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## Hadron Spectrum from Lattice QCD

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A brief review is given of the lattice QCD calculation of the hadron spectrum. The status of current attempts toward inclusion of dynamical up, down and strange quarks is summarized focusing on our own work. Recent work on the possible existence of pentaquark states are assessed. We touch upon the PACS-CS Project for building our next machine for lattice QCD, and conclude with a near-term physics and machine prospects.

*Keywords:* lattice QCD; hadron spectrum; pentaquark

### 1. Introduction

The spectrum of hadrons is a fundamental entity for lattice QCD elucidation of the dynamics of the strong interactions. Calculating the spectrum of standard hadrons such as pion, nucleon,  $J/\psi$ ,  $\Upsilon$ , and  $D$ 's and  $B$ 's provides a basic mean for confirmation of the theory and a benchmark for the lattice calculational methods. Finding the spectrum of non-standard hadrons, *e.g.*, glueballs, hybrids and multi-quark states, offers chances for predictions and thereby further tests of QCD. For these reason the hadron spectrum has been repeatedly and routinely calculated over the years with increasing accuracy and sophistication.

At present, the focus of spectrum calculations and many other subjects with phenomenological impact centers around dynamical simulations including the sea quark effects of all three light quarks, up, down and strange. There are two major attempts, one using the staggered quark action<sup>2</sup> and the other using the Wilson-clover quark action<sup>1</sup>. In addition, a third attempt with the domain-wall quark action, made possible with the commissioning of QCDOC with 10Tflops-class capability, are beginning<sup>3</sup>. In Sec. 2 we discuss results from our own attempt on the light hadron spectrum and quark masses, and recent preliminary results on heavy quark systems.

Search for possible pentaquark states has occupied a lot of experimental and theoretical effort. A number of lattice QCD simulations have also been made to see if there is signal indicating their theoretical presence. A crucial and special

2 *Akira Ukawa*

issue with the pentaquark concerns distinguishing bound or resonance states from scattering states. Recent lattice studies focused on applying theoretical criteria on this point, and we shall review them in Sec. 3. We conclude with a brief summary in Sec. 4.

## 2. $N_f = 2 + 1$ lattice QCD simulations with the Wilson-clover quark action

There are two major lattice QCD research collaborations in the Tsukuba area in Japan, the CP-PACS Collaboration based at University of Tsukuba and the JLQCD Collaboration based at KEK. Since 2001 the two collaborations have jointly pursued a project to carry out simulations including dynamical up, down and strange quarks. The strategy adopted, and the necessary preparations carried out prior to actual runs, is as follows: (i) use Iwasaki RG-improved gluon action to span a range of lattice spacing toward coarse lattices and avoid the artificial phase transition observed for the plaquette gluon action<sup>4</sup>, (ii) use Wilson-clover quark action with a fully  $O(a)$ -improved clover coefficient calculated with the Schrödinger functional methods for three dynamical flavors<sup>5</sup>, and (iii) apply the polynomial HMC to handle the strange quark<sup>6</sup> in addition to the standard HMC for the up and down quarks which are treated as degenerate.

In the table below we list the parameters of our simulations. Runs are made at three values of lattice spacing equally spaced in  $a^2$ . At each lattice spacing, five values are taken for the degenerate up and down quark mass in the range  $m_{PS}/m_V \approx 0.6 - 0.8$ , and two values for the strange quark mass at  $m_{PS}/m_V \approx 0.7$ . Hadron masses calculated for these quark masses are fitted with a general quadratic polynomial of VWI quark masses  $m^{VWI} = (K^{-1} - K_c^{-1})/2$ , and are extrapolated to the physical point defined either by the experimental  $\pi, \rho$  and  $K$  meson masses ( $K$ -input) or  $\pi, \rho$  and  $\phi$  meson masses ( $\phi$ -input). The agreement of the lattice spacing determined by the two types of input as seen in Table 2 provides a simple but important check on the internal consistency of the  $N_f = 2 + 1$  calculation, which is not realized in quenched and  $N_f = 2$  simulations.

### 2.1. Light quark sector

In Fig. 1 we plot the continuum extrapolation of meson masses in quenched(triangles),  $N_f = 2$ (squares) and the present  $N_f = 2 + 1$ (circles)

Parameters of our simulations					
$\beta$	size	$a$ [fm] ( $K$ -input)	$a$ [fm] ( $\phi$ -input)	trajectory	
1.83	$16^3 \times 32$	0.1222(17)	0.1233(30)	7000 – 8600	
1.90	$20^3 \times 40$	0.0993(19)	0.0995(19)	5000 – 9200	
2.05	$28^3 \times 56$	0.0758(48)	0.0755(48)	3000 – 4000	

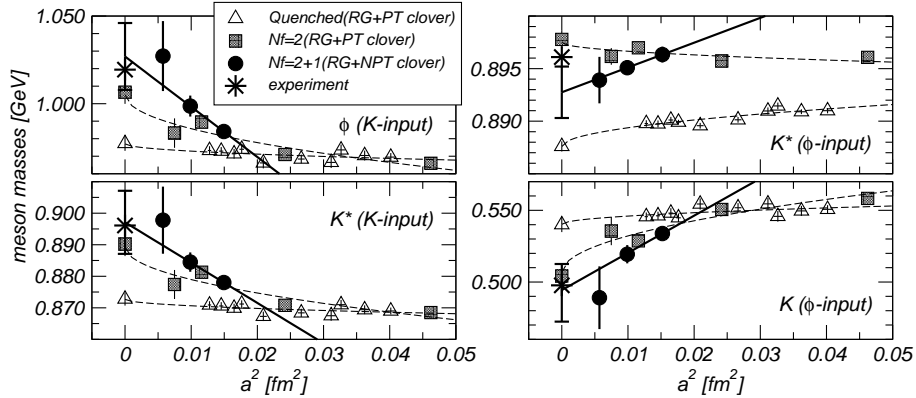


Fig. 1. Continuum extrapolation of meson masses, compared with those for quenched and  $N_f = 2$  QCD<sup>7</sup>. Note that the quenched and  $N_f = 2$  simulations are made with the one-loop perturbatively  $O(a)$ -improved clover action. Thus extrapolations are made linearly in  $a$ .

simulations<sup>8</sup>. The solid lines for the  $N_f = 2 + 1$  data are pure quadratic fits to the results for the two coarse lattice spacings for which runs and measurements have been completed. The agreement with experiment in the continuum limit is encouraging, but we need further data at the finest lattice spacing for a solid conclusion.

Figure 2 shows the continuum extrapolation for the light quark masses<sup>8</sup>. Values sizably smaller than the quenched estimate as has been strongly suggested in the previous  $N_f = 2$  simulations are confirmed. Again, we wait completion of the analyses at the finest lattice spacing to quote the final result for light quark masses in the continuum limit.

An important issue with the analyses in the light quark sector concerns chiral extrapolation. Since Wilson-clover quark action involves explicit chiral symmetry breaking, the chiral behavior of physical quantities deviate from that in the continuum. The Wilson chiral perturbation theory<sup>9</sup> provides the procedure to work out the chiral behavior for finite lattice spacings for Wilson-type actions, and work is under way to calculate the consequence of the approach for the spectroscopic quantities including pseudo scalar meson masses, decay constants, and quark masses on the one hand, and to analyze data based on those results.

## 2.2. Heavy quark sector

Calculating the physical quantities of hadrons containing heavy quarks is important to constrain the parameters of the Standard Model and to help explore physics at finer scales. A serious obstacle is large systematic errors of form  $O(m_H a)$  where  $m_H$  denotes the heavy quark mass, which are large at currently accessible lattice spacings. A standard lore for overcoming this problem is to resort to heavy quark effective theory approach such as NRQCD. We wish to pursue a different approach

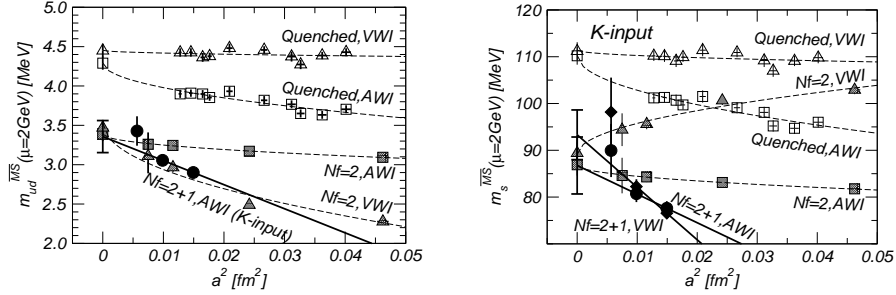


Fig. 2. Continuum extrapolations of the up, down and strange quark masses obtained with the  $K$ -input. The data at the finest lattice is not included in the continuum extrapolations. For comparison, results for quenched and  $N_f = 2$  QCD are overlaid.

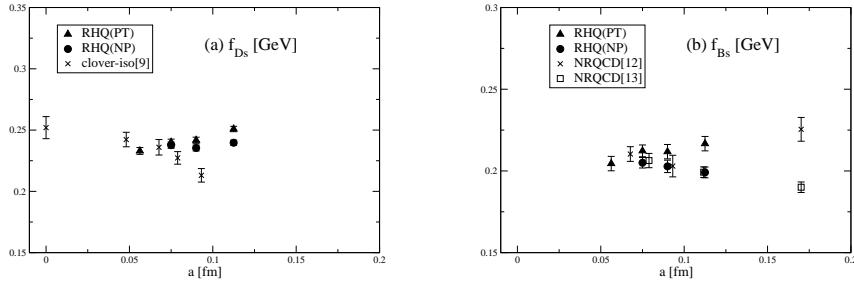


Fig. 3. Decay constant for (a)  $D_s$  and (b)  $B_s$  in quenched QCD from relativistic heavy quark approach<sup>11</sup> as compared to those of effective theories<sup>12,13,14</sup>.

in which parameters of the quark action are tuned demanding Lorentz invariance of on-shell quantities for arbitrary large  $m_H a$  to  $O(a)^{10}$ . This approach allows the continuum extrapolation to be taken in contrast to effective theory approaches.

We have been testing this approach for charm and bottom systems on  $N_f = 2$  and quenched configurations prior to applying it to  $N_f = 2 + 1$  QCD. In Fig. 3 we show results for the decay constant of  $D_s$  and  $B_s$  mesons in quenched QCD<sup>11</sup> and compare them with those of effective theory approaches<sup>12,13,14</sup>. Mild cutoff dependence of the present results and consistency at finite lattice spacings indicate success of our approach.

### 2.3. The next step

At present the largest limitation in our data is a rather large value of the up and down quark mass  $m_{ud}$ . In terms of the physical strange quark mass  $m_s^{phys}$ , it only goes down to  $m_{ud} \approx m_s^{phys}/2$  while experimentally  $m_{ud} \approx m_s^{phys}/25$ . We wish to reduce the value to at least  $m_{ud} \approx m_s^{phys}/5$  or below in order to control the chiral behavior.

This requires both an enhancement of the computing power and an improvement

of the algorithm. The situations in both respects are very promising at present. For the latter, the domain-decomposition acceleration of HMC<sup>15</sup> offers a very promising resolution. On the latter, in December 2004, Japanese Government formally approved our proposal to develop a massively parallel cluster at the Center for Computational Physics, University of Tsukuba. We list in table below the specification of the cluster, which is named PACS-CS (Parallel Array Computer System for Computational Sciences). The development is well under way, and the installation and start of operation are scheduled in June-July of 2006. We plan to combine these developments to break through the current limitations in our Wilson-clover program.

### 3. Lattice pentaquark search

A standard lattice QCD calculation of the mass of a hadron starts with a preparation of the operators having the desired quantum numbers and an examination of the large-time behavior of the two-point function to extract the ground state signal. With lattice pentaquark searches, we have to distinguish a possible pentaquark bound state or resonance from the nucleon-kaon scattering state. Hence, we need to disentangle at least two states in the correlation function, and we also have to distinguish a bound state/resonance from scattering states. In addition, since the spin-parity of the pentaquark state is yet unknown, we have to explore over a large operator space.

The initial lattice studies<sup>16</sup> did not explore these points in detail. Subsequent calculations<sup>17,18</sup> addressed them using two techniques. For multi-state analyses, a variational method using a set of operators and diagonalizing the normalized correlator matrix has been known for a long time. This method has been employed in a number of recent pentaquark searches, and has been reasonably successful. For distinguishing a scattering state, a basic strategy is to examine the spatial size dependence of the energy eigenvalues obtained in the multi-state analysis. A scattering state, if the interactions between the scattering hadrons are weak as in the case of the nucleon and kaon for the pentaquark case, would show a size dependence

Design specification of PACS-CS		
Number of nodes	2560	Node configuration
Peak performance	14.3 Tflops	single LV Xeon 2.8GHz 5.6Gflops
Total memory	5 TByte	2 GByte memory 6.4GByte/s
Total disk space	0.41 PByte	160 GByte×2(RAID1) local disk
Interconnect	16 × 16 × 10	Interconnect bandwidth
	3-dim hyper-crossbar	250MByte/s×2/link
OS	Linux and SCore	750MByte/s×2/node
Programming	Fortran90, C, C++, MPI	
System size	59 racks	Estimated power 545 kW

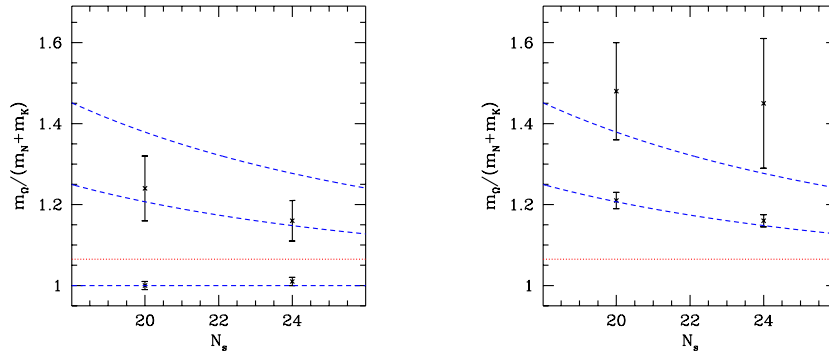


Fig. 4. Spatial size dependence of energies in the pentaquark channel in quenched QCD<sup>18</sup>.

expected from the sum of two energies  $\sqrt{m^2 + (2\pi\ell/L)^2}$  with  $\ell = 1, 2, 3, \dots$ . Another criteria is to look at the residue of the state in question in the two point function :  $\langle O(t)O(0) \rangle \xrightarrow{\infty} Z \exp(-mt)$ . For a scattering state we expect the overlap of the two particles decreasing as  $Z \propto 1/V$ .

In Fig. 4 we reproduce the spatial size dependence of the energies in the  $1/2^-$  and  $1/2^+$  channel calculated with five operator basis in quenched QCD<sup>18</sup>. The dashed lines indicate the size dependence expected for scattering states, and the horizontal dotted lines show the experimental value quoted for the  $\Theta^+$  state. As the figure indicates, the authors conclude that there is no evidence for the existence of pentaquark states in their data.

While conclusions are not unanimous among the studies<sup>17,18</sup>, it is our view that the data published to date are more consistent with the absence of pentaquark states at present. Caution should be stated that all data so far have been generated in quenched QCD, mostly at a fixed lattice spacing and at relatively large quark masses. Truly realistic tests with dynamical light quarks with large volume and small lattice spacing require further work in the future.

#### 4. Conclusions

Currently lattice QCD simulations are in a state of transition. Realistic simulations including dynamical up, down and strange quarks are becoming routine, and chirally invariant quark actions are beginning to be used increasingly frequently. This trend will accelerate as computers with 10 Tflops-class capability, starting with QCDOC and followed by ApeNEXT, PACS-CS and also BlueGene/L at several institutions, becomes available. We hope that single hadron properties will be fully understood, inviting us to venture into the World beyond, in the near future.

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## References

1. T. Kaneko *et al.* [CP-PACS and JLQCD Collaborations], Nucl. Phys. **B** (Proc. Suppl.) **129** (2004) 188; T. Ishikawa *et al.* [CP-PACS and JLQCD Collaborations], Nucl. Phys. **B** (Proc. Suppl.) **140** (2005) 225.
2. C. Aubin *et al.* [HPQCD Collaboration, MILC Collaboration and UKQCD Collaboration], Phys. Rev. **D70** (2004) 031504(R); Phys. Rev. **D70** (2004) 114501; A. Kronfeld, these proceedings.
3. R. Mawhinney, parallel talk at Lattice 2005 (to appear in the proceedings).
4. S. Aoki *et al.* [JLQCD Collaboration], To be published in Phys. Rev. **D**.
5. S. Aoki *et al.* [CP-PACS and JLQCD Collaborations], [hep-lat/0508031](#).
6. S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. **D65** (2002) 094507.
7. A. Ali Khan *et al.* [CP-PACS Collaboration], Phys. Rev. **D65** (2002) 054505; Erratum, **D67** (2003) 059901.
8. T. Ishikawa *et al.*, [CP-PACS and JLQCD Collaboration], [hep-lat/0509142](#) (2005).
9. S. Aoki, Phys. Rev. **D68** (2003) 054508; S. Aoki *et al.*, [hep-lat/0509049](#).
10. S. Aoki, Y. Kuramashi and S. Tominaga, Prog. Theor. Phys. **109** (2003) 383.
11. Y. Kuramashi, parallel talk at Lattice 2005 (to appear).
12. A. Jüttner and J. Rolf, Phys. Lett. **B560** (2003) 59.
13. S. Collins *et al.*, Phys. Rev. **D63** (2001) 034505.
14. K-I. Ishikawa *et al.*, [JLQCD Collaboration], Phys. Rev. **D61** (2000) 074501.
15. M. Lüscher, [hep-lat/0409106](#); these proceedings.
16. F. Csikor, Z. Fodor, S. D. Katz and T. G. Kovacs, JHEP **0311**, 070 (2003); S. Sasaki, Phys. Rev. Lett. **93**, 152001 (2004).
17. N. Mathur *et al.*, Phys. Rev. **D70**, 074508 (2004); N. Ishii *et al.*, Phys. Rev. **D71**, 0340001 (2005); T. W. Chiu and T. H. Hsieh, Phys. Rev. **D72**, 034505 (2005); [hep-ph/0501227](#); B. G. Lasscock *et al.*, [hep-lat/0504015](#); Phys. Rev. **D72**, 104502 (2005); C. Alexandrou and A. Tsapalis, [hep-lat/0503013](#); T. T. Takahashi, T. Umeda, T. Onogi and T. Kunihiro, Phys. Rev. **D71**, 114509 (2005); K. Holland and K. Juge, [hep-lat/0504007](#).
18. F. Csikor, Z. Fodor, S.D. Katz, T.G. Kovács, and B.C. Tóth, [hep-lat/0503012](#).