$B \to \pi l\overline{\nu}$ Form Factors with NRQCD Heavy Quark and Clover Light Quark Actions *

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We report results on semileptonic $B \to \pi l\overline{\nu}$ decay form factors near $q^2_{\rm max}$ using NRQCD heavy quark and clover light quark actions and currents improved through $O(\alpha a)$. An inconsistency with the soft pion relation $f^0(q^2_{\rm max}) = f_B/f_\pi$ found in a previous work is confirmed, and a possible solution with nonperturbative renormalization is discussed. We find that $f^+(q^2)$ is well described by the B^* pole near $q^2_{\rm max}$, and its $1/M_B$ scaling is also consistent with the prediction of the pole dominance model.

1. Introduction

The $B \to \pi l\overline{\nu}$ form factors are relevant for the extraction of the CKM matrix element $|V_{ub}|$ through the exclusive decay. While lattice QCD computation can cover only the region near $q^2 = q^2_{\rm max}$ with reasonable statistical and discretization errors, it is still useful once the experiments reach sufficiently high statistics to measure the partial decay rate in the same region. In this article we report preliminary results of our study of the $B \to \pi l\overline{\nu}$ form factors using NRQCD for heavy quark.

2. Simulation

We employ $O(1/M)$ NRQCD for heavy quark. For the light quark the SW clover quark action is used, with the clover coefficient $c_{\rm sw}$ determined by mean field improved perturbation theory at one-loop order. The heavy-light vector current involved in the matrix element is renormalized to $O(\alpha_s a)$ using the one-loop calculation of Morningstar and Shigemitsu [1], and of Ishikawa [2] including mixings with higher dimensional operators.

The calculation of $f^0(q_{\rm max})$ was performed at β=5.7 and 5.9 on $12^3 \times 32$ and $16^3 \times 40$ lattices with 232 and 222 gauge configurations, respectively. We took five different heavy quark masses covering m_b and four different light quark masses ranging from m_s to $m_s/2$. For $f^+(q^2)$ we only analyzed the $\beta = 5.7$ data, so far. The momentum combinations p_B=(0,0,0), k_π=(0,0,0), (1,0,0), (1,1,0), and p_B=(1,0,0), k_π=(0,0,0) in units of $2\pi/(12a)$ are considered at $\kappa_l = 0.1369$, which is around m_s.

3. Results for $f^0(q^2_{\rm max})$

Let us first present the results for $f^0(q^2_{\rm max})$, for which the soft pion theorem $f^0(q^2_{\rm max}) = f_B/f_\pi$ in the chiral limit provides an important check of the lattice calculation. The $1/M_B$ dependence of $\sqrt{M_B}\ f^0(q^2_{\rm max})\ (\alpha_s(M_B)\ /\alpha_s(M_B^{\rm phys}))^{2/\beta_0}$ and a comparison with $\sqrt{M_B}\ f_B/f_\pi\ \ (\alpha_s(M_B)\ \ /\alpha_s(M_B^{\rm phys}))^{2/\beta_0}$ is shown in

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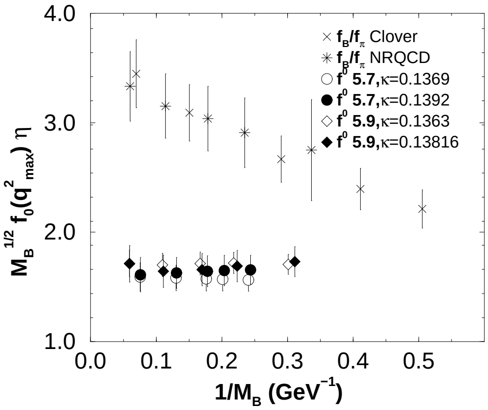

Figure 1. Comparison of $\sqrt{M_B} f^0(q^2_{\rm max})\eta$ with $\sqrt{M_B} f_B/f_\pi \eta$, where η in defined as $(\alpha_s(M_B)/\alpha_s(M_B^{\rm phys}))^{2/\beta_0}$. $f^0(q^2_{\rm max})$ with heaviest and with lightest light quark mass is plotted at β=5.7 and 5.9. Data for f_B/f_π are taken from our previous works: heavy clover [4] and NRQCD [5].

Fig. 1. The data for $f^0(q^2_{\rm max})$ at two β values show nice scaling, while a clear disagreement with f_B/f_π is observed, confirming the point made by the Hiroshima Group [3].

The chiral (soft pion) limit is not taken for the points given in the plot, which is a possible reason of the violation of the soft pion relation. We find, however, that the light quark mass dependence of $f^0(q^2_{\rm max})$ is consistent with a constant within statistical error, and a polynomial chiral extrapolation in m_q and m_q^2 gives a consistent result. With the present statistics, we are not able to fit our data with both $\sqrt{m_q}$ and m_q in the fitting function.

Another possible reason for the disagreement with the soft pion relation is the large uncertainty in the matching constants. Since the vector and axial vector heavy-light currents are involved on the two sides of the equality, perturbative errors in the matching between the continuum and lattice operators could be important. Naively this error is $O(\alpha_s^2)$, and hence should be small at $\beta \sim 6.0$. Nonetheless, the large one-loop correction in the renormalization constant Z_A^{HL} suggests that there could be large higher order corrections.

In order to see how such higher order effect contributes, we computed the ratio of the renormalization constants Z_A^{HL}/Z_V^{HL} nonperturbatively in the static limit, using the chiral Ward-Takahashi identity

$$Z_A Z_V^{HL} \int d^4y \langle (\partial_\mu A_\mu - 2m_q P)(y) V_0^{HL}(x) \mathcal{O}\rangle$$
$$= \quad -Z_A^{HL} \langle A_0^{HL}(x)\mathcal{O}\rangle, \qquad (1)$$

where A_μ and P denote light-light axial-current and pseudoscalar density, with Z_A the renormalization factor for A_μ, and V_0^{HL} and A_0^{HL} are the heavy-light (static-light in this particular case) currents. We performed simulations on a $12^3 \times 32$ lattice at $\beta = 6.0$ following the methods of Maiani-Martinelli [6]. For the operator $\mathcal{O}$ we took a heavy-light meson interpolation operator with wall source. The clover coefficient c_{sw} for the light quark was chosen to be the nonperturbative value from [7]

Combining our result for $Z_A^{HL}/(Z_V^{HL} Z_A)$ from (1) with the nonperturbative value of [7] for Z_A, we obtained $Z_A^{HL}/Z_V^{HL} = 0.72(2)$ to be compared with the perturbative result 0.87(3). We find that the nonperturbative value for Z_A^{HL}/Z_V^{HL} is about 20% smaller than the corresponding one-loop result. While this explains part of the discrepancy between f_B/f_π and $f^0(q^2_{\rm max})$, the reduction is not sufficient to remove the disagreement seen in Fig. 1.

In our study of the renormalization constant, the light-light and static-light current did not include corrections from higher dimensional operators. Since these corrections are known to give a large contribution in the calculation of f_B [1], it would be important to perform a study of renormalization constant with improved curents.

4. Results for $f^+(q^2)$

We next study the q^2 and $1/M_B$ dependence of the form factor $f^+(q^2)$. For the large q^2 region, the form factor $f^+(q^2)$ should be well approximated by the pole dominance model with the B^* pole, since B^* is almost degenerate with B in the heavy quark limit and the pole is very close to $q^2_{\rm max}$. The pole model predicts

$$1/f^+(q^2) = -c_1(q^2 - M_{B^*}^2) + O((q^2 - M_{B^*}^2)^2), \qquad (2)$$

where the coefficient c_1 is written in terms of the dimensionless $B^*B\pi$ coupling g and the B^* me-

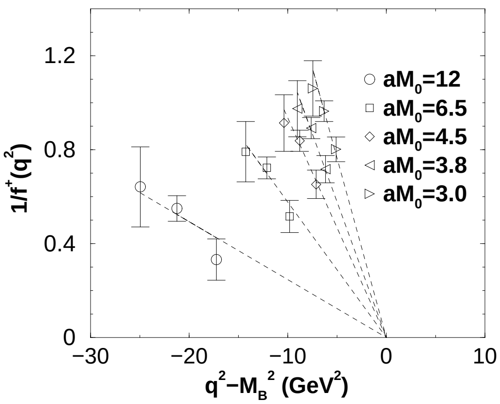

Figure 2. Pole fit of $1/f^+(q^2)$. Lattice results at five different heavy quark mass are shown at β=5.7. The B^* pole is almost degenerated with B, and located at the origin in the plot. aM_0 is the bare heavy quark mass.

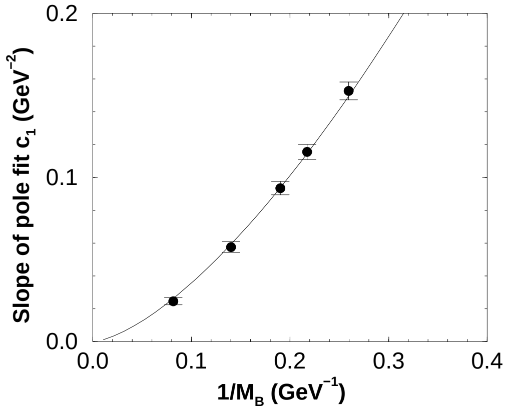

Figure 3. $1/M_B$ scaling of the pole fit coefficient.

son decay constant f_{B^*} as $c_1 = f_\pi/(f_{B^*}M_{B^*}^2 g)$. Figure 2 shows our data for $1/f^+(q^2)$ near $q^2_{\rm max}$ for $\kappa = 0.1369$ at $\beta = 5.7$. We find that the pole fit indeed explains the data quite well for each value of the heavy quark mass.

The heavy quark scaling is also predicted within the pole dominance model. Using $f_{B^*} \sim M_B^{-1/2}$ and $g \sim$ constant, we obtain $c_1 \sim M_B^{-3/2}$. The slope obtained with the fit (2) is plotted against $1/M_B$ in Fig. 3 together with a curve representing $1/M_B^{3/2}$. We confirm that the heavy quark scaling is nicely satisfied. Furthermore, from this fit, we obtain g=0.33(4), which is consistent with the value extracted from $D^* \to D\pi$ [8] $g = 0.27(6)$ and with the recent lattice study [9] $g = 0.42(8)$.

5. Conclusions

We consider that the problem of the soft pion relation should be resolved before a prediction of the $B \to \pi l \overline{\nu}$ form factors from lattice QCD can be made. We have investigated the possibility that the perturbative matching contains a large systematic error, and found that a nonperturbative value shows a non-negligible difference from the one-loop result.

Checking the q^2 dependence and the $1/M_B$ scaling of $f^+(q^2)$ is a necessary step toward a prediction of partial decay rate. We have found that the pole dominance model provides an excellent way to fit the observed shape of $f^+(q^2)$ in the large q^2 region. The $1/M$ scaling is also consistent with the pole model prediction.

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