

6 付録

6.1 付録 A

((72) 式の証明)

(証明) (31) 式と補助定理 4 から (71) 式は次のように求められる：

(i) $2 \lfloor H \rfloor \leq (N-1)$ の場合：

$$\begin{aligned}
& \sum_{k=-\infty}^{\infty} \int_{-kh}^{(N-1-k)h} \left| {}^m \tilde{\psi}_0(t) - {}^m \psi_0(t) \right|^2 dt \\
&= \sum_{k=-\infty}^{-\lfloor H \rfloor} \int_{-kh}^{(N-1-k)h} \left| {}^m \psi_0(t) \right|^2 dt + \sum_{k=-\lfloor H \rfloor+1}^{\lfloor H \rfloor-1} \int_{-Hh}^{(N-1-k)h} \left| {}^m \psi_0(t) \right|^2 dt \\
&\quad + \sum_{k=\lfloor H \rfloor}^{N-\lfloor H \rfloor-1} \left\{ \int_{-kh}^{-Hh} \left| {}^m \psi_0(t) \right|^2 dt + \int_{Hh}^{(N-1-k)h} \left| {}^m \psi_0(t) \right|^2 dt \right\} \\
&\quad + \sum_{k=N-\lfloor H \rfloor}^{N+\lfloor H \rfloor-2} \int_{-kh}^{-Hh} \left| {}^m \psi_0(t) \right|^2 dt + \sum_{k=N+\lfloor H \rfloor-1}^{\infty} \int_{-kh}^{(N-1-k)h} \left| {}^m \psi_0(t) \right|^2 dt \\
&\leq \sum_{k=-\infty}^{-\lfloor H \rfloor} \int_{-kh}^{(N-1-k)h} \frac{({}^m W)^2}{h^2} ({}^m w)^{2t/h} dt + \sum_{k=-\lfloor H \rfloor+1}^{\lfloor H \rfloor-1} \int_{Hh}^{(N-1-k)h} \frac{({}^m W)^2}{h^2} ({}^m w)^{2t/h} dt \\
&\quad + \sum_{k=\lfloor H \rfloor}^{N-\lfloor H \rfloor-1} \left\{ \int_{-kh}^{-Hh} \frac{({}^m W)^2}{h^2} ({}^m w)^{-2t/h} dt + \int_{Hh}^{(N-1-k)h} \frac{({}^m W)^2}{h^2} ({}^m w)^{2t/h} dt \right\} \\
&\quad + \sum_{k=N-\lfloor H \rfloor}^{N+\lfloor H \rfloor-2} \int_{-kh}^{Hh} \frac{({}^m W)^2}{h^2} ({}^m w)^{-2t/h} dt + \sum_{k=N+\lfloor H \rfloor-1}^{\infty} \int_{-kh}^{(N-1-k)h} \frac{({}^m W)^2}{h^2} ({}^m w)^{-2t/h} dt \\
&= \frac{(N-1)({}^m W)^2 ({}^m w)^{2H}}{h \ln ({}^m w)}. \tag{111}
\end{aligned}$$

(ii) $2 \lfloor H \rfloor \geq (N+1)$ の場合：

$$\begin{aligned}
& \sum_{k=-\infty}^{\infty} \int_{-kh}^{(N-1-k)h} \left| {}^m \tilde{\psi}_0(t) - {}^m \psi_0(t) \right|^2 dt \\
&= \sum_{k=-\infty}^{-\lfloor H \rfloor} \int_{-kh}^{(N-1-k)h} \left| {}^m \psi_0(t) \right|^2 dt + \sum_{k=-\lfloor H \rfloor+1}^{N-\lfloor H \rfloor-1} \int_{Hh}^{(N-1-k)h} \left| {}^m \psi_0(t) \right|^2 dt
\end{aligned}$$

$$\begin{aligned}
& + \sum_{k=N-\lfloor H \rfloor}^{\lfloor H \rfloor-1} \int_{-kh}^{(N-1-k)h} \left| {}^m\tilde{\psi}_0(t) - {}^m\psi_0(t) \right|^2 dt \\
& + \sum_{k=\lfloor H \rfloor}^{N+\lfloor H \rfloor-2} \int_{-kh}^{Hh} \left| {}^m\psi_0(t) \right|^2 dt \\
& + \sum_{k=N+\lfloor H \rfloor-1}^{\infty} \int_{-kh}^{(N-1-k)h} \left| {}^m\psi_0(t) \right|^2 dt \\
\leq & \sum_{k=-\infty}^{-\lfloor H \rfloor} \int_{-kh}^{(N-1-k)h} \frac{({}^mW)^2}{h^2} ({}^mw)^{2t/h} dt + \sum_{k=-\lfloor H \rfloor+1}^{N-\lfloor H \rfloor-1} \int_{Hh}^{(N-1-k)h} \frac{({}^mW)^2}{h^2} ({}^mw)^{2t/h} dt \\
& + \sum_{k=N-\lfloor H \rfloor}^{\lfloor H \rfloor-1} \int_{-kh}^{(N-1-k)h} 0 dt + \sum_{k=\lfloor H \rfloor}^{N+\lfloor H \rfloor-2} \int_{-kh}^{-Hh} \frac{({}^mW)^2}{h^2} ({}^mw)^{-2t/h} dt \\
& + \sum_{k=N+\lfloor H \rfloor-1}^{\infty} \int_{-kh}^{(N-1-k)h} \frac{({}^mW)^2}{h^2} ({}^mw)^{-2t/h} dt \\
= & \frac{(N-1)({}^mW)^2({}^mw)^{2H}}{h \ln({}^mw)}. \tag{112}
\end{aligned}$$

(iii) $2\lfloor H \rfloor = N$ の場合 :

$$\begin{aligned}
& \sum_{k=-\infty}^{\infty} \int_{-kh}^{(N-1-k)h} \left| {}^m\tilde{\psi}_0(t) - {}^m\psi_0(t) \right|^2 dt \\
= & \sum_{k=-\infty}^{-\lfloor H \rfloor} \int_{-kh}^{(N-1-k)h} \left| {}^m\psi_0(t) \right|^2 dt + \sum_{k=-\lfloor H \rfloor+1}^{\lfloor H \rfloor-1} \int_{Hh}^{(N-1-k)h} \left| {}^m\psi_0(t) \right|^2 dt \\
& + \sum_{k=N-\lfloor H \rfloor}^{N+\lfloor H \rfloor-2} \int_{-kh}^{-Hh} \left| {}^m\psi_0(t) \right|^2 dt + \sum_{k=N+\lfloor H \rfloor-1}^{\infty} \int_{-kh}^{(N-1-k)h} \left| {}^m\psi_0(t) \right|^2 dt \\
\leq & \sum_{k=-\infty}^{-\lfloor H \rfloor} \int_{-kh}^{(N-1-k)h} \frac{({}^mW)^2}{h^2} ({}^mw)^{2t/h} dt + \sum_{k=-\lfloor H \rfloor+1}^{\lfloor H \rfloor-1} \int_{Hh}^{(N-1-k)h} \frac{({}^mW)^2}{h^2} ({}^mw)^{2t/h} dt \\
& + \sum_{k=N-\lfloor H \rfloor}^{N+\lfloor H \rfloor-2} \int_{-kh}^{Hh} \frac{({}^mW)^2}{h^2} ({}^mw)^{-2t/h} dt + \sum_{k=N+\lfloor H \rfloor-1}^{\infty} \int_{-kh}^{(N-1-k)h} \frac{({}^mW)^2}{h^2} ({}^mw)^{-2t/h} dt \\
= & \frac{(N-1)({}^mW)^2({}^mw)^{2H}}{h \ln({}^mw)}. \tag{113}
\end{aligned}$$

(111),(112),そして(113)式より,(72)式が導かれる。(Q.E.D)

6.2 付録 B

((82) 式の証明)

(証明) 補助定理 1 における (30) 式より, (81) 式は次のように求められる:

(i) $2\lfloor H \rfloor \leq (N-1)$ の場合:

$$\begin{aligned}
& \sum_{k=-\infty}^{\infty} \int_{-kh}^{(N-1-k)h} |{}^m \tilde{\psi}_0(t)|^2 dt \\
&= \sum_{k=-\infty}^{-\lfloor H \rfloor} \int_{-kh}^{(N-1-k)h} 0 dt + \sum_{k=-\lfloor H \rfloor+1}^0 \int_{-kh}^{Hh} |{}^m \psi_0(t)|^2 dt \\
&+ \sum_{k=1}^{\lfloor H \rfloor-1} \left\{ \int_{-kh}^0 |{}^m \psi_0(t)|^2 dt + \int_0^{Hh} |{}^m \psi_0(t)|^2 dt \right\} \\
&+ \sum_{k=\lfloor H \rfloor}^{N-\lfloor H \rfloor-1} \left\{ \int_{-Hh}^0 |{}^m \psi_0(t)|^2 dt + \int_0^{Hh} |{}^m \psi_0(t)|^2 dt \right\} \\
&+ \sum_{k=N-\lfloor H \rfloor}^{N-1} \left\{ \int_{-Hh}^0 |{}^m \psi_0(t)|^2 dt + \int_0^{(N-1-k)h} |{}^m \psi_0(t)|^2 dt \right\} \\
&+ \sum_{k=N}^{N+\lfloor H \rfloor-2} \int_{-Hh}^{(N-1-k)h} 0 dt + \sum_{k=N+\lfloor H \rfloor-1}^{\infty} \int_{-kh}^{(N-1-k)h} 0 dt \\
&\leq \sum_{k=-\lfloor H \rfloor+1}^0 \int_{-kh}^{Hh} ({}^m U)^2 ({}^m u)^{2t/h} dt \\
&+ \sum_{k=1}^{\lfloor H \rfloor-1} \left\{ \int_{-kh}^0 ({}^m U)^2 ({}^m u)^{-2t/h} dt + \int_0^{Hh} ({}^m U)^2 ({}^m u)^{2t/h} dt \right\} \\
&+ \sum_{k=\lfloor H \rfloor}^{N-\lfloor H \rfloor-1} \left\{ \int_{-Hh}^0 ({}^m U)^2 ({}^m u)^{-2t/h} dt + \int_0^{Hh} ({}^m U)^2 ({}^m u)^{2t/h} dt \right\} \\
&+ \sum_{k=N-\lfloor H \rfloor}^{N-1} \left\{ \int_{-Hh}^0 ({}^m U)^2 ({}^m u)^{-2t/h} dt + \int_0^{(N-1-k)h} ({}^m U)^2 ({}^m u)^{2t/h} dt \right\} \\
&= \frac{T ({}^m U)^2 \{1 - ({}^m u)^{2H}\}}{\ln ({}^m u)}. \tag{114}
\end{aligned}$$

(ii) $(N+1)/2 \leq \lfloor H \rfloor \leq (N-2)$ の場合:

$$\begin{aligned}
& \sum_{k=-\infty}^{\infty} \int_{-kh}^{(N-1-k)h} |{}^m\tilde{\psi}_0(t)|^2 dt \\
&= \sum_{k=-\infty}^{-[H]} \int_{-kh}^{(N-1-k)h} 0 dt + \sum_{k=-[H]+1}^0 \int_{-kh}^{Hh} |{}^m\psi_0(t)|^2 dt \\
&\quad + \sum_{k=1}^{N-[H]-1} \left\{ \int_{-kh}^0 |{}^m\psi_0(t)|^2 dt + \int_0^{Hh} |{}^m\psi_0(t)|^2 dt \right\} \\
&\quad + \sum_{k=N-[H]}^{[H]-1} \left\{ \int_{-kh}^0 |{}^m\psi_0(t)|^2 dt + \int_0^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt \right\} \\
&\quad + \sum_{k=[H]}^{N-1} \left\{ \int_{-Hh}^0 |{}^m\psi_0(t)|^2 dt + \int_0^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt \right\} \\
&\quad + \sum_{k=N}^{N+[H]-2} \int_{-Hh}^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt + \sum_{k=N+[H]-1}^{\infty} \int_{-kh}^{(N-1-k)h} 0 dt \\
&\leq \sum_{k=-[H]+1}^0 \int_{-kh}^{Hh} ({}^mU)^2 ({}^mu)^{2t/h} dt \\
&\quad + \sum_{k=1}^{N-[H]-1} \left\{ \int_{-kh}^0 ({}^mU)^2 ({}^mu)^{-2t/h} dt + \int_0^{Hh} ({}^mU)^2 ({}^mu)^{2t/h} dt \right\} \\
&\quad + \sum_{k=N-[H]}^{[H]-1} \left\{ \int_{-kh}^0 ({}^mU)^2 ({}^mu)^{-2t/h} dt + \int_0^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{2t/h} dt \right\} \\
&\quad + \sum_{k=[H]}^{N-1} \left\{ \int_{-Hh}^0 ({}^mU)^2 ({}^mu)^{-2t/h} dt + \int_0^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{2t/h} dt \right\} \\
&\quad + \sum_{k=N}^{N+[H]-2} \int_{-Hh}^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{-2t/h} dt \\
&= \frac{T({}^mU)^2 \{1 - ({}^mu)^{2H}\}}{\ln({}^mu)}. \tag{115}
\end{aligned}$$

(iii) $[H] \geq (N+1)$ の場合 :

$$\begin{aligned}
& \sum_{k=-\infty}^{\infty} \int_{-kh}^{(N-1-k)h} |{}^m\tilde{\psi}_0(t)|^2 dt \\
&= \sum_{k=-\infty}^{-[H]} \int_{-kh}^{(N-1-k)h} 0 dt + \sum_{k=-[H]+1}^{N-[H]-1} \int_{-kh}^{Hh} |{}^m\psi_0(t)|^2 dt \\
&\quad + \sum_{k=N-[H]}^0 \int_{-kh}^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt
\end{aligned}$$

$$\begin{aligned}
& + \sum_{k=1}^{N-1} \left\{ \int_{-kh}^0 |{}^m\psi_0(t)|^2 dt + \int_0^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt \right\} \\
& + \sum_{k=N}^{\lfloor H \rfloor - 1} \int_{-kh}^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt + \sum_{k=\lfloor H \rfloor}^{N+\lfloor H \rfloor - 2} \int_{-Hh}^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt \\
& + \sum_{k=N+\lfloor H \rfloor - 1}^{\infty} \int_{-kh}^{(N-1-k)h} 0 dt \\
\leq & \sum_{k=-\lfloor H \rfloor + 1}^{N-\lfloor H \rfloor - 1} \int_{-kh}^{Hh} ({}^mU)^2 ({}^mu)^{2t/h} dt \\
& + \sum_{k=N-\lfloor H \rfloor}^0 \int_{-kh}^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{2t/h} dt \\
& + \sum_{k=1}^{N-1} \left\{ \int_{-kh}^0 ({}^mU)^2 ({}^mu)^{-2t/h} dt + \int_0^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{2t/h} dt \right\} \\
& + \sum_{k=N}^{\lfloor H \rfloor - 1} \int_{-kh}^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{-2t/h} dt \\
& + \sum_{k=\lfloor H \rfloor}^{N+\lfloor H \rfloor - 2} \int_{-Hh}^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{-2t/h} dt \\
= & \frac{T({}^mU)^2 \{1 - ({}^mu)^{2H}\}}{\ln({}^mu)}. \tag{116}
\end{aligned}$$

(iv) $\lfloor H \rfloor = N/2$ の場合 :

$$\begin{aligned}
& \sum_{k=-\infty}^{\infty} \int_{-kh}^{(N-1-k)h} |{}^m\tilde{\psi}_0(t)|^2 dt \\
= & \sum_{k=-\infty}^{-\lfloor H \rfloor} \int_{-kh}^{(N-1-k)h} 0 dt + \sum_{k=-\lfloor H \rfloor + 1}^0 \int_{-kh}^{Hh} |{}^m\psi_0(t)|^2 dt \\
& + \sum_{k=1}^{\lfloor H \rfloor - 1} \left\{ \int_{-kh}^0 |{}^m\psi_0(t)|^2 dt + \int_0^{Hh} |{}^m\psi_0(t)|^2 dt \right\} \\
& + \sum_{k=\lfloor H \rfloor}^{N-1} \left\{ \int_{-Hh}^0 |{}^m\psi_0(t)|^2 dt + \int_0^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt \right\} \\
& + \sum_{k=N}^{N+\lfloor H \rfloor - 2} \int_{-Hh}^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt \\
& + \sum_{k=N+\lfloor H \rfloor - 1}^{\infty} \int_{-kh}^{(N-1-k)h} 0 dt
\end{aligned}$$

$$\begin{aligned}
&\leq \sum_{k=-[H]+1}^0 \int_{-kh}^{Hh} ({}^mU)^2 ({}^mu)^{2t/h} dt \\
&\quad + \sum_{k=1}^{[H]-1} \left\{ \int_{-kh}^0 ({}^mU)^2 ({}^mu)^{-2t/h} dt + \int_0^{Hh} ({}^mU)^2 ({}^mu)^{2t/h} dt \right\} \\
&\quad + \sum_{k=[H]}^{N-1} \left\{ \int_{-Hh}^0 ({}^mU)^2 ({}^mu)^{-2t/h} dt + \int_0^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{2t/h} dt \right\} \\
&\quad + \sum_{k=N}^{N+[H]-2} \int_{-Hh}^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{-2t/h} dt \\
&= \frac{T ({}^mU)^2 \{1 - ({}^mu)^{2H}\}}{\ln({}^mu)}. \tag{117}
\end{aligned}$$

(v) $[H] = N - 1$ の場合 :

$$\begin{aligned}
&\sum_{k=-\infty}^{\infty} \int_{-kh}^{(N-1-k)h} |{}^m\tilde{\psi}_0(t)|^2 dt \\
&= \sum_{k=-\infty}^{-[H]} \int_{-kh}^{(N-1-k)h} 0 dt + \sum_{k=-[H]+1}^0 \int_{-kh}^{Hh} |{}^m\psi_0(t)|^2 dt \\
&\quad + \sum_{k=N-[H]}^{[H]-1} \left\{ \int_{-kh}^0 |{}^m\psi_0(t)|^2 dt + \int_0^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt \right\} \\
&\quad + \sum_{k=[H]}^{N-1} \left\{ \int_{-Hh}^0 |{}^m\psi_0(t)|^2 dt + \int_0^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt \right\} \\
&\quad + \sum_{k=N}^{N+[H]-2} \int_{-Hh}^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt \\
&\quad + \sum_{k=N+[H]-1}^{\infty} \int_{-kh}^{(N-1-k)h} 0 dt \\
&\leq \sum_{k=-[H]+1}^0 \int_{-kh}^{Hh} ({}^mU)^2 ({}^mu)^{2t/h} dt \\
&\quad + \sum_{k=N-[H]}^{[H]-1} \left\{ \int_{-kh}^0 ({}^mU)^2 ({}^mu)^{-2t/h} dt + \int_0^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{2t/h} dt \right\} \\
&\quad + \sum_{k=[H]}^{N-1} \left\{ \int_{-Hh}^0 ({}^mU)^2 ({}^mu)^{-2t/h} dt + \int_0^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{2t/h} dt \right\} \\
&\quad + \sum_{k=N}^{N+[H]-2} \int_{-Hh}^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{-2t/h} dt
\end{aligned}$$

$$= -\frac{T({}^mU)^2 \{1 - ({}^mu)^{2H}\}}{\ln({}^mu)}. \quad (118)$$

(vi) $[H] = N$ の場合 :

$$\begin{aligned}
& \sum_{k=-\infty}^{\infty} \int_{-kh}^{(N-1-k)h} |{}^m\tilde{\psi}_0(t)|^2 dt \\
&= \sum_{k=-\infty}^{-[H]} \int_{-kh}^{(N-1-k)h} 0 dt + \sum_{k=-[H]+1}^{N-[H]-1} \int_{-kh}^{Hh} |{}^m\psi_0(t)|^2 dt \\
&\quad + \sum_{k=N-[H]}^0 \int_{-kh}^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt \\
&\quad + \sum_{k=1}^{N-1} \left\{ \int_{-kh}^0 |{}^m\psi_0(t)|^2 dt + \int_0^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt \right\} \\
&\quad + \sum_{k=N}^{N+[H]-2} \int_{-Hh}^{(N-1-k)h} |{}^m\psi_0(t)|^2 dt \\
&\quad + \sum_{k=N+[H]-1}^{\infty} \int_{-kh}^{(N-1-k)h} 0 dt \\
&\leq \sum_{k=-[H]+1}^{N-[H]-1} \int_{-kh}^{Hh} ({}^mU)^2 ({}^mu)^{2t/h} dt \\
&\quad + \sum_{k=N-[H]}^0 \int_{-kh}^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{2t/h} dt \\
&\quad + \sum_{k=1}^{N-1} \left\{ \int_{-kh}^0 ({}^mU)^2 ({}^mu)^{-2t/h} dt + \int_0^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{2t/h} dt \right\} \\
&\quad + \sum_{k=N}^{N+[H]-2} \int_{-Hh}^{(N-1-k)h} ({}^mU)^2 ({}^mu)^{-2t/h} dt \\
&= -\frac{T({}^mU)^2 \{1 - ({}^mu)^{2H}\}}{\ln({}^mu)}. \quad (119)
\end{aligned}$$

よって (114) から (119) 式より (82) 式が導かれる。 (Q.E.D)

6.3 付録 C

((85) 式の証明)

(証明) 付録 A における ${}^m\tilde{\psi}_0(t), {}^m\psi_0(t), \frac{({}^mW)^2}{h^2}$, そして $({}^mw)$ を ${}^m\tilde{\psi}_0(t), {}^m\psi(t), ({}^mU)^2$, そして $({}^mu)$ でそれぞれ置きかえることで (84) 式が得られる. さらに $T = (N-1)h$ を (84) に適用すると (85) 式が導出される.