

SENIORITY AND UNEMPLOYMENT

by

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ABSTRACT

This paper explains why unemployed workers fail to exert downward pressure on wages sufficiently to eliminate unemployment. It shows that rationing of unemployment by seniority effectively contains the downward pressure. Seniority, guaranteeing to current junior workers future employment and so future income at their present firm, gives rise to a high reservation wage for starting to work for other firms. The high reservation wage keeps unemployed workers from underbidding wages. They simply stay unemployed. By contrast, if rationed by lottery, unemployment creates sufficient downward pressure on wages to eliminate itself almost entirely.

I Introduction

A popular explanation of unemployment is to attribute it to the union's power over wage determination. McDonald and Solow (1981) and Hart (1982) are recent representative contributions along this line. However, as Friedman (1976) points out in criticizing Keynesian theory, the explanation is incomplete since it begs a further question of why downward pressure on wage rates does not develop in the presence of unemployment. Unemployed workers resulting from a high union wage rate may become outsiders to other unions, and then underbid union wage rates there, even if they are not allowed legally or morally to underbid the wage rate of their *own* union. Even anticipation of the underbidding keeps unions from setting their union wages at high levels. This paper addresses this problem of the underbidding in unemployment theory. It presents a model of the labor market which takes into full account the possibility of the underbidding by unemployed

workers, yet shows that high union wage rates causing unemployment are sustainable. Further, the model allows for fluctuation of employment in response to changes in demand for the firms' products as it occurs over business cycle.

In the minimum wage and development literature such as Mincer (1976) and Harris and Todaro (1970), a high non—equilibrium wage is sustainable because it is offset by a higher unemployment rate (so a lower chance of employment) resulting from the high wage. This theory applies when wages are set legally as the minimum wages, and hence the underbidding by unemployed workers is precluded. But, this paper considers the case where wages are set by unions, and hence the underbidding of the wages by union outsiders is possible.

To be complete as an explanation of unemployment, the union wage setting must be accompanied by a specification of a rationing scheme of unemployment that restrains unemployed workers from putting downward pressure on wages as outsiders. This paper focuses on what kind of rationing scheme diffuses the downward pressure on union wage rates. The focus represents the principal difference between the model of this paper and other models that also assume the union wage setting or other non—competitive wage determination such as bargaining. In the literature of unemployment, it is common to assume rationing of unemployment by lottery. This paper instead assumes rationing by seniority as is prevalent in the U.S. labor market, and then shows that the seniority rationing effectively restrains the downward pressure on wages from unemployed workers. To make clearer the importance of seniority for sustainable high union wage rates and unemployment, this paper also shows that, when rationed by

lottery, unemployment almost disappears in the otherwise same structure.

The basic reason for sustainable high union wage rates and unemployment under the seniority rationing in this paper is the following. Junior workers will eventually become senior workers by staying with the same union. Then, the seniority rationing guarantees future jobs and so, in effect, future income to present junior workers (uncertainty over future wage levels due to economic changes apart). The future income thus guaranteed under the seniority rationing at the present firm gives rise to a high reservation wage even with small unemployment compensations and small moving cost for workers. The high reservation wage keeps unemployed workers from underbidding wage rates of other unions and working for other firms.

Grossman (1983) analyzes wage and unemployment in a particular union in relation to seniority, especially whether a union with a more rigorous seniority rule sets its wage rate higher for a given demand than otherwise. He does not, however, fully consider the question of this paper, namely, whether high union wage rates are sustainable in face of the threat of the underbidding by outsiders, especially whether the seniority rationing is effective in sustaining high union wages. For, first, he does not take account of the underbidding of union wages by outsiders, although he does so indirect arbitrage between expected utilities inside and outside a particular union through expansion and shrinkage of the union membership. Second, in Grossman's model, the union expands, that is, new workers are allowed to join the union only to the extent that demand for workers forthcoming at the union wage rate exceeds the number of union members. Then, even the indirect arbitrage is taken into account only to the limited degree, and only

in an unimportant case in which the union wage rate is sufficiently low rather than high.

In a different structure from the one in this paper, Osborne (1984) studies the threat of cheaper unemployed workers (strike breakers in his term) to a high union wage rate. In his paper, a sufficiently high reservation wage of unemployed workers (the cost of strike breakers in his term) prevents the firm from hiring unemployed workers, and sustains a high union wage rate¹. The high reservation wage itself is, however, not accounted for, but simply assumed in his paper. By contrast, a high reservation wage of unemployed workers itself is the subject of explanation in this paper. The difference is important because one of the mysteries of unemployment is why the reservation wage of unemployed workers does not fall to a sufficiently low level to create demand for them.

Why the seniority rule is in place is an important question. Presumably, one of the reasons is that workers accumulate more and deeper skills, and become more valuable to employers as they work longer in the same firm or profession. Then, employers naturally lay off and recall workers according to seniority. As well as Grossman (1983), however, this paper does not formalize this explanation of the seniority rule, but simply presupposes the rule. Since the cause of the rule is, though an important and interesting subject to explore, a separate issue from its effects on unemployment, this paper avoids the problem of causes, and concentrates on effects. This allows us to reduce complication in exposition, and to place a sharper focus on the effect of the seniority rule on unemployment.

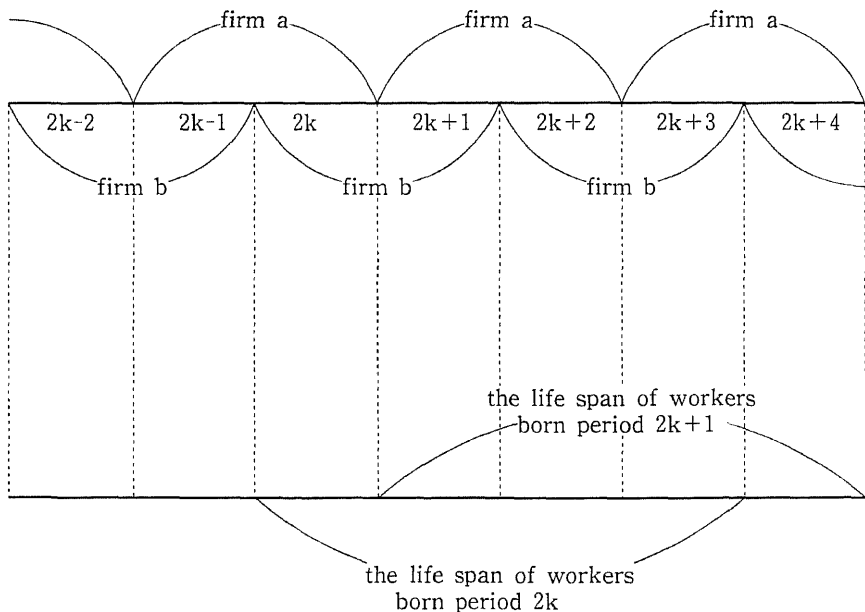
This paper is organized as follows. Section II presents a model of the labor market under the seniority rule. Section III demonstrates and explains

unemployment in the model. It also shows that the lottery rationing cannot sustain high union wage rates. Appendix proves a theorem in Section III.

II A Model of the Labor Market under Seniority Rationing

This paper supposes an overlapping structure of jobs in two firms, firm a and b. Jobs in firm a and b start at the beginnings of period $2k+1$ and $2k+2$, extend two periods, and end at the ends of period $2k+2$ and $2k+3$, respectively (see Figure 1). This rather artificial overlapping structure should be a natural formalization of the labor market with unsynchronized indivisible job tenures. Workers become sufficiently productive only after they get accustomed to their jobs. Thus, firms require workers to stay with themselves for some length of time². Employment relationship between a firm and workers is then indivisible in terms of its time length, and extends

Figure 1



over some interval of time. This paper refers to this interval as a job tenure. Job tenures in different firms are not synchronized. In consequence, they overlap each other. The above—mentioned structure should be the minimum to model extensive and indivisible job tenures and their overlapping.

Jobs in firm a and b continue for two periods in the assumed structure. After a simple change in interpretation, the model of this paper admits of jobs of one period duration as well as those of two period duration. This change does not imply changes in the substantive conclusion of this paper. We will come to this point later.

Given the two period duration of jobs, the usual assumption of two period living individuals do not suffice for the purpose of the paper. As workers become senior workers only after one job tenure is finished, the youngest senior workers at firm a and b are two period old. If workers live only two or three periods, senior workers cannot work at firm a or b since jobs there extend two periods³. Consequently, workers must be assumed to live four periods (see Figure 1). A worker who is just born, has lived one, two, and three periods is referred to as zero, one, two, and three years old.

Junior Members

Each firm has a union. The union of firm a (b) is referred to as union a (b). $N/4$ workers are born each period. All workers born at period $2k+1$ ($2k+2$) join union a (b) when they are born, though they may leave it later. Junior members of union a consist exclusively of these newly born workers. One year old workers, who were born one period earlier, that is, period $2k$, do not join union a as junior workers. This is because the one year old workers are too old to become senior workers of union a in future, and are not necessarily employed in the present period by firm a under the seniority

rationing, while as outsiders of union a, they are employed by the firm for certainty at the wage only slightly lower than the union a wage. Two year old workers are not junior, but senior workers of union a if they belong to it. Three year old workers are not eligible for jobs that will extend two future periods. Hence, letting P_{aj} be the number of junior members in union a, one has

$$(1) \quad P_{aj} = N/4.$$

In this paper, expositions made with regard to index a(b) are symmetrically applicable to b(a). In consequence, notations and equations are paired. For instance, P_{aj} and P_{bj} are paired, and they denote the number of junior workers in union a and b, respectively. Equation (1), $P_{aj} = N/4$, is paired with $P_{bj} = N/4$. Given the explanation of one of paired symbols and equations, the other in a pair should be self-explanatory. This paper introduces only one of a pair formally, and the other without an explanation. The unexplained one of paired equations will be referred to by putting a prime to the equation number of the explained equation of the pair. For instance, (1') refers to $P_{bj} = N/4$.

Behavior of Unemployed Workers

At the beginning of period $2k+1$, the firm a's job tenure starts. Firm a does not necessarily employ all junior workers of union a, that is, workers born period $2k+1$. When unemployed, these workers have two options at period $2k+2$ when they become one year old: either to stay with union a and firm a until the beginning of period $2k+3$ to be senior workers of union a, or to become outsiders of union b at the beginning of period $2k+2$ and to start to work for firm b. They will not become junior workers of union b as remarked already.

For the reason to be explained later⁴, if the unemployed workers choose the first option, they can expect that firm a will certainly employ them from period $2k+3$. Then, they will earn the wage rate of union a, W_a , in period $2k+3$ and $2k+4$. They earn unemployment compensations, W_c , at age two (period $2k+2$) while unemployed. Let r be the interest rate, and R be $1/(1+r)$. It follows that, choosing the first option period $2k+2$, the unemployed one year old workers of union a anticipate to receive $W_c + \{R+R^2\} W_a$ in the present value as of period $2k+2$.

If they choose the second option, the unemployed one year old workers receive the wage rate for outsiders at firm b, W_{ob} , in period $2k+2$ and $2k+3$ over which a firm b's job tenure extends. Because the wage rate for outsiders is determined to equate their supply with demand for them, unemployed one year old workers are certainly employed and receive W_{ob} if they become outsiders.

Turning to work for firm b as outsiders of union b, unemployed one year old workers of union a incur negligible moving cost, t . The cost may be transportation costs due to a location change, or psychological costs due to necessary adaptation to new environment. It must be stressed that the cost is so negligible that the cost itself does not discourage any unemployed workers from underbidding the wage rate at another firm, and working for it. The assumption of the negligible moving cost is introduced to ensure the uniqueness of equilibrium.

After having chosen the second option period $2k+2$ and working for firm b period $2k+2$ and $2k+3$, workers born period $2k+1$ are three years old at the beginning of period $2k+4$. They are no longer qualified for jobs at firm b whose new job tenure will continue until the end of period $3k+5$. They

retire and receive W_c as unemployment compensations. Accordingly, the payoff from working for firm b as outsiders of union b from period $2k+2$ is $(1+R)W_{ob} + R^2W_c - t$ in the present value as of the beginning of period $2k+2$.

Which options unemployed one year old workers of union a choose at period $2k+2$ depends upon the payoffs from the two options. When $W_c + \{R+R^2\} W_a$ is larger (smaller) than $\{1+R\} W_{ob} + R^2W_c - t$, the workers choose to stay with union a until period $2k+3$ (to work for firm b as outsiders of union b from period $2k+2$). Let E_{a0} be the number of zero year old workers of union a who are employed by firm a at period $2k+1$ when they are born. Hence, $N/4 - E_{a0}$ is the number of workers who were born period $2k+1$, and not employed by firm a at that period. Among them, Y_b become outsiders of union b at period $2k+2$ when they become one year old. All workers are the same, and so behave the same way in the same circumstance. Then, one has

$$(2) \quad Y_b = \begin{cases} N/4 - E_{a0} & \text{if } W_c + \{R+R^2\} W_a < \{1+R\} W_{ob} + R^2W_c - t, \\ 0 & \text{otherwise.} \end{cases}$$

W_c has been described as unemployment compensations. Alternatively, one may introduce one period jobs besides the two period ones, and interpret W_c as income from the one period job. Workers may take up the jobs when unemployed by firm a and b. The argument of this paper proceeds almost the same way, and the substantive conclusion of the paper does not change regardless of the two interpretations of W_c . Hence, the paper continues to refer to the variable as unemployment compensations. A possible conceptual issue associated with the second interpretation of W_c shall be discussed in Section III.

Whichever way interpreted, W_c must be assumed very small. Otherwise, unemployed workers become content with receiving W_c , and then there is trivially no incentive for them to underbid high union wage rates. This is contrary to the purpose of the paper.

Senior Members

Let P_{as} represent the number of senior workers in union a. Senior workers of union a at period $2k+1$ are a fraction of workers born two periods earlier, namely, period $2k-1$. Others, after having found themselves unemployed by firm a at period $2k-1$ when they were born, chose the second option period $2k$ and started to work for firm b from the period. They will stay with firm b until the job tenure there ends, that is, until the end of period $2k+1$. As they are, thus, not in a position to come back to firm a at the beginning of period $2k+1$, they do not belong to union a then. Let E_{b1} be the number of these workers. Then, one has,

$$(3) \quad P_{as} = N/4 - E_{b1}.$$

By definition, if P_a denotes the number of union a members,

$$(4) \quad P_a = P_{aj} + P_{as}$$

holds.

The Firm b's Employment Policy

Now, consider firm b's employment policy at period $2k+2$ under the seniority rationing. Let $L(W)$ be firm b's demand for workers. $L'(W) < 0$. There are two kinds of workers for firm b: union b members and outsiders who are one year old and have come from union a. The cost of employing the former is the union b wage rate W_b , and that of employing the latter is W_{ob} . Let E_{b2} and E_{b0} be the levels of firm b's employment for two year old (and so senior), and zero year old (and so junior) members of union b. As

was just introduced above, E_{b1} denotes the firm b's employment for outsiders coming from union a.

Consider first firm b's employment policy under $W_b > W_{ob}$. Since union members are more expensive than outsiders of union b, the firm demands none of union b members, but outsiders. Therefore, one has

$$(5) \quad E_{b2} = E_{b0} = 0, \text{ and } E_{b1} = L(W_{ob}).$$

If $W_b < W_{ob}$, firm b does not demand more expensive outsiders. It employs $L(W_b)$ union b members. On account of the seniority rationing, the firm employs senior workers (two year old union b members) first. The firm employs junior workers (zero year old members of union b) only to the extent that $L(W_b)$ exceeds the number of senior workers. Mathematically, one has

$$(6) \quad E_{b2} = \min \{L(W_b), P_{bs}\}, \quad E_{b0} = L(W_b) - E_{b2}, \text{ and } E_{b1} = 0.$$

Next, consider the case with $W_b = W_{ob}$. In this case, the costs of union members and outsiders are the same. Then, union b reasonably requires that firm b should treat junior workers and outsiders equally. Then, if outsiders are employed prior to senior workers, junior workers must be so in violation of the seniority rule. Therefore, the firm employs senior workers of union b prior to both junior workers and outsiders, and it distributes remaining jobs evenly among junior workers and outsiders of the union. Accordingly, one has

$$(7) \quad E_{b2} = \min \{L(W_b), P_{bs}\}, \quad E_{b0} = c_b P_{bj}, \text{ and } E_{b1} = c_b Y_b$$

where c_b is defined by

$$(8) \quad c_b = \{L(W_b) - E_{b2}\} / \{Y_b + P_{bj}\}.$$

The Outsider's Market

Demand and supply of outsiders must balance:

$$(9) \quad Y_b = E_{b1}.$$

Equation (9) means that outsiders coming from union a compete with each other to lower their wage, so that all outsiders from union a should be employed by firm b. Equations (5) through (8) define firm b's demand for junior, senior workers and outsiders, given the union wage rate and the outsider's wage rate. Equation (9) specifies determination of the outsider's wage rate at firm b.

The Union's Optimization Problem

Given the firm's demand seen above, union b sets its wage rate unilaterally in such a way as to maximize the wage revenue for its members, that is, $W_b(E_{b0} + E_{b2})^5$. Though frequently assumed in the literature such as Grossman(1983), Hart (1982), Oswald(1982), and Solow(1986), the assumption of the unilateral wage setting by the union is admittedly a gross abstraction of negotiation procedure between a firm and its union. Besides, this assumption implies an inefficient outcome (see Leontief (1946), Hall and Lilien (1979), and McDonald and Solow (1981)). However, the simple assumption makes more traceable the economic mechanism to be elucidated in this paper, while sophisticated bargaining assumptions should produce complicated models with a less sharp focus on the basic point of this paper.

In setting its wage rate, union b must provide jobs at firm b to at least half of its members. Otherwise, its wage policy is unable to gather the support of at least half of its members, and is not to be ratified by its members. This paper refers to this constraint on the union's optimization problem as union democracy.

It follows that the solution of the following optimization problem characterizes the behavior of union b:

$$(10) \quad \max W_b \{E_{b0} + E_{b2}\} \quad \text{subject to } P_b/2 \leq E_{b0} + E_{b2} \leq P_b.$$

The seniority rule requires that senior workers of union b should be employed prior to its junior members, while senior workers of union b are no more than half of union b members by (1), (3), and (4). Then, the constraint of (10) means all the senior workers are always employed; that is,

$$(11) \quad E_{b2} = P_{bs}.$$

Because of (5), the union democracy implies $W_b \leq W_{ob}$ ⁶; union b has to set its wage rate no higher than outsider's wage rate to induce firm b to employ at least half of its members. Thus, union democracy means the union wage policy that is never underbid. Then, the question is not whether union wages are actually underbid by outsiders, but how low the threat of the underbidding by outsiders forces unions to set their wages. Put another way, the question is if unions must set their wage rates so low as to bring out nearly full employment of their members in preventing the underbidding by outsiders.

III Unemployment in Equilibrium of the Unions' Policies

We will demonstrate that when the seniority rationing is in place, even high union wage rates are not underbid by outsiders, and so that large unemployment is sustainable.

I make the following assumption:

Assumption 1 $WL(W)$ is strictly concave, and reaches its maximum at W^*

Let $L^* = L(W^*)$. By Assumption 1, $WL(W)$ is increasing for $W < W^*$, and

decreasing for $W > W^*$. Let W^0 and W^1 be defined by $L(W^0) = N/4$ and $L(W^1) = N/2$.

The subsequent part of this paper allows for shifts of $L(W)$. Let s be the shift parameter of $L(W)$, and capture the state of the product markets for firms. L^* , W^* , W^0 , and W^1 depend upon s . We assume $\partial L(W, s) / \partial s > 0$. Then, $dW^0/ds > 0$, and $dW^1/ds > 0$. The assumption also implies $dW^*L^*/ds > 0$, but does not necessarily both of $dW^*/ds > 0$ and $dL^*/ds > 0$. This paper assumes both of them to hold simultaneously as it is a very plausible case⁷. Let s' and s'' be defined by $L^*(s') = N/4$ and $L^*(s'') = N/2$.

The second assumption this paper uses is:

Assumption 2 $L(t) > N$.

This assumption implies that the moving cost is so small that it in itself does not keep unemployed one year old workers from moving to work for another firm as outsiders. When the wage rate is below t so that moving to work for another firm is not worthwhile, it follows from the assumption that demand for labor exceeds the total labor force. This is impossible. Therefore, Assumption 2 implies that the wage rate net of the moving cost is always positive, and that the moving cost itself does not prevent unemployed workers from moving to other firms.

This paper also assumes:

Assumption 3 $r = 0$.

The assumption of the zero interest rate is unrealistic, but it is useful in

avoiding mathematical complications, and highlighting the effect of the seniority rationing on unemployment. Seniority rationing is based upon workers' appreciation of what is expected to happen in future. Meaning that workers value future income and the present one equally, the assumption represents the extreme case of the appreciation. This assumption thus shows the effect of the seniority rationing on unemployment in the most accentuated manner.

Let E_i for $i=a,b$ be the number of workers employed by firm i ; that is, $E_i = E_{i0} + E_{i1} + E_{i2}$. Let E be $E_a + E_b$, namely, the total employment. Under Assumption 1,2, and 3, the following theorem holds:

Theorem The total employment E and the union wage rate $W_i (i=a,b)$ are given by:

$$E=N/2 \text{ and } W_i=W^0(s) \text{ for } s \leq s',$$

$$E=2L^*(s) \text{ and } W_i=W^*(s) \text{ for } s' \leq s \leq s'',$$

$$E=N \text{ and } W_i=W^1(s) \text{ for } s'' \leq s.$$

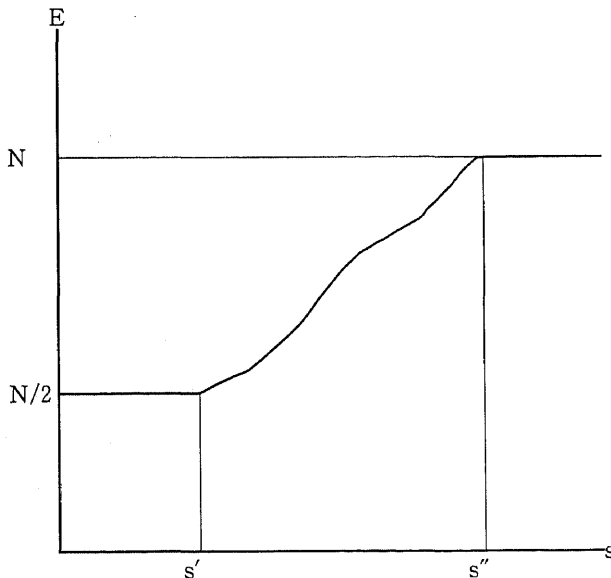
Figure 2 depicts the theorem. As the state of the firms' product markets improves, the equilibrium level of employment increases. The theorem means that the seniority rationing effectively contains the downward pressure on wages which would otherwise develop from frustrations of unemployed workers, and so that the seniority rationing creates large unemployment when demand for firms' products is not large enough.

Here, I will demonstrate the theorem rather casually. (A formal proof of the theorem is delegated to Appendix.)

The basic reason for sustainable high union wage rates in the theorem is

that the seniority rationing gives rise to a high reservation wage of unemployed workers. Since presently senior workers will retire in the future, present junior workers are certain that they will be employed as senior workers by the presently associated firm in future under the seniority rule. It follows that the seniority rationing together with union democracy guarantees to junior workers future income at their present firm. On the other hand, job tenures in different firms overlap each other on account of the unsynchronized and indivisible tenures. Consequently, once they succeed in being employed by other firms, unemployed junior workers will not be in a position to come back to their present firm in future for working there as senior workers. The guaranteed income under the seniority rationing, then, constitutes a high reservation wage for working elsewhere, even if the moving cost for workers and unemployment compensations (or income from

Figure 2



temporary jobs) are very small. With a high reservation wage, unemployed junior workers of a union do not have incentive to underbid the wage rates of other unions to be employed. Therefore, unions can sustain high union wage rates, and, in consequence, large unemployment becomes sustainable.

For a more detailed, though still informal, account of the theorem, consider the most interesting case of $N/4 < L^*(s) < N/2$, namely, $W^1(s) < W^*(s) < W^0(s)$. Let us first observe that union b does not set its wage rate W_b above $W_a + t/2$; that is,

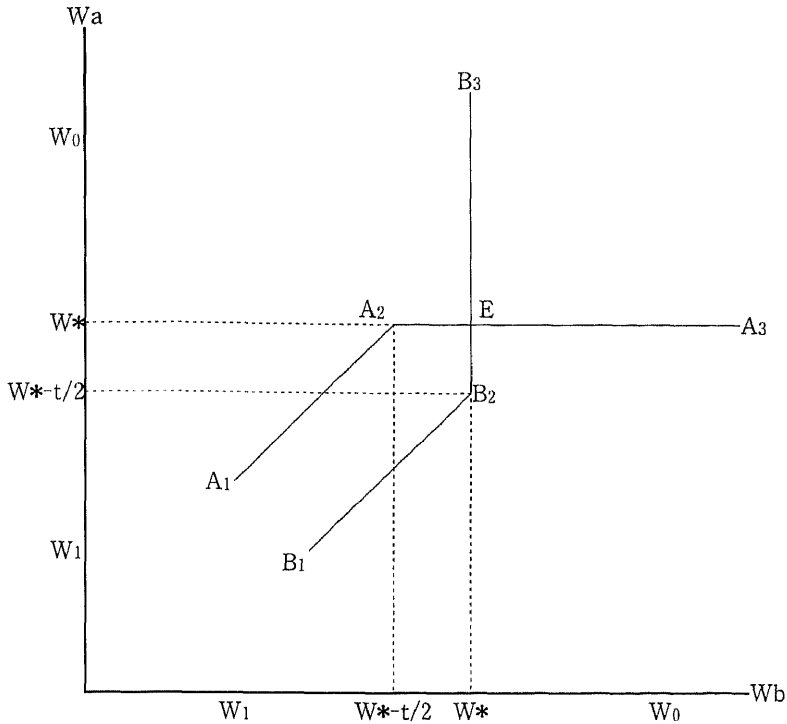
$$(12) \quad W_b \leq W_a + t/2$$

holds.

Suppose the contrary. Then, $W_b > W_{ob}$ must hold. Otherwise, by (2), (5), and (6) and Assumption 3, the market of outsiders at firm b is not in equilibrium; unemployed one year old members of union a become outsiders to union b, while firm b' does not demand all of them⁸. By (5), however, $W_b > W_{ob}$ implies that firm b employs none of union b members. This contradicts the constraint in the optimization problem (10); the union wage policy of setting W_b above W_{ob} does not create a job for its members, and cannot be ratified by the union members, and be put into effect. Thus, allowable union b wage policies must satisfy (12).

Given (12), $W_b \leq W_{ob} \leq W_a + t/2$ must hold in order that the outsider's market at firm b is in equilibrium. In this situation, there is no supply of outsiders to firm b; by (2), none of unemployed junior(one year old)workers of union a becomes an outsider of union b. It then follows from the equilibrium condition of the outsider's market, (9), that firm b employs no outsider. Firm b employs $L(W_b)$ union b members, and hence the total wage for the union b members is $W_b L(W_b)$.

Figure 3



Suppose $W^* \leq W_a + t/2$. In this case, $W_b = W^*$ satisfies (12). Because $WL(W)$ reaches its maximum at W^* by Assumption 1, and because the total wage for the union b members is, as just seen, $W_b L(W_b)$ when (12) holds, union b sets its wage rate at W^* in this case. The line B_2B_3 in Figure 3 depicts this wage policy of union b.

Next consider the case of $W^* > W_a + t/2$. Because union b must set its wage rate below $W_a + t/2$ with the total wage $W_b L(W_b)$, and because, by Assumption 1, $WL(W)$ is increasing for $W^* > W$, union b sets its wage rate at $W_a + t/2$. The segment B_1B_2 in Figure 3 depicts this wage policy.

By the symmetrical argument, one can derive the union a's wage policy

given the union b's policy. The line $A_1A_2A_3$ in Figure 3 depicts the policy. Point E in the figure represents the Nash equilibrium of union a and b's policies. At the equilibrium, $W_a = W_b = W^*$ holds. With these wage policies, neither firm a nor firm b employ outsiders. Therefore, the level of total employment in the economy is $2L(W^*)$ as the theorem asserts for $N/4 < L^* < N/2$. Since $2L(W^*)$ is less than N under that condition, there is unemployment at the equilibrium. The high union wage rates creating unemployment cannot be underbid by unemployed workers to eliminate unemployment. Note that the negligible, but positive t assures uniqueness of the Nash equilibrium in Figure 3.

Lottery Rationing and Unemployment

We now consider what happens if employment is rationed by lottery instead of seniority in the otherwise same structure as the above. Answering the question illuminates how critical the seniority rationing is in creating unemployment. The answer to this question is that, under the lottery rationing, unemployment exerts enough downward pressure on wages to eliminate itself almost entirely when the moving cost is negligible.

Even when employment is rationed by lottery, equation (1), (3), (4), (5), (9), and (10) in Section II still apply. But, in the model of the lottery rationing, (6) should be replaced by:

$$(13) \quad E_{b2} = c_b P_{bs}, \quad E_{b0} = c_b P_{bj}, \quad \text{and} \quad E_{b1} = 0,$$

where c_b is now defined by $L(W_b)/P_b$. Equation (7) and (8) must be also replaced by

$$(14) \quad E_{b2} = c_b P_{bs}, \quad E_{b0} = c_b P_{bj}, \quad \text{and} \quad E_{b1} = c_b Y_b$$

where c_b is defined by $L(W_b)/\{P_b + Y_b\}$. Instead of (2), the following (15) describes the decision of unemployed workers on whether to become out-

siders:

$$(15) \quad Y_b = \begin{cases} N/4 - E_{a0} & \text{if } W_{ra} < W_{ob}, \\ 0 & \text{otherwise.} \end{cases}$$

where W_{ra} is the reservation wage of unemployed one year old members of union a for working for firm b as outsiders of union b.

Equation (11) and (12) are irrelevant to the present case.

Let us see that unemployment is negligible in the model when t is negligible. If $W_{ra} < W_b$, firm b employs outsiders at the wage rate between W_{ra} and W_b , but none of union b members. This contradicts the union democracy. Therefore, union b must set its wage rate at a level no higher than W_{ra} , that is, $W_{ra} \geq W_b$ must hold. Then, because of (15), no unemployed worker from union a wants to take up a job at firm b as an outsider of union b, and hence firm b employs only union b members. In addition, then, all one year old members of union a remain in union a until they become two year old, although they are not guaranteed jobs for age two and three. Accordingly, by (3) and (4), $P_a = N/2$.

The above results imply W_b maximizes $WL(W)$ subject to $W \leq W_{ra}$ and $N/4 \leq L(W) \leq N/2$. To shorten the argument, let me concentrate on the case of $W^1 < W^* < W^0$, namely, $N/4 < L(W^*) < N/2$. Then, by Assumption 1,

$$(16) \quad W_b = \min \{W_{ra}, W^*\}.$$

Because of $P_a = N/2$, and because firm a employs no outsider, two year old members of union a are employed by firm a with the probability of $L(W_a)/(N/2)$ under the lottery rationing. When not employed by the firm for age two and three, they earn W_c . Then, for unemployed one year old members of union a, the expected wage rate from staying with union a until age two is $2W_a L(W_a)/N + W_c(1 - 2L(W_a)/N)$. Under Assumption 3, this plus

half of the negligible moving cost is the reservation wage for taking up jobs at firm b as an outsider; namely, $W_{ra} = 2W_a L(W_a)/N + W_c(1 - 2L(W_a)/N) + t/2$.

Therefore, on account of (16), the symmetric Nash equilibrium is either $W_a = W_b = W^*$, or the solution of

$$(17) \quad W = 2WL(W)/N + W_c(1 - 2L(W)/N) + t/2.$$

When t tends to zero, the solution of (17) does so to the solution of $N/2 = L(W)$, that is, W^1 . By $W^1 < W^*$, W_a and W_b tend to W^1 , and hence E tends to N when t tends to zero. Thus, if the moving cost for workers is negligible, unemployment is negligible and almost all workers are employed when unemployment is rationed by lottery. This is in a sharp contrast to sustainable large unemployment under seniority rationing.

Intuition behind the virtual disappearance of unemployment under the lottery rationing is the following. Senior and junior workers are treated equally in the queue of employment under the lottery rationing. It follows that, under the rationing scheme, workers do not become more likely to be employed by their presently associated firm simply by staying longer with that firm and its union. Consequently, presently unemployed workers are not guaranteed future jobs and so future income at their presently associated firm under the lottery rationing. In other words, their expected income from staying with the same union rather than working elsewhere is low. The low expected income makes unemployed workers willing to work for other firms at low wages as outsiders of unions of other firms. Then, outsiders coming from other unions underbid a high wage rate of a union, and work for the firm of that union. Thus, unions setting wage rates at high levels are unable to assure jobs for their members. Unions are hence forced

to set their wages at low levels. At low union wage rates, only a negligible fraction of union members are unemployed, and unemployment becomes negligible.

Unemployment and Temporary Jobs

If W_c represents the wage income from one period jobs, one year old workers not employed by firm a and b are taken to work temporarily elsewhere. Some may then criticize that full employment realizes even under the seniority rule when one period jobs are available. This criticism is merely semantic, and not substantive⁹.

With the literal interpretation of unemployment that underlies the above criticism, unemployment does not exist by definition. "Unemployed" workers (or their wives) occasionally take up temporary, low paid jobs to make living. "Unemployed" workers receiving unemployment compensations, or using up savings are said to prefer leisure to work. To be meaningful, the term of unemployment should be interpreted less narrowly in conformity with its every day usage.

We observe that, while some workers are still working at high wage rates for jobs their skills are meant for, their usual colleagues with similar skills are unable to do similar jobs, but are taking up temporary, low skill, low-paid jobs, or receiving unemployment compensations, or using up their savings. The same worker moves back and forth between the two different conditions over business cycle. The literal unemployment does not exist, but such different treatments of similar (same) workers by the market do. Whether or not unemployment is a right word to describe the treatments, these phenomena of the different treatments are a serious economic problem to be accounted for, and are usually referred to as unemployment.

Unemployment in this sense is the subject of explanation in theory of unemployment.

When W_c represents income from a temporary job, workers employed by neither firm a nor firm b take up one period jobs. Given the difference in the durations of the jobs, and in view of the reason for the two period duration of jobs at firm a and b mentioned in Section II, the two period jobs at firm a and b must not be thought of as belonging to the same category as the one period job. A natural understanding should be to think that the difference captures the dual labor market of Doeringer and Piore (1971). Then, the two period jobs at firm a and b represent relatively secure, high skill jobs of the primary labor market. They are typically those in the manufacturing sector. On the other hand, the one period jobs represent temporary, low skill jobs with low wage rates belonging to the secondary labor market. They are, for instance, vendors in fast food shops. In a concrete example, unemployed one year old workers who take up one period jobs are those who could be otherwise working at high wage rates for highly skilled jobs in manufacturing industries, but are actually working as vendors at hamburger shops for low wage rates. Therefore, when W_c is interpreted as income from a one period job, unemployment in the above sense, which is the real question to answer, exists in the model of this paper, and this paper has shown that such unemployment cannot be eliminated by the threat of the underbidding by unemployed outsiders.

IV Conclusion

As the criticism of Friedman (1976) implies, macroeconomic theories in the Keynesian tradition do not take account of individual rationality on the

part of unemployed workers. The theories cannot be complete without a specification of a rationing scheme of unemployment that would diffuse downward pressure on wages which the individual rationality of unemployed workers would imply.

The model of this paper takes account of individual rationality of workers, yet shows that unemployment results. The model demonstrates that the seniority rationing of unemployment diffuses the downward pressure and so that unemployment is sustainable. It also shows that, when unemployment is rationed by seniority, an economy functions in a manner that an increase in demand for the firms' products brings about a decrease in unemployment.

This paper has also shown that, in contrast to the seniority rationing, the lottery rationing is unable to keep unemployed workers from putting downward pressure on wages, and that unemployment is negligible under lottery rationing.

APPENDIX

This appendix proves the theorem in the main text. In the following, i and j represent either a or b .

Lemma 1 $W_{oi} \geq W_i$.

Proof Suppose the contrary. By (5), then, $E_{i0} + E_{i2} = 0$, which is less than $P_i/2$. This violates the constraint of (10). (Q.E.D.)

Lemma 2 $Y_i = 0$.

Proof By Lemma 1, $W_{ob} \geq W_b$. If this inequality holds strictly, $E_{b1} = 0$ by (6). Hence, by (9), $Y_b = 0$ holds; the lemma holds for $i=b$. Therefore, we assume $W_{ob} = W_b$.

When $W_{ob} = W_b$, $E_{b1} = c_b Y_b$ by (7). On the other hand, by (9), $Y_b = E_{b1}$. Therefore, $Y_b = 0$ or $c_b = 1$. If the former holds, the lemma is established for $i=b$. Hence, let $c_b = 1$ and $Y_b > 0$. It follows from (1'), (7), (8), (11) and $c_b = 1$ that $E_{b0} = N/4$ and $E_b = L(W_b) = P_{bs} + N/4 + Y_b$.

By (3') and (9'), $P_{bs} = N/4 - Y_a$. Therefore, by (2'), either $P_{bs} = N/4$ or $P_{bs} = E_{b0}$. Then, because of $E_{b0} = N/4$, $P_{bs} = N/4$. Accordingly, one has $E_b = L(W_b) = N/2 + Y_b$, and hence $W_b < W^1$.

From (5'), (6'), (7'), and (8'), $E_a \geq L(W_a)$. Then, $N \geq E_a + E_b \geq L(W_a) + N/2 + Y_b$. Hence, $W_a > W^1 > W_b = W_{ob}$. By (2) and Assumption 3, this implies $Y_b = 0$.

One can prove the lemma for $i=a$ similarly. (Q.E.D.)

Lemma 3 $P_{is} = N/4$, $P_i = N/2$, and $E_i = L(W_i) = E_{i0} + E_{i2}$. Furthermore, $W^1 \leq W_i \leq W^0$

Proof By Lemma 2, $Y_b = 0$. By (9), then, $E_{b1} = 0$. Therefore, by (3), $P_{as} = N/4$. Hence, by (1) and (4), $P_a = N/2$.

By Lemma 1, $W_{oa} \geq W_a$. Hence, by (6'), (7') and (8'), $E_a = L(W_a) = E_{a2} + E_{a0} + E_{a1}$. From Lemma 2 and (9'), $E_{a1} = 0$. Therefore, $E_a = L(W_a) = E_{a2} + E_{a0}$. Then, by (10) $W^1 \leq W_a \leq W^0$

Similarly, one can show the lemma for $i=b$. (Q.E.D.)

Lemma 4 Suppose $W_i \leq W_j + t/2$ where i and j are not identical. Then, W_i maximizes $WL(W)$ subject to $P_i/2 \leq L(W) \leq P_i$ and $W \leq W_j + t/2$.

Proof By Lemma 3, $L(W_b) = E_{b2} + E_{b0}$. Hence, by the constraint of (10), $P_b/2 \leq L(W_b) \leq P_b$ must hold. If this lemma is not true, there would be W' that satisfies

$P_b/2 \leq L(W') \leq P_b$ and $W' \leq W_a + t/2$ and $W'L(W') > W_bL(W_b)$.

Let the union b wage be W' rather than W_b . It follows from (2), (6), and Assumption 3 that with the union b wage rate at W' and so no more than $W_a + t/2$, firm b employs none of outsiders, but $L(W')$ union b members. The value of the union b objective function then equals $W'L(W')$. This contradicts the definition of W_b on account of $W'L(W') > W_bL(W_b) = W_b(E_{b2} + E_{b0})$.

One can prove the lemma similarly when $i=a$ and $j=b$. (Q.E.D.)

Proposition 1 If $L^*(s) \leq N/4$ (equivalently, $s \leq s'$ under $dL^*(s)/ds > 0$), $E_i = N/4$, $W_i = W^0(s)$, and $E = N/2$.

Proof Without loss of generality, $W_a \geq W_b$. Then, $W_a + t/2 > W_b$. Hence, by Lemma 3 and 4, W_b maximizes $WL(W)$ subject to $N/4 \leq L(W) \leq N/2$ and $W \leq W_a + t/2$. On account of $L^* \leq N/4$ (namely, $W^0 \leq W^*$) and Assumption 1, then, $W_b = \min \{W_a + t/2, W^0\}$. By $W_a \geq W_b$, then, $W_b = W^0$ and so $L(W_b) = N/4$.

By $W_b = W^0$ and $W_a \leq W^0$ (due to Lemma 3 and the union democracy), $W_a \leq W_b + t/2$. From this, $W_a = W^0$ follows similarly to the above.

The rest of the proposition follows easily from $W_a = W_b = W^0(s)$. (Q.E.D.)

Proposition 2 If $N/4 < L^*(s) < N/2$ (equivalently, $s' < s < s''$ under $dL^*/ds > 0$), $W_i = W^*(s)$, and $E_i = L^*(s)$, and $E = 2L^*(s)$.

Proof Again, without loss of generality, $W_a \geq W_b$, and so $W_a + t/2 \geq W_b$. Then, it follows from Lemma 4 and the assumption of the present proposition that $W_b = \min \{W^*, W_a + t/2\}$. By $W_a \geq W_b$, $W_b = W^*(s)$.

By Lemma 3 and (11), $L(W_b) = E_{b0} + N/4$. By $W_b = W^*$, $L(W_b) = L^* < N/2$. Therefore, $N/4 - E_{b0} > 0$. If $W_{oa} > W_b + t/2$, then, $Y_a > 0$. This contradicts Lemma 2. Therefore, $W_{oa} \leq W_b + t/2$. By Lemma 1, hence, $W_b + t/2 \geq W_a$. Similarly to the

first part of this proof, then, $W_a = W^*(s)$.

The rest of the proposition follows immediately from the above results. (Q.E.D.)

Proposition 3 If $N/2 \leq L^*(s)$ (equivalently, $s'' \leq s$ under $dL^*/ds > 0$), $E_1 = N/2$, $W_1 = W^1(s)$, and $E = N$.

Proof $N/2 \leq L^*$ is equivalent to $W^1 \geq W^*$. By Assumption 1, $WL(W)$ is decreasing for $W \geq W^1$. Accordingly, union a does not set its wage higher than W^1 . Then, by Lemma 3, $W_a = W^1$ and $L(W_a) = N/2$ hold. (Q.E.D.)

The theorem of the main text is combination of Proposition 1, 2, and 3.

NOTES

1. With a small cost of strike breakers, the condition of Lemma 1 in Osborne (1983) does not hold. Thus, the firm always uses strike breakers when their cost is low.
2. To ensure workers to stay with the firm, the wage rate may rise as workers work for the same firm longer. To simplify exposition, this paper introduces a certain length of the employment relationship without such a wage system.
3. One may think that if the reason for extensive and indivisible job tenures is time needed to get workers accustomed to their jobs, job tenures for experienced workers among senior ones can be less than two periods long. However, technological changes, though implicit in this paper, demand even experienced workers' getting newly accustomed to their jobs when new job tenures begin.
4. See an explanation preceding equation (11).

5. For a use of the same union objective function, see Hart (1982).

6. Someone criticized that $W_b \leq W_{ob}$ was opposite to actual observations. Indeed, one observes union wage rates to be higher than those of outsiders in the circumstances where outsiders are indeed working. However, this paper attempts to show that the seniority rule prevents the market for outsiders from developing, namely, that the market is in equilibrium without demand for and supply of outsiders under the rule. Without the market of outsiders, their wage rates are not posted explicitly and hence not visible. W_{ob} in the inequality is not an observed price of actual outsiders, but a theoretical or implicit price in the minds of would-be outsiders and would-be employers. Hence, the actually observed margins of union wages over outsider's wages are irrelevant to the question of this paper.

7. For $L(W,s) = -aW + s$, $dW^*/ds > 0$ and $dL^*/ds > 0$ hold simultaneously.

8. If $E_{ao} = N/4$, by (2), supply of outsiders is none, and the market clears even with $W_{ob} > W_b > W_a + t/2$. But, this does not happen under $N/4 < L^* < N/2$. The proof for this fact is delegated to the appendix. Firm b does not employ all of outsiders unless $c_b = 1$. But, when $c_b = 1$, $Y_b = 0$ as seen in the appendix. Given $W_{ob} > W_a + t/2$, this is possible only when $E_{ao} = N/4$.

9. See Solow (1986) and Stiglitz (1987, p35) for similar views on unemployment.

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