Pion-nucleon σ term in lattice QCD

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We calculate both the connected and disconnected contributions to the π-N σ term in quenched lattice QCD with the Wilson quark action on a $12^3 \times 20$ lattice at $\beta = 5.7$ with the lattice spacing $a \approx 0.14$ fm. The latter is evaluated with the variant wall source method, previously applied successfully for π-π scattering lengths and the η' meson mass. We found the disconnected contribution to be about twice as large as the connected one. The value for the full π-N σ term $\sigma = 40$–60 MeV is consistent with the experimental estimates. The nucleon matrix element of the strange quark density $\bar{s}s$ is fairly large in our result.

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The study of the π-N σ term defined as the nucleon matrix element of the scalar density

$$\sigma = \frac{m_u + m_d}{2} \langle N|\bar{u}u + \bar{d}d|N\rangle \tag{1}$$

has a long history and still shows some controversy. Current algebra and PCAC (partial conservation of axial vector current) relate the σ term to the quantity $\Sigma = f_\pi^2 \bar{D}^+(2m_\pi^2)$ where $\bar{D}^+(2m_\pi^2)$ is the crossing even π-N scattering amplitude at the Cheng-Dashen point $t = 2m_\pi^2$, and a dispersion analysis of π-N scattering leads to the value $\Sigma = 64(8)$ MeV [1]. On the other hand, treating flavor symmetry breaking to first order, one finds $\sigma = \sigma_0/(1-y)$ with $\sigma_0 \approx 25$ MeV and $y = 2\langle N|\bar{s}s|N\rangle/\langle N|\bar{u}u + \bar{d}d|N\rangle$ [2]. The two estimates cannot be reconciled unless the nucleon has an unexpectedly large strangeness content $y \approx 0.6$. A possible resolution is that the variation of the π-N amplitude from the Cheng-Dashen point $t = 2m_\pi^2$ to $t = 0$, where the σ term (1) is defined, is substantial, resulting in a smaller value $\sigma \approx 45$ MeV [3]. Combined with the suggestion [4] that one-loop chiral perturbative corrections raise the value of σ_0 to $\sigma_0 = 35$ MeV, this would imply a more reasonable value $y \approx 0.2$ of the nucleon strangeness content.

In principle, lattice QCD provides a more direct and more satisfactory route for resolving the issue through a numerical calculation of the σ term matrix element. A serious technical obstacle, however, has been that evaluation of the disconnected amplitude of quark loops and nucleon propagators projected onto the zero-momentum state shown in Fig. 1(a) would require a prohibitively large number of quark matrix inversions if one were to carry it out with the conventional method of point source. In this work we show that the variant of the method of wall source without gauge fixing, which has been successfully applied in the calculation of π-π scattering lengths [5] and the η' meson mass [6], also works well for the disconnected contribution to the σ term. Employing the Wilson quark action, we find an encouraging result, $\sigma = 43(7)$ MeV, from an estimate of the matrix element and the up- and down-quark masses, or $\sigma = 58(11)$ MeV from that of $m_N\sigma/m_\pi^2$, albeit within quenched QCD at a fairly strong coupling of $\beta = 5.7$ with the lattice spacing $a \approx 0.14$ fm.

Let us note that previous estimates of the connected contribution in quenched QCD yielded values in the range 15–25 MeV [7]. The derivative $m_q dm_N/dm_q$ in quenched QCD, equal to the σ term via the Hellmann-Feynman theorem, also gave similar values [8]. These results were considered reasonable; sea quarks are absent in the quenched approximation, and hence one expects $\sigma_{\text{connected}} \approx \sigma_0 = 25$ MeV. This already suggested that the disconnected contribution is substantial. Indeed we find $\sigma_{\text{disconnected}}/\sigma_{\text{connected}} = 2.23(52)$. Previous attempts for estimating the disconnected contribution,

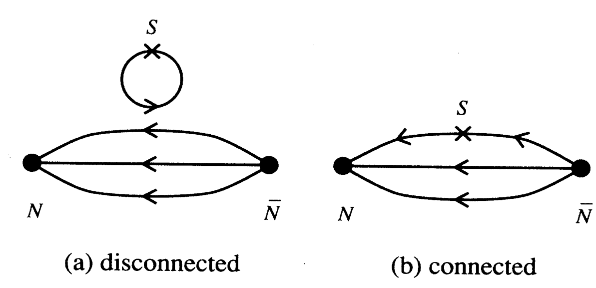

FIG. 1. (a) Disconnected and (b) connected contributions to the nucleon-scalar density three-point function.

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made either indirectly through a comparison of the slope $m_q dm_N/dm_q$ and the connected contribution in full QCD [9] or through noisy estimators for the quark loop [10,11], indicated a similar magnitude of the disconnected contribution, albeit with large errors. We should add that the large disconnected contribution poses a problem regarding the strangeness content, which we shall comment upon toward the end of this work.

To extract the nucleon matrix element of the scalar density $S = \bar{u}u + \bar{d}d$ we calculate the ratio of the three-point function of the nucleon and scalar density to the nucleon two-point function, each projected onto the zero-momentum state [7]:

$$R(t) = \frac{\langle N(t) \sum_{t' \neq 0} S(t') \bar{N}(0) \rangle}{\langle N(t) \bar{N}(0) \rangle} \underset{\text{large } t}{\sim} \text{const} + Z_S^{-1} \langle N | \bar{u}u + \bar{d}d | N \rangle t \ , \quad (2)$$

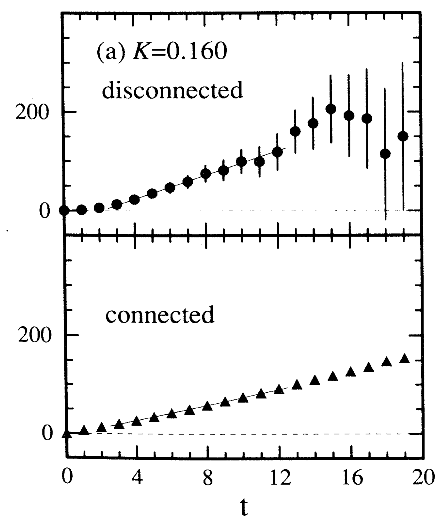
with Z_S the lattice renormalization factor for the scalar density. The $t = 0$ time slice is excluded from the sum in the numerator for the reason explained below. This procedure affects only the constant term in (2) since the term linear in t arises from the sum over the range $t \gg t' \gg 0$ where the nucleon dominates the three valence quark state. The connected amplitude [Fig. 1(b)] can be calculated by the conventional source method [12]. To handle the disconnected piece [Fig. 1(a)] we prepare a quark propagator evaluated with unit source at every space-time site (except for the $t = 0$ time slice) without gauge fixing [6]:

$$G(\mathbf{n}', t') = \sum_{(\mathbf{n}'', t'' \neq 0)} G(\mathbf{n}', t'; \mathbf{n}'', t'') \ . \quad (3)$$

The product of the nucleon propagator and $\sum_{(\mathbf{n}', t' \neq 0)} \mathrm{Tr}[G(\mathbf{n}', t')]$ equals the disconnected amplitude up to gauge-variant nonlocal terms which cancel out in the average over gauge configurations. The superior feature of this method is that it requires only two quark matrix inversions for each gauge configuration. We note that with the exclusion of the $t'' = 0$ term in (3) and that of the $t' = 0$ in the sum $\sum_{(\mathbf{n}', t' \neq 0)} \mathrm{Tr}[G(\mathbf{n}', t')]$, the Fierz mixing of the quark propagator (3) for the disconnected loop and nucleon valence quark propagators is automatically avoided. This is the reason why the sum in (2) is restricted to $t' \neq 0$.

We have applied the method for the Wilson quark action in quenched QCD at $\beta = 5.7$ on a $12^3 \times 20$ lattice. We analyzed 300 configurations for the hopping parameter $K = 0.160$, 0.164, and 400 configurations for 0.1665, generated with the single plaquette action separated by 1000 pseudo-heat-bath sweeps. In order to avoid contamination from the negative-parity partner of the nucleon propagating backward in time we employ the Dirichlet boundary condition in the temporal direction for quark propagators, fixing gauge configurations on the $t = 0$ time slice to the Coulomb gauge in order to enhance nucleon signals. We used the relativistic nucleon operator $N = ({}^t q C^{-1} \gamma_5 q) q$. Errors are estimated by the single elimination jackknife procedure.

In Fig. 2 the ratio $R(t)$ is plotted for the connected and disconnected amplitudes as a function of t for the case of $K = 0.160$ and 0.1665. For the connected amplitude we observe a clear linear increase with very small errors of less than 1–2 % up to $t \approx 10$ as has been known from previous work [7,9]. Signals are also reasonable, albeit worse, for the disconnected amplitude, and are consistent with a linear behavior in t up to $t \approx 10$. The slope of the disconnected amplitude is even larger than that of the connected one, indicating a substantial contribution from the disconnected amplitude to the σ term. To extract the scalar density matrix element we fit the data to the linear form (2) with the fitting range chosen to be



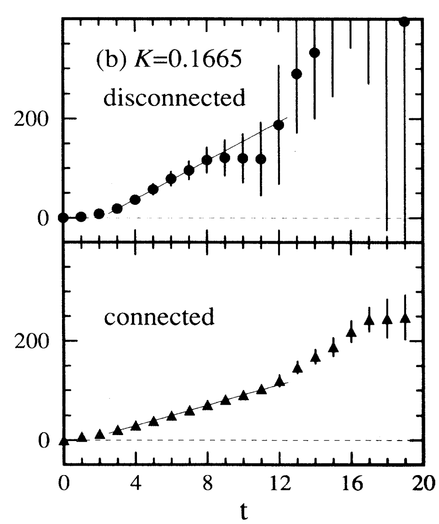


FIG. 2. Ratio $R(t)$ for the disconnected (circles) and connected (triangles) amplitudes at $\beta = 5.7$ on a $12^3 \times 20$ lattice. Solid lines are linear fits over $4 \leq t \leq 9$. (a) $K = 0.160$ and (b) $K = 0.1665$.

TABLE I. Disconnected and connected contributions to the σ term matrix element $\langle N|\bar{u}u+\bar{d}d|N\rangle$ at $\beta = 5.7$ on a $12^3\times20$ lattice in quenched QCD. Results are corrected by the tadpole-improved Z factor. Values at K_c are obtained by a linear fit in $1/K$.

K	0.160	0.164	0.1665	K_c=0.1694
m_q	0.173	0.097	0.051	
No. conf.	300	300	400	
m_π/m_ρ	0.85	0.74	0.60	
m_N	1.2957(79)	1.080(10)	0.926(13)	0.783(16)
$\langle N\|\bar{u}u+\bar{d}d\|N\rangle_{\rm conn}$	2.323(15)	2.413(30)	2.693(81)	2.615(61)
$\langle N\|\bar{u}u+\bar{d}d\|N\rangle_{\rm disc}$	3.56(76)	4.58(92)	5.14(1.10)	5.8(1.4)

$4 \le t \le 9$. In Table I, we tabulate the fitted values of the matrix elements corrected by the tadpole-improved renormalization factor in the modified minimal subtraction ($\overline{\rm MS}$) scheme at the scale $\mu = 1/a$ given by [13]

$$Z_S = \left[1 + \alpha_s\left(\frac{2}{\pi}\ln a\mu - 0.0098\right)\right]\left(1 - \frac{3K}{4K_c}\right), \quad (4)$$

where we used $\alpha_{\overline{\rm MS}}(1/a) = 0.2207$ for α_s.

We present the quark mass dependence of the matrix element $\langle N|\bar{u}u+\bar{d}d|N\rangle$ in Fig. 3 as functions of the bare quark mass $m_q = (1/K - 1/K_c)/2$ using $K_c = 0.1694$ [14]. The disconnected contribution (circles) is about twice as large as the connected contribution and increases almost linearly toward small quark masses, while the connected contribution remains constant. Extrapolating linearly in the quark mass, we find $\langle N|\bar{u}u+\bar{d}d|N\rangle = 5.8(1.4)$ for the disconnected contribution, 2.62(6) for the connected piece, and 8.6(1.4) for the sum at $m_q = 0$. The value for the connected contribution is reasonably consistent with the value of the derivative $dm_N/dm_q = 2.97(11)$ obtained by fits of the nucleon mass in the range $K = 0.160 - 0.1665$ where our calculations are made.

An estimate of the σ term in physical units requires the value of the bare mass $\hat{m} = (m_u + m_d)/2$ for the up and down quark and the physical scale of lattice spacing. (The renormalization factor for the quark mass equals the inverse of that for the scalar density to all orders in perturbation theory, and hence the two factors cancel out in the σ term. This point is taken into account in our estimate of σ below.) From fits of the meson spectrum obtained on our set of gauge configurations, we find $m_\pi^2 = 2.710(32)\hat{m}$, $m_\rho = 0.5294(86) + 1.593(61)\hat{m}$, from which we obtain $\hat{m} = 0.00342(12)$ and $a^{-1} = 1.45(2)$ GeV using $m_\pi = 140$ MeV and $m_\rho = 770$ MeV. These values yield $\sigma = 43(7)$ MeV, showing an encouraging agreement with the dispersion theoretic estimate of the σ term. Alternatively one can form the dimensionless ratio $m_N\sigma/m_\pi^2$ at each quark mass and extrapolate to the chiral limit. The ratio turned out to depend little on m_q, and a linear extrapolation gives $m_N\sigma/m_\pi^2 = 2.80(53)$ or $\sigma = 58(11)$ MeV with the physical nucleon and pions masses. The difference between the two values of σ originates from the fact that $m_N/m_\rho = 1.48(4)$ at $\beta = 5.7$, taken for the present simulation, is larger than the experimental value 1.22.

Let us remark that scaling violation effects and the use of quenched QCD are the obvious sources of systematic uncertainties in the results above. In this connection we note that the physical value of the light quark mass for Wilson quark action shows a substantial decrease toward smaller lattice spacing in quenched QCD and that the full QCD values are a factor 2–3 smaller than those for quenched QCD [15]. Whether these affects the physical value of the σ term for smaller lattice spacings and in full QCD poses an important issue for further studies.

We now turn our attention to the nucleon matrix element of the strange quark density $\langle N|\bar{s}s|N\rangle$. This matrix element receives contribution only from the disconnected amplitude. For an extraction of its physical value one needs to extrapolate the valence quark mass to the chiral limit keeping the strange quark mass fixed. In Table II, we summarize the results for the matrix element $\langle N|\bar{s}s|N\rangle$ corrected by the tadpole-improved Z factor relevant for such an analysis. Results of a linear extrapolation to the chiral limit for the valence quark are also listed in Table II. Fitting the extrapolated values for three values of K_s we obtain $\langle N|\bar{s}s|N\rangle_{K_{\rm val}=K_c} = 2.93(93) - 1.1(7.9)m_s$.

The strange quark mass may be estimated by gen-

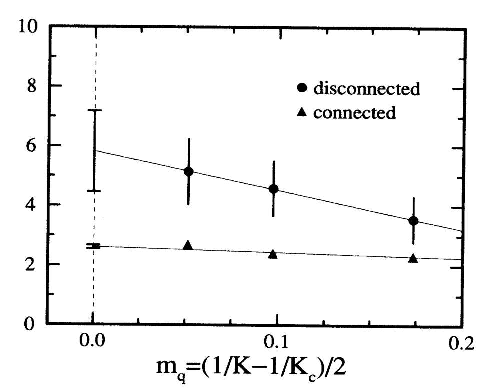


FIG. 3. Disconnected (circles) and connected (triangles) contribution to the matrix element $\langle N|\bar{u}u+\bar{d}d|N\rangle$ as a function of the quark mass $m_q = (1/K - 1/K_c)/2$ in lattice units. Solid lines are linear fits for extrapolation to the chiral limit $m_q = 0$.

TABLE II. $\langle N|\bar{s}s|N\rangle$ as a function of the hopping parameters of valence and strange quarks obtained at $\beta = 5.7$ on a $12^3\times 20$ lattice in quenched QCD with 300 gauge configurations. Results are corrected by the tadpole-improved Z factor.

	$\langle N\|\bar{s}s\|N\rangle$		
$K_{\rm val}$	$K_s = 0.160$	$K_s = 0.164$	$K_s = 0.1665$
0.160	1.78(38)	2.02(38)	2.20(39)
0.164	2.04(47)	2.29(46)	2.49(45)
0.1665	2.50(55)	2.81(55)	2.57(55)
K_c=0.1694	2.67(69)	2.99(68)	2.77(68)

eralizing the relation $m_\pi^2 = 2.710(32)\hat{m}$ to $m_K^2 = 2.710(32)(\hat{m} + m_s)/2$ and using the experimental ratio $m_K/m_\rho = 0.64$, which yields $m_s = 0.0831(30)$ or $K_s = 0.1648$. We then obtain $\langle N|\bar{s}s|N\rangle = 2.84(44)$. Two quantities of physical interest that can be estimated from this result are the strange quark contribution to the nucleon mass $m_s\langle N|\bar{s}s|N\rangle/m_N = 0.302(48)$ and the K-N σ term

$$\begin{aligned}\sigma_{KN} &= (\hat{m} + m_s)\langle N|\bar{u}u + \bar{d}d + 2\bar{s}s|N\rangle/4 \\ &= 0.310(37) = 451(54)\ \text{MeV}\ .\end{aligned} \tag{5}$$

While these results appear quite reasonable, we need to note that the value of the matrix element $\langle N|\bar{s}s|N\rangle$ itself is large. Combined with our result $\langle N|\bar{u}u + \bar{d}d|N\rangle = 8.6(1.4)$ for the up and down quark, we find

$$y = 2\langle N|\bar{s}s|N\rangle/\langle N|\bar{u}u + \bar{d}d|N\rangle = 0.66(15)\ , \tag{6}$$

while a phenomenological estimate gives $y \approx 0.2$ [3]. We have not identified an apparent origin of systematic errors that lead to the result, perhaps except for a possible scaling violation as our simulation is made at a rather strong coupling. This point should be examined through a repetition of the calculation with a smaller lattice spacing, which we leave for future investigations.

To summarize we have shown that the method of wall source without gauge fixing [5,6] helps to overcome the computational difficulty of yet another quantity, the disconnected contribution to the π-N σ term, for which we obtained σ = 40–60 MeV with a 15% error at a lattice spacing $a \approx 0.14$ fm in quenched QCD. Our results also show a large strangeness content in the nucleon.

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