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Quenched QCD with domain-wall fermions on coarse lattices *

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We investigate the existence of chiral zero modes at $a^{-1} \simeq 1$ GeV in quenched domain-wall QCD. Simulations are carried out for the plaquette and an RG-improved gauge actions on a $12^3 \times 24 \times N_s$ lattice with $N_s = 10 - 50$. We find that the pion mass in the chiral limit remains non-vanishing as $N_s \to \infty$ for both gauge actions. Possible origins of this non-vanishing pion mass are discussed.

1. INTRODUCTION

The domain-wall fermion formulation of QCD (DWQCD) [1,2] is expected to realize exact chiral symmetry on the lattice without species doubling at finite lattice spacing. This represents an appealing possibility, particularly for investigations of problems such as weak matrix elements sensitive to chiral symmetry [3]. Therefore, a number of studies have been made [4].

Simulations in DWQCD, however, requires considerable computing power. Even if the size of the extra dimension N_s can be taken as small, i.e. $N_s = O(10)$, lattice spacings much finer than $a^{-1} \approx 2$ GeV will be difficult to simulate even in quenched QCD. Hence simulations on coarse lattices down to $a^{-1} \approx 1$ GeV will be needed for reliable continuum extrapolations.

A first step in DWQCD at such a strong coupling will be to ensure the existence of chiral zero modes. Here we report our preliminary results on this problem. Our study is made for the plaquette action, and also for an RG-improved action [5] to examine the effects of reduced discretization errors.

*Talk presented by Y. Aoki at Lattice 99, Pisa, Italy.

2. PARAMETERS

We employ the fermion action identical to that in Ref. [2], with the domain wall height M and bare quark mass m_f. The gauge coupling is chosen to be $\beta = 5.65$ for the plaquette action and $\beta = 2.2$ for the RG action; these values correspond to the scale $a^{-1} \approx 1$ GeV determined from the string tension. Simulations are made on an $12^3 \times 24 \times N_s$ lattice with $N_s = 10, 20, 30$ and 50.

In the free fermion case the chiral zero mode exists over the range $0 < M < 2$. Since this range will be shifted in the interacting theory, we employ $M = 1.3, 1.7, 2.1$ and 2.5. For each of these values of M, we take $m_f = 0.1, 0.05$ and 0.03 and calculate the pion mass for both degenerate and non-degenerate quark and antiquark pairs. For each parameter point we have typically 20 configurations.

3. RESULTS

In Fig. 1 the pion mass squared m_π^2 is plotted as a function of the averaged bare quark mass m_f^{av} at $M = 1.7$ in the case of the plaquette gauge action. Since the linearity of m_π^2 in m_f^{av} is well satisfied, we adopt a linear chiral extrapolation in

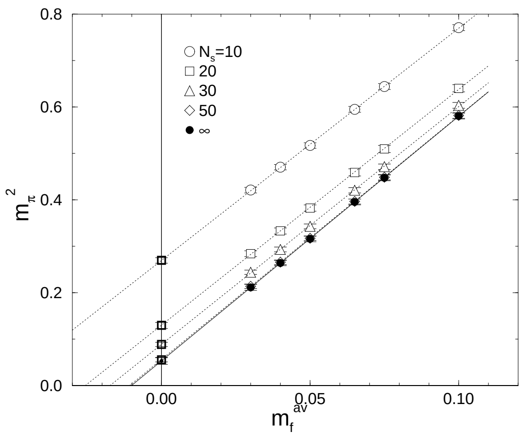

Figure 1. Pion mass squared as a function of m_f^{av} at $M = 1.7$ for the plaquette action. Solid circles show the $N_s \to \infty$ limit of open symbols for each m_f. Lines show the linear fits.

m_f^{av}. As it can be seen in the Fig. 1, we find a non-zero value for m_π^2 at $m_f^{av} = 0$ for each N_s.

This non-zero value, however, is expected to vanish exponentially as $N_s \to \infty$. To see whether this occurs, we plot m_π^2 at $m_f^{av} = 0$ by thick square symbols in Fig. 2(a) as a function of N_s. Fitting these points by the form $\alpha e^{-\xi N_s}$ yields the dotted line with $\chi^2/dof = 21.4$. An alternative fit, allowing a constant, $c + \alpha e^{-\xi N_s}$ gives the solid line with $\chi^2/dof = 1.7$ and $c = 0.0532(75)$. Clearly the latter fit better reproduces the behavior of our data. A similar phenomenon has been previously reported also at $\beta = 5.7$ [6].

In order to confirm the existence of a non-zero c, we attempt to interchange the order of the limits $m_f^{av} \to 0$ and $N_s \to \infty$. As shown by solid lines going through open circles in Fig. 2(a), we first make a fit of form $m_\pi^2(m_f^{av}, N_s) = c'(m_f^{av}) + \alpha e^{-\xi N_s}$ to make an $N_s \to \infty$ extrapolation for each value of m_f^{av}. The results, shown by solid circles in Fig. 1, exhibits a linear behavior in m_f^{av}. A linear chiral extrapolation $m_\pi^2(m_f^{av}, N_s = \infty) = c'(m_f^{av}) = d + \gamma m_f^{av}$ then yields $d = 0.0531(70)$, which agrees well with the value of c previously obtained. The commutativity of the two limits is summarized in Fig. 2(a), where the two values obtained for $m_\pi^2(m_f^{av} = 0, N_s = \infty)$ are marked by "X" and "Y".

The above analyses strongly support the conclusion that m_π^2 at $m_f^{av} = 0$ does not vanish in the limit $N_s = \infty$ at $a^{-1} \simeq 1$ GeV for the 4-dimensional lattice size of $12^3 \times 24$. We find this conclusion to apply as well to the RG-improved gauge action, as shown in Fig. 2(b).

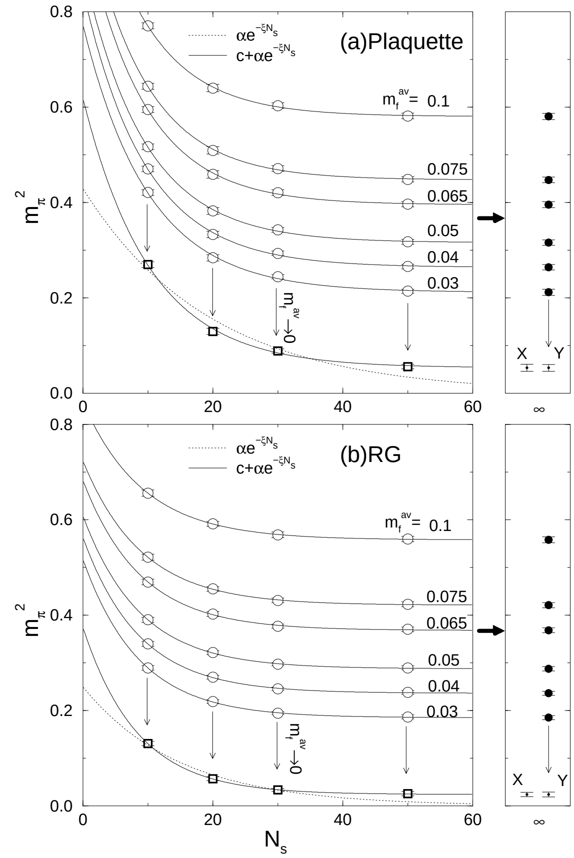

Figure 2. Pion mass squared as a function of N_s at $M = 1.7$ for the plaquette (a) and RG (b) actions. Two limits $m_f^{av} \to 0$ and $N_s \to \infty$, and their combinations in two possible orders denoted by "X" and "Y" are shown.

In Fig. 3 the residual $m_\pi^2(m_f^{av} = 0, N_s = \infty)$ is plotted as a function of M for the plaquette and RG-improved gauge actions. We observe that a non-vanishing residual remains under the variation of M for both actions. An improvement may be seen for the RG action, however, in that the magnitude of the residual is reduced by about a factor of two compared to that for the plaquette action.

Finally the decay rate ξ extracted from the fits

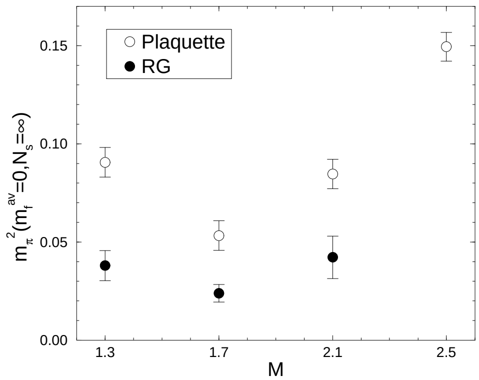

Figure 3. Summary of the squared pion mass in the double limit $m_f \to 0$ and $N_s \to \infty$.

is shown in Fig.4 as a function of m_f^{av} at $M = 1.7$. For both actions, the chiral extrapolation of ξ from $m_f^{av} \neq 0$ (open circles) smoothly agrees with the one directly obtained at $m_f^{av} = 0$ assuming $c \neq 0$ (filled circle), but disagrees with the value from the $c = 0$ fit. Furthermore the values of ξ varies little with m_f^{av}. This is consistent with the expectation that the decay rate is governed by the transfer matrix in the extra dimension, which is independent of m_f.

4. DISCUSSION

Our study of domain-wall QCD with the plaquette gauge action has shown that the pion mass in the chiral limit remains non-zero for an infinitely sized extra dimension at a strong gauge coupling corresponding to $a^{-1} \simeq 1$ GeV on a $12^3 \times 24$ lattice. We have also found that this conclusion remains unchanged for the RG-improved gauge action.

One possibility for the origin of a non-zero pion mass is finite spatial size effects, which is being checked by increasing the spatial size from 12 to 16. Another possibility is that it is an artifact of the linear chiral extrapolation, which does not take into account the possible presence of quenched chiral logarithms. To explore this possibility, the chiral breaking term in the axial Ward-Takahashi identity [2], which we expect to be free from the quenched singularities, is currently being investigated.

Finally it is possible that no chiral zero modes exist at $a^{-1} \simeq 1$ GeV, and therefore the domain-wall formalism fails to realize chiral symmetry in the region of strong coupling. A plausible explanation for this failure might be that the range of M for zero modes to exist disappears for stronger couplings, where there is no gap of eigenvalues for the 4-dimensional Wilson operator [7].

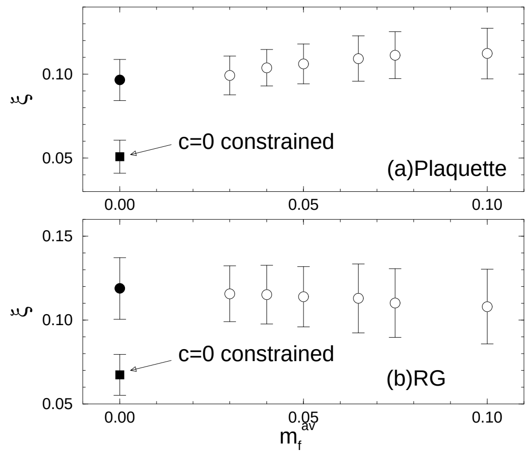

Figure 4. Decay rate ξ from the curves of Figs. 2 for the plaquette (a) and RG (b).

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