# Chapter 7

# CONCLUSION

We proposed a signal representation based on quaternion for three-dimensional measurement signals. Quaternion measurement signal $q_s(t)$ is defined as

$$q_s(t) = s_0 + s_x(t)i + s_y(t)j + s_z(t)k.$$

Here, $s_x(t)$, $s_y(t)$ and $s_z(t)$ are three scalar functions which stand for a triaxial measurement signal. $s_0$ is an empty signal.

Then, in order to handle the quaternion signal $q_s(t)$ in the frequency domain, a transformation and its inverse one was derived. They can be written simply as

$$Q_s(f) = \int_{-\infty}^{+\infty} q_s(t) e^{2\pi f t \boldsymbol{p}}\, dt,$$
$$q_s(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} Q_s(f) e^{-2\pi f t \boldsymbol{p}}\, df.$$

This transformation is very similar to complex Fourier transform. The difference is just a symbol "$\boldsymbol{p}$". This transformation allow us to grasp a quaternion signal with a concept of amplitude and phase. Such concepts are to be familiar for us.

Nextly, the relation between a periodic component of quaternion signal and a virtual elliptic orbit in quaternion space was pointed out. Basing on this relation, we proposed three evaluation indices to grasp the information about three-dimensional measurement signal. They are $TEI_\alpha$, $TEI_\gamma$ and $\boldsymbol{TEI}_\phi$ (See Eq.(3.24)). These indices relate the shape of an ellipse and its attitude, and are computed from the quaternion spectral coefficients.

Then, to check a concept mentioned above and show an entrance of concrete application, we considered two filter systems. One is the "Filter Bank". Another is the "FIR" filter extended for quaternion signal. Comparing the scalar processing, these filter system could indicate some different function when proper quaternion gains are given.

At last, as sample applications for triaxial measurement signal, we produced a small sensor ball to record the three-dimensional movements and 3D vibration scope to visualize three-dimensional vibration. We think that both of these systems may have some possibilities to develop as diagnosis system for the machine in live motion.

Though this study is still the primitive step, the quaternion representation will leads the measurement signal processing of three dimensions to a new point of view.