Conclusions

We have presented our final results on the quenched light hadron spectrum and related quantities. In the course of analyses we found that our data for light hadron masses are consistent with predictions of QχPT. The effect of QχPT singularities is small, however, and the continuum results do not noticeably shift from those obtained with polynomial chiral extrapolations. Our results show that the quenched light hadron spectrum clearly and systematically deviates from the experimental spectrum. The discrepancy is much larger than our statistical error of 1% for mesons and 2–3% for baryons.

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