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付録 A

アナライジングウェーブレットの不確定性関係式

A.1 証明

アナライジングウェーブレット$\psi(t)$は2乗可積分関数であり，規格化条件(2.3)を満足している．さて，任意の2つの関数$\varphi(t)$, $\phi(t)$に対して次の関係式が成立する:

$$\left|\int_R \mathrm{d}t\ \varphi^*(t)\phi(t)\right|^2 \le \int_R \mathrm{d}t\ |\varphi(t)|^2 \int_R \mathrm{d}t\ |\phi(t)|^2. \tag{A.1}$$

これはCauchy-Schwarzの不等式として知られる．ただし，*は複素共役でRは定義域全体を表す．ここで，$\varphi(t) \to t\psi(t)$, $\phi(t) \to \psi'(t)$とおけば($\psi'(t)$はtに関する1回微分)，関係式

$$\left|\int_{-\infty}^{\infty} \mathrm{d}t\ t\psi^*(t)\psi'(t)\right|^2 \le \int_{-\infty}^{\infty} \mathrm{d}t\ t^2|\psi(t)|^2 \int_{-\infty}^{\infty} \mathrm{d}t\ |\psi'(t)|^2 \tag{A.2}$$

を得る．(A.2)右辺の第1積分は(2.8)により$(\Delta t)^2$に等しい．右辺第2積分はParsevalの公式により

$$\int_{-\infty}^{\infty} \mathrm{d}t\ |\psi'(t)|^2 = \int_{-\infty}^{\infty} \mathrm{d}\omega\ \omega^2|\hat{\psi}(\omega)|^2 = (\Delta\omega)^2 \tag{A.3}$$

となる．ここで$\hat{\psi}(\omega)$は$\psi(t)$のフーリエ変換(2.7)であり，また関係式(2.9)を用いた．よって(A.2)右辺は$(\Delta t)^2(\Delta\omega)^2$となる．次に，関数$\psi(t)$を一般に振幅と位相を用いて$\psi(t) = a(t)\exp(i\theta(t))$と表せば(A.2)左辺の被積分関数は部分積分を用いて

$$\begin{aligned} t\psi^*(t)\psi'(t) &= ta(t)a'(t) + it\theta'(t)a^2(t) \\ &= \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}ta^2(t) - \frac{1}{2}a^2(t) + it\theta'(t)a^2(t) \end{aligned} \tag{A.4}$$

となる．(A.4)右辺第1項はその積分が0となる完全微分であり，第2項は規格化条件より1/2，第3項は共分散関数$c \equiv \int_R \mathrm{d}t\ t\theta'(t)a^2(t)$となる．よって(A.2)は

$$\left|-\frac{1}{2} + ic\right|^2 \le (\Delta t)^2(\Delta\omega)^2 \tag{A.5}$$

となる. こうして関係式

$$\left(\frac{1}{2}\right)^2 + c^2 \leq \Delta t^2 \Delta\omega^2 \tag{A.6}$$

を得る. ここで$c \geq 0$より任意のアナライジングウェーブレット$\psi(t)$に対して, t及びωの標準偏差間には不確定性関係式

$$\frac{1}{2} \leq \Delta t \Delta\omega \tag{A.7}$$

が成立する. ■

A.2 双曲線関数

双曲線関数(2.15)は時間と空間の関数である非線形シュレーディンガー方程式

$$i\psi_t + \frac{1}{2}\psi_{xx} + |\psi|^2\psi = 0 \tag{A.8}$$

の1–ソリトン解に他ならない(Hasegawa and Kodama, 1995). 一般に, ガリレイ変換

$$x \to x - \kappa t \tag{A.9}$$

(ここでκは定数)に対して不変な1–ソリトン解は

$$\psi(x,t) = \eta\,\mathrm{sech}\{\eta(x-\kappa t - x_0)\}\exp\left\{i\kappa x + i\frac{(\eta^2-\kappa^2)}{2}t + i\sigma\right\} \tag{A.10}$$

と与えられる. ここでη, x_0, σは任意定数である. このとき, $\psi(x,t)$のフーリエ変換は

$$\begin{aligned}\hat{\psi}(p,t) &= \sqrt{\frac{\pi}{2}}\,\mathrm{sech}\{\frac{\pi}{2\eta}(p-\kappa)\} \\ &\quad \times \exp\left\{-i\frac{(\kappa p - \eta^2 - \kappa^2)}{2}t - i(p-\kappa)x_0 + i\sigma\right\}\end{aligned} \tag{A.11}$$

となる(Saji and Konno, 1998b). 一般性を失うことなく(A.10), (A.11)において$\sigma = 0$, $t = 0$とし, また規格化因子C_0を導入する. このとき, 位置xとその2乗x^2に関する平均(期待値)はそれぞれ

$$\langle x\rangle = 2C_0^2\eta x_0, \tag{A.12}$$

$$\langle x^2\rangle = \frac{C_0^2\pi^2}{6\eta} + 2C_0^2\eta x_0^2 \tag{A.13}$$

と与えられる. 同様にして運動量pとその2乗p^2に関する期待値は

$$\langle p\rangle = 2C_0^2\eta\kappa, \tag{A.14}$$

$$\langle p^2\rangle = \frac{2}{3}C_0^2\eta^3 + 2C_0^2\eta\kappa^2 \tag{A.15}$$

と与えられる. それゆえ, xに関する標準偏差とpに関する標準偏差は以下の関係式

$$\Delta x \equiv \sqrt{\langle x^2\rangle - \langle x\rangle^2}, \tag{A.16}$$

$$\Delta p \equiv \sqrt{\langle p^2\rangle - \langle p\rangle^2} \tag{A.17}$$

を用いて

$$(\Delta x)^2 = C_0^2(\frac{\pi^2}{6\eta} + 2\eta x_0^2 - 4C_0^2\eta^2 x_0^2), \tag{A.18}$$

$$(\Delta p)^2 = C_0^2(\frac{2}{3}\eta^3 + 2\eta\kappa^2 - 4C_0^2\eta^2\kappa^2) \tag{A.19}$$

と求められる. ここで規格化条件

$$\int_{-\infty}^{\infty} \mathrm{d}x|\psi(x,t)|^2 = \int_{-\infty}^{\infty} \mathrm{d}\omega|\hat{\psi}(p,t)|^2 = 1 \tag{A.20}$$

により$C_0^2 = 1/2\eta$となる. よって(A.18), (A.19)は

$$(\Delta x)^2 = \frac{\pi^2}{12\eta^2}, \tag{A.21}$$

$$(\Delta p)^2 = \frac{\eta^2}{3} \tag{A.22}$$

となる. こうして, 1–ソリトン解(A.10)に対する不確定性関係式

$$\Delta x\, \Delta p = \frac{\pi}{6} \tag{A.23}$$

を得る(Saji and Konno, 1998b). (A,10)の変数(x, p)を(t, ω)に読み替え, $x_0 = 0$, $\kappa = p_0$, $\eta = \sqrt{p_0/\gamma'}$とすれば(A.10)は双曲線関数(2.15)に他ならない. よって, (A.23)は双曲線関数(2.15)の不確定性関係式である.

付録 B

Pearson 系確率分布関数

B.1 一般形式

方程式(3.10) の一般形式として以下の微分方程式

$$\begin{aligned}\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{(x+a)}{b_0 + b_1 x + b_2 x^2 + \cdots + b_{j'-1}x^{j'-1}} \\ &= \frac{(x+a)}{\sum_j b_j x^j}\end{aligned} \tag{B.1}$$

を考察する. ここでa, b_j (j=0, 1,..., $j'-1$) は定数である. (B.1) 式は以下のように変形できる:

$$(b_0 + b_1 x + b_2 x^2 + \cdots + b_{j'-1}x^{j'-1})\frac{\mathrm{d}y}{\mathrm{d}x} = (x+a)y. \tag{B.2}$$

ここで, 両辺にx^n (n=0, 1, 2,..., $n'-1$) を掛けてxに関して積分をする:

$$\int \mathrm{d}x\, x^n (b_0 + b_1 x + b_2 x^2 + \cdots)\frac{\mathrm{d}y}{\mathrm{d}x} = \int \mathrm{d}x\, x^n (x+a)y. \tag{B.3}$$

このとき, 左辺は部分積分により

$$\begin{aligned}&x^n(b_0 + b_1 x + b_2 x^2 + \cdots + b_{j'-1}x^{j'-1})y \\ &\quad - \int \mathrm{d}x\, \{n x^{n-1} b_0 + (n+1)x^n b_1 + (n+2)x^{n+1} b_2 + \cdots + (n+j'-1)x^{n+j'-2} b_{j'-1}\}y\end{aligned} \tag{B.4}$$

となる. ここでxのn次積率

$$m_n \equiv \int \mathrm{d}x\, x^n y \tag{B.5}$$

を導入し, またxに比べyが十分に速く収束する, すなわち (B.4) の第一項目が0となる

$$x^n(b_0 + b_1 x + b_2 x^2 + \cdots + b_{j'-1}x^{j'-1})y \to 0 \tag{B.6}$$

とすれば(B.5) を用いて関係式

$$m_n a + n m_{n-1} b_0 + (n+1) m_n b_1 + \cdots + (n+j'-1) m_{n+j'-2} b_{j'-1} = -m_{n+1} \tag{B.7}$$

を得る. (B.7) は$j'+1$個の未知数, n個の等式からなる連立一次方程式であり, ここで積率に関する$(n')\times(j'+1)$行列

$$\mathbf{A}=\begin{pmatrix} m_0 & 0 & m_0 & \dots & j'm_{j'-1} \\ m_1 & m_0 & 2m_1 & \dots & (1+j')m_{j'} \\ m_2 & 2m_1 & 3m_2 & \dots & (2+j')m_{j'+1} \\ \vdots & & & \dots & \vdots \\ m_{n'-1} & (n'-1)m_{n'-2} & (n')m_{n'-1} & \dots & (n'+j'-1)m_{j'+n'-2} \end{pmatrix}, \tag{B.8}$$

定数係数ベクトル$\mathbf{v}=(a, b_1, b_2,..., b_{j'-1})^t$, 積率ベクトル$\mathbf{m}=(m_1, m_2,..., m_{n'})^t$を導入すれば(B.7) は

$$\mathbf{Av}=\mathbf{m} \tag{B.9}$$

と表わされる. 行列$\mathbf{A}$が正則行列のとき, 逆行列$\mathbf{A}^{-1}$が存在するので, 定数係数ベクトル$\mathbf{v}$は

$$\mathbf{v}=\mathbf{A}^{-1}\mathbf{m} \tag{B.10}$$

と積率の関数で与えられる. こうして微分方程式(B.1) で記述される関数$y=f(x)$の定数係数a, b_j (j=0, 1,..., $j'-1$) はxの高次積率によって表される. ここで, 一般性を失うことなくm_0, m_1は

$$m_0 = \int \mathrm{d}x\, y = 1 \tag{B.11}$$

$$m_1 = \int \mathrm{d}x\, xy = 0 \tag{B.12}$$

と設定できる. (B.11) は実関数$y=f(x)$の規格化条件に他ならない. また, (B.12) は実関数$y=f(x)$に対してスケール変換(平均値を0にする平行移動) を施すことにより容易に実現できる. このようにしておくことにより (B.10) の計算が容易になる. 以下では(B.11), (B.12) を仮定し, 実際にj'=0, 1, 2の場合について考察する.

(i)j'=0 の時

$$\mathbf{v}=\begin{pmatrix} 0 \\ -m_2 \end{pmatrix} \tag{B.13}$$

より微分方程式は2次の積率を用いて

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{x}{m_2} \tag{B.14}$$

で与えられる.

(ii)j'=1 の時

$$\mathbf{A}=\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ m_2 & 0 & 3m_2 \end{pmatrix} \tag{B.15}$$

を用いて

$$\mathbf{v}=\begin{pmatrix} \frac{m_3}{2m_2} \\ -m_2 \\ -\frac{m_3}{2m_2} \end{pmatrix} \tag{B.16}$$

と求められる. よって解くべき微分方程式は

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{\frac{2m_2}{m_3}x+1}{\frac{2m_2^2}{m_3}+x} \tag{B.17}$$

と与えられる.

(iii)$j'=2$の時

$$\mathbf{A}=\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3m_2 \\ m_2 & 0 & 3m_2 & 4m_3 \\ m_3 & 3m_2 & 4m_3 & 5m_4 \end{pmatrix} \tag{B.18}$$

を用いて

$$\mathbf{v}=\begin{pmatrix} \frac{m_3(m_4+3m_2^2)}{2(5m_2m_4-6m_3^2-9m_2^3)} \\ -\frac{m_2(4m_2m_4-3m_3^2)}{2(5m_2m_4-6m_3^2-9m_2^3)} \\ -\frac{m_3(m_4+3m_2^2)}{2(5m_2m_4-6m_3^2-9m_2^3)} \\ -\frac{2m_2m_4-3m_3^2-6m_2^3}{2(5m_2m_4-6m_3^2-9m_2^3)} \end{pmatrix} \tag{B.19}$$

と求められる. よって微分方程式は4次積率までを用いて

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{x+\frac{m_3(m_4+3m_2^2)}{2(5m_2m_4-6m_3^2-9m_2^3)}}{\frac{m_2(4m_2m_4-3m_3^2)}{2(5m_2m_4-6m_3^2-9m_2^3)}+\frac{m_3(m_4+3m_2^2)}{2(5m_2m_4-6m_3^2-9m_2^3)}x+\frac{2m_2m_4-3m_3^2-6m_2^3}{2(5m_2m_4-6m_3^2-9m_2^3)}} \tag{B.20}$$

と表される. ここでパラメータ$\beta_1=m_3^2/m_2^3$, $\beta_2=m_4/m_2^2$を用いて(B.20)を書き直すと

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{x+\frac{\sqrt{m_2}\sqrt{\beta_1}(\beta_2+3)}{2(5\beta_2-6\beta_1-9)}}{\frac{m_2(4\beta_2-3\beta_1)}{2(5\beta_2-6\beta_1-9)}+\frac{\sqrt{m_2}\sqrt{\beta_1}(\beta_2+3)}{2(5\beta_2-6\beta_1-9)}x+\frac{2\beta_2-3\beta_1-6}{2(5\beta_2-6\beta_1-9)}x^2} \tag{B.21}$$

となる. $j'=2$の場合, 微分方程式(B.1)の解は右辺分母の2次方程式$b_0+b_1x+b_2x^2=0$の解の種類に依存して異なる. そこで2次方程式の解は判別式

$$b_1^2-4b_0b_2 \tag{B.22}$$

の符号判定により, あるいは物理量

$$K=\frac{b_1^2}{4b_0b_2} \tag{B.23}$$

の符号判定により鑑別できる. ここで(B.19)の結果を(B.23)に代入すればKは

$$K=\frac{\beta_1(\beta_2+3)^2}{4(2\beta_2-3\beta_1-6)(4\beta_2-3\beta_1)} \tag{B.24}$$

となる．こうして，判別基準量Kによって微分方程式(B.1)の解の種類を同定することが可能である．こうして$j'=2$のときの解集合$\{y=f(x)\}$はxの高次積率により同定される．実際に，この解集合を確率分布関数系として見れば，非正規性を持った12の確率分布関数に分類でき(Kendall and Buckland, 1982; 東京工業大学統計工学研究会, 1953)，また個々の関数型は評価基準量Kで同定できることから，これらの解集合は積率推定法を用いたPDF推定に応用されている．

B.2 導出

微分方程式(B.1)を

$$\frac{\mathrm{d}\log y}{\mathrm{d}x} = \frac{x+a}{b_0+b_1x+b_2x^2} \tag{B.25}$$

と変形する．

B.2.1 Pearson I 型確率分布関数

(B.25)の右辺分母の関数が，2つの異符号実数解を持つ場合には判別式より以下の関係式

$$b_2b_0 < 0 \tag{B.26}$$

が成り立つ．すなわち判別基準量Kでは

$$K < 0 \tag{B.27}$$

に対応する．このとき，それぞれの解を$-A_1(<0)$, $A_2(>0)$とすれば(B.25)右辺は

$$\begin{aligned}\frac{x+a}{b_0+b_1x+b_2x^2} &= \frac{x+a}{b_2(x^2+\frac{b_1}{b_2}x+\frac{b_0}{b_2})}\\ &= \frac{x+a}{b_2(x-A_2)(x+A_1)}\\ &= \frac{1}{b_2}\left\{\frac{A_1-a}{A_1+A_2}\frac{1}{x+A_1}-\frac{A_2+a}{A_1+A_2}\frac{1}{A_2-x}\right\}\end{aligned} \tag{B.28}$$

と変形できる．ここで(B.28)を(B.25)に代入して両辺をxで積分すれば

$$\log y = \frac{1}{b_2}\frac{A_1-a}{A_1+A_2}\log(x+A_1)+\frac{1}{b_2}\frac{A_2+a}{A_1+A_2}\log(A_2-x)+c \quad (-A_1 \le x \le A_2) \tag{B.29}$$

となる($c=\log f_0$は定数)．(B.29)を整理すれば微分方程式(B.25)の解は

$$y = f_0(x+A_1)^{\frac{1}{b_2}\frac{A_1-a}{A_1+A_2}}(A_2-x)^{\frac{1}{b_2}\frac{A_2+a}{A_1+A_2}} \quad (-A_1 \le x \le A_2) \tag{B.30}$$

と与えられる．ここで変数変換$x = x' - a$, 及び新たなパラメータ$a_1 = A_1 - a$, $a_2 = A_2 + a$, $q_1 = a_1/(a_1 + a_2)$, $q_2 = a_2/(a_1 + a_2)$ を導入して(B.30)を整理すればPearson I 型曲線

$$y = f_0'\left(1 + \frac{x'}{a_1}\right)^{q_1}\left(1 - \frac{x'}{a_2}\right)^{q_2} \quad (a_1 \leq x' \leq a_2) \tag{B.31}$$

を得る．ここで$f_0' = f_0(a_1)^{q_1}(a_2)^{q_2}$とする．

規格化条件(B.11)より(B.31)は

$$\begin{aligned}
1 &= \int_{a_1}^{a_2} \mathrm{d}x'\, f_0'\left(1 + \frac{x'}{a_1}\right)^{q_1}\left(1 - \frac{x'}{a_2}\right)^{q_2} \\
&= \int_{a_1}^{a_2} \mathrm{d}x'\, \frac{f_0'}{(a_1)^{q_1}(a_2)^{q_2}}(a_1 + x')^{q_1}(a_2 - x')^{q_2} \\
&\quad \text{変数変換}\, z = \frac{x' + a_1}{a_1 + a_2}\,\text{を導入} \\
&= \int_0^1 \mathrm{d}z\, \frac{f_0'(a_1 + a_2)}{(a_1)^{q_1}(a_2)^{q_2}}\{z(a_1 + a_2)\}^{q_1}\{(1 - z)(a_1 + a_2)\}^{q_2} \\
&= \frac{f_0'(a_1 + a_2)^{q_1 + q_2 + 1}}{(a_1)^{q_1}(a_2)^{q_2}} \int_0^1 \mathrm{d}z\, z^{q_1}(1 - z)^{q_2} \\
&= \frac{f_0'(a_1 + a_2)^{q_1 + q_2 + 1}}{(a_1)^{q_1}(a_2)^{q_2}} B(q_1 + 1, q_2 + 1) \\
&= \frac{f_0'(a_1 + a_2)^{q_1 + q_2 + 1}}{(a_1)^{q_1}(a_2)^{q_2}} \frac{\Gamma(q_1 + 1)\Gamma(q_2 + 1)}{\Gamma(q_1 + q_2 + 2)} \qquad \text{(B.32)}
\end{aligned}$$

となる．よってf_0'は

$$f_0' = \frac{(a_1)^{q_1}(a_2)^{q_2}}{(a_1 + a_2)^{q_1 + q_2 + 1}} \frac{\Gamma(q_1 + q_2 + 2)}{\Gamma(q_1 + 1)\Gamma(q_2 + 1)} \tag{B.33}$$

と与えられる．ここで

$$\begin{aligned}
B(q_1 + 1, q_2 + 1) &\equiv \int_0^1 \mathrm{d}z\, z^{q_1}(1 - z)^{q_2} \qquad \text{(B.34)} \\
&= \frac{\Gamma(q_1 + 1)\Gamma(q_2 + 1)}{\Gamma(q_1 + q_2 + 2)} \qquad \text{(B.35)}
\end{aligned}$$

を用いた．ゆえにPearson I 型確率分布関数は

$$y = f(x') = \frac{(a_1)^{q_1}(a_2)^{q_2}}{(a_1 + a_2)^{q_1 + q_2 + 1}} \frac{\Gamma(q_1 + q_2 + 2)}{\Gamma(q_1 + 1)\Gamma(q_2 + 1)}\left(1 + \frac{x'}{a_1}\right)^{q_1}\left(1 - \frac{x'}{a_2}\right)^{q_2} \quad (a_1 \leq x' \leq a_2) \tag{B.36}$$

である．また変数変換$z = x + a_1/a_1 + a_2$, 定数$\mu = q_1 + 1$, $\nu = q_2 + 1$ を導入して(B.36)を書き直せば

$$y = f(x) = \frac{\Gamma(\mu + \nu)}{\Gamma(\mu)\Gamma(\nu)} x^{\mu - 1}(1 - x)^{\nu - 1} \quad (0 \leq x \leq 1) \tag{B.37}$$

となる．すなわちPearson I 型確率分布はベータ分布に他ならない．

B.2.2 Pearson IV 型確率分布関数

(B.25) の右辺分母に関数が虚数解を持つ場合, すなわち

$$b_1^2 < 4b_2b_0 \tag{B.38}$$

の時Kは

$$0 < K < 1 \tag{B.39}$$

に対応する. このとき, (B.25) の右辺は以下のように変形できる;

$$\begin{aligned} \text{右辺} &= \frac{x+a}{b_2\{(x+\frac{b_1}{2b_2})^2 - \frac{b_1^2}{4b_2^2} + \frac{b_0}{b_2}\}} \\ &= \frac{1}{b_2}\frac{x'+A}{x'^2+\alpha^2}. \end{aligned} \tag{B.40}$$

ここで新しい変数$x' = x + b_1/2b_2$, $\alpha = -b_1^2/4b_2^2 + b_0/b_2$, $A = a - b_1/2b_2$ を用いた. (B.40) を用いて(B.25) の両辺をx'に関して積分すれば

$$\begin{aligned} \log y &= \frac{1}{b_2}\left\{\int \mathrm{d}x' \frac{x'}{x'^2+\alpha^2} + \int \mathrm{d}x' \frac{A}{x'^2+\alpha^2}\right\} \\ &= \frac{1}{2b_2}\log(x'^2+\alpha^2) + \frac{A}{b_2\alpha}\arctan\left(\frac{x'}{\alpha}\right) + f_0 \end{aligned} \tag{B.41}$$

となる (ここで$f_0 = c/b_2$). (B.41) の両辺を整理してPearson IV 型曲線

$$y = f_0'\left(1+\frac{x'^2}{\alpha^2}\right)^{-\mu}\exp\left\{-\nu\arctan\left(\frac{x'}{\alpha}\right)\right\} \tag{B.42}$$

を得る. ここで, $f_0' = e^{f_0}\alpha^{-b_2}$, $\mu = -1/2b_2$, $\nu = -A/\alpha b_2$ である.

規格化条件(B.11) より

$$\begin{aligned} 1 &= \int_{-\infty}^{\infty}\mathrm{d}x'\, f_0'\left(1+\frac{x'^2}{\alpha^2}\right)^{-\mu}\exp\left\{-\nu\arctan\left(\frac{x'}{\alpha}\right)\right\} \\ &\quad \tan\theta = \frac{x'}{\alpha}\text{とすれば} \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\mathrm{d}\theta\,\alpha f_0'\cos^{2\mu-2}\theta\exp(-\nu\theta) \\ &\quad \text{ここで}\phi = \theta + \frac{\pi}{2}\text{とする} \\ &= \alpha f_0'\exp\left(\nu\frac{\pi}{2}\right)\int_0^{\pi}\mathrm{d}\phi\,\cos^{2\mu-2}\left(\phi-\frac{\pi}{2}\right)\exp(-\nu\phi) \\ &= \alpha f_0'\exp\left(\nu\frac{\pi}{2}\right)\int_0^{\pi}\mathrm{d}\phi\,\sin^{2\mu-2}\phi\exp(-\nu\phi) \\ &= \alpha f_0' f(r,\nu) \end{aligned} \tag{B.43}$$

を得る．ここで$r=2\mu-2$, また関数

$$f(r,\nu) \equiv \exp\left(\nu\frac{\pi}{2}\right)\int_0^{\pi} d\phi\ \sin^r\phi\exp(-\nu\phi) \tag{B.44}$$

を導入した．よって

$$f_0' = \frac{1}{\alpha f(r,\nu)} \tag{B.45}$$

と与えられる．ゆえにPearson IV 型確率分布関数は

$$y = f(x) = \frac{1}{\alpha f(r,\nu)}\left(1+\frac{x^2}{\alpha^2}\right)^{-\mu}\exp\left\{-\nu\arctan\left(\frac{x}{\alpha}\right)\right\} \quad (-\infty \le x \le \infty) \tag{B.46}$$

である．

B.2.3 Pearson VI 型確率分布関数

(B.25) の右辺分母の関数が同符号の2実数解$A_1 < A_2$を持つ場合，すなわち

$$b_1^2 > 4b_2b_0 > 0 \tag{B.47}$$

のとき対応するKは

$$K > 1 \tag{B.48}$$

である．I 型のときと同様にして(B.25) の右辺は以下のように変形できる;

$$\begin{aligned} \text{右辺} &= \frac{x+a}{b_2(x+A_1)(x+A_2)} \\ &= \frac{1}{b_2}\left\{-\frac{A_1-a}{A_2-A_1}\frac{1}{x+A_1}+\frac{A_2-a}{A_2-A_1}\frac{1}{x+A_2}\right\}. \end{aligned} \tag{B.49}$$

(B.49) を用いて(B.25) をxに関して積分を実行すれば

$$y = f_0(x+A_1)^{-\frac{1}{b_2}\frac{A_1-a}{A_1-A_2}}(x+A_2)^{\frac{1}{b_2}\frac{A_2-a}{A_2-A_1}} \tag{B.50}$$

を得る($f_0 = e^c$, cは積分定数)．ここで，変数変換$x' = x + A_1$, 及び$A = A_1 - A_2$, $\mu = (A_1-a)/b_2A$, $\nu = -(A_2-a)/b_2A$ を行い(B.50) の両辺を整理すればPearson VI 型曲線

$$y = f_0(x')^{-\mu}(x'-A)^{\nu} \tag{B.51}$$

を得る．

規格化条件(B.11) より

$$\begin{aligned} 1 &= \int_A^{\infty} dx'\ f_0(x')^{-\mu}(x'-A)^{\nu} \\ &= \int_A^{\infty} dx'\ f_0 \quad a^{q_2-q_1}\left(\frac{x}{a}\right)^{-q_1}\left(\frac{x}{a}-1\right)^{q_2} \end{aligned}$$

$$
\begin{aligned}
& \text{変数変換}\frac{1}{z}=\frac{x}{a}\text{を実行すると} \\
= \;& f_0 a^{q_2-q_1+1}\int_0^1 \mathrm{d}z\, z^{q_1-q_2-2}(1-z)^{q_2} \\
= \;& f_0 a^{q_2-q_1+1} B(q_2+1, q_1-q_2-1) \\
= \;& f_0 a^{q_2-q_1+1}\frac{\Gamma(q_2+1)\Gamma(q_1-q_2-1)}{\Gamma(q_1)}
\end{aligned}
\tag{B.52}
$$

となる. ここでベータ関数(B.34), (B.35)を用いた. こうしてf_0は

$$
f_0=\frac{a^{q_1-q_2-1}\Gamma(q_1)}{\Gamma(q_2+1)\Gamma(q_1-q_2-1)} \tag{B.53}
$$

と与えられ, Pearson VI 型確率分布関数は

$$
y=f(x)=\frac{a^{q_1-q_2-1}\Gamma(q_1)}{\Gamma(q_2+1)\Gamma(q_1-q_2-1)}x^{-\mu}(x-a)^{\nu} \quad (a\le x\le\infty) \tag{B.54}
$$

である.

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