APPENDIX

A.1 Antenna Impedance

The antenna is essentially a transducer between the radio or radar systems and the propagation medium. Therefore the antenna designer must be concerned with the characteristics of the electromagnetic fields transmitted or received and the characteristics of a load connected to the antenna. Typically the antenna will be connected to the transmitter or receiver by a transmission line, which may take the form of wires, coaxial cable, dielectric or metallic waveguide, or one of the newer forms of transmission line such as stripline. It is generally desirable to achieve the maximum power transfer from the transmission line to the antenna, and vice versa, without the distortion of the information conveyed.

The impedance concept can be very useful for a certain class of antennas in defining the required characteristics at the input terminal of an antenna. If the impedance of the transmission line can be defined, then the design objective for the antenna impedance will be to match this value, thereby ensuring a maximum power transfer on the basis of the power transfer theorem.

The impedance concept can be particularly useful for lower frequency antennas where a pair of input terminals can be readily defined, the impedance is single valued and is relatively easy to measure. The concept does not fail at higher frequencies, but there may be practical difficulties in defining and measuring this quantity.

For example, at microwave frequencies the antenna may be connected to a waveguide transmission line. The impedance of a waveguide is not single valued since, unlike coaxial cable, the electric and magnetic fields inside the waveguide are not purely transverse. An impedance concept is based upon the transverse components of the electromagnetic fields of the fundamental mode in the waveguide, but in practice the input terminal to the antenna is often a waveguide flange and it is more convenient to employ waveguide-matching techniques to measure the **voltage-standing-wave ratio (VSWR) or return loss**. These measurements can be converted into an effective input impedance and displayed on a Smith chart if required.

Where it is applicable the impedance concept is helpful as an aid to the understanding and design of antenna systems. In these cases the antenna input impedance can be considered as a two terminal network terminating the physical antenna. In general this impedance can be considered to be comprised of two parts: a self-impedance and a mutual impedance, such that,

input impedance = self impedance + mutual impedance

The *self impedance* is the impedance which would be measured at the input terminals of the antenna in free space, that is in the absence of any other antennas or reflecting obstacles. The *mutual impedance* accounts for the influence of coupling to the antenna from any source outside. Clearly nearby objects are potentially greater sources of coupling and for many antennas the mutual impedance is effectively zero, either because the antenna is sufficiently isolated in space or because the influence of nearby objects is much less than the self impedance.

On the other hand some antennas rely on the mutual coupling between elements to produce the desired specifications. A classic example is the Yagi-Uda antenna where all but one of the elements are passive and unconnected to any other elements but have currents induced on them by mutual coupling from the one driven element. In array antennas the mutual coupling may not be desired but will often be a significant factor in the total radiation characteristics. The calculation of mutual impedance is usually theoretically complicated because the coupled antennas are in their reactive near-field regions and the geometry of the antennas is often difficult to model analytically.

The self impedance of an antenna has both a resistive and a reactive component, i.e. by employing complex algebra it has the form

self impedance = (antenna resistance) + j (self reactance)

The *self reactance* arises from the reactive energy which is stored in the near-field region surrounding the antenna, while the *antenna resistance* accounts for all the power absorbed by the antenna. The power absorbed includes that which is ultimately radiated by the antenna and thus the antenna resistance comprises a so-called *radiation resistance* R_r and a *loss resistance* R_L which accounts for the dissipative and ohmic losses in the antenna structure.

Hence

antenna resistance = $R_r + R_L$

where the radiation resistance is defined as the equivalent resistance which would dissipate a power equal to that radiated, P_r, when carrying the current I_0 flowing at the input terminals, i.e.

$$R_r = P_r/I_0^2 \tag{A.1}$$

For an efficient antenna it is necessary that the radiation resistance be much greater than the loss resistance. For a practical thin half-wave dipole for example, the radiation resistance may have a value of approximately 73Ω, with a loss resistance of perhaps 2Ω. On the other hand a high frequency band notch antenna on an aircraft may have a radiation resistance of 0.01Ω with a loss resistance of several ohms. Many standard textbooks have dealt with the calculations of the radiation resistance for cylindrical rods and wires.

Measurements of input impedance can be performed using conventional impedance bridge techniques and this is common practice at the lower frequencies. At higher frequencies measurements of reflection coefficients or voltage-standing- wave-ratios (VSWR) are favoured. Provided that the characteristics of the transmission line are known, these measurements can be converted to an impedance if required.

For example, if an antenna of impedance Z_L terminates a transmission line of characteristic impedance Z_0 [I] and an impressed sinusoidal voltage at the input to the the antenna (V) gives rise to a reflected voltage (V') (see Fig. A.1) then the reflection coefficient Γ of the antenna is simply given by

$$\Gamma = (|V|/|V'|)e^{j\theta} \tag{A.2}$$

where θ is the phase difference between the transmitted and reflected voltages.

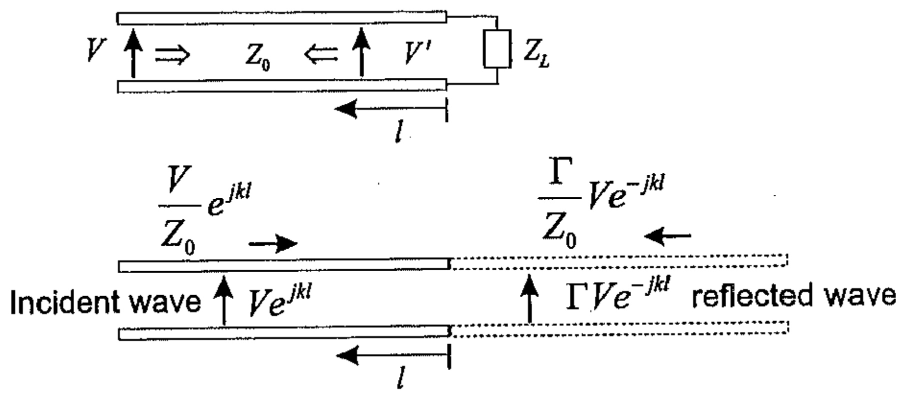

Fig. A.1: The voltage and current on the transmission line.

The voltage and current on the transmission line can be expressed as [II]

$$V(l) = V(e^{jkl} + \Gamma e^{-jkl}) \tag{A.3}$$

$$I(l) = \frac{V}{Z_0}(e^{jkl} - \Gamma e^{-jkl}) \tag{A.4}$$

The voltage standing-wave ratio (VSWR) is a measure of the ratio of the maximum and minimum voltages set up on the transmission line. In terms of

[I]The characteristics impedance Z_0 of a transmission line is defined as the ratio of the voltage to the current established on an unlimited transmission line. The wave on the unlimited transmission line is called traveling wave and no reflection exists on the transmission line.

[II]The incident wave is supposed to come from the region $l < 0$.

the reflection coefficient the VSWR will have the value

$$\mathbf{VSWR} = (1+|\Gamma|)/(1-|\Gamma|) \quad \text{(A.5)}$$

and will take values in the range of unity (i.e. perfect match) to infinity. Typically microwave horn antennas have VSWR values in the range 1.01 to 1.5, while a broad-band antenna may operate with VSWR values of the order of 1.5 to 2.5. A VSWR of 5.8 ($\Gamma^2 = 1/2$) implies that one half of the incident power is reflected from the antenna to the transmission line.

The antenna impedance (Z_L) is given by

$$Z_L = Z_0(1+\Gamma)/(1-\Gamma) \quad \text{(A.6)}$$

and it must be noted that this is a complex quantity which requires that the phase angle of the reflection coefficient be established. The return loss, which is often specified as a performance parameter in microwave applications, is given by

$$\textbf{return loss} = 20\log_{10}|\Gamma| \text{ decibels} \quad \text{(A.7)}$$

Return losses [III] [IV] of -15dB [V] are typical for many antennas but values of the order of -30dB [VI] or more may be demanded for a high performance satellite communications ground station.

[III] When $|\Gamma| = 1/3$, VSWR = 2, return loss = -9.54dB.
[IV] When $|\Gamma| \to 0$, VSWR $\to 1$, return loss $\to -\infty$ dB.
[V] When return loss = -15dB, $|\Gamma| = 0.177$, VSWR = 1.43.
[VI] When return loss = -30dB, $|\Gamma| = 1/30$, VSWR = 1.07.

A.2 Antenna Polarization

The polarization of an antenna is the polarization of the wave radiated by the antenna in a given direction.

If the electric and magnetic field vector of an electromagnetic wave lie in a fixed plane at all times, it is called a *plane polarized wave*. The tip of the instantaneous electric field vector traces out a figure with time; we refer to this phenomena simply as the polarization of the electric field vector. There may be a random component to this figure (a nonperiodic behavior), but we will not consider such randomly polarized wave components because antennas cannot generate them. For a completely polarized wave the figure traced out is, in general, an ellipse.

There are some important special cases of the polarization ellipse. If the path of the electric field vector is back and forth along a line, it is said to be linearly polarized. See Figs. A.2(a) and A.2(b). An example is the electric field from an ideal dipole or any linear current. If the electric field vector remains constant in length but rotates around in a circular path, it is called *circularly polarized*. The radian frequency of the rotation is ω and occurs in one of two directions, referred to as the *sense of rotation*. If the wave is traveling toward the observer and the vector rotates counterclockwise, it is *right-hand polarized*. The right-hand rule applies here: with the thumb of the right hand in the direction of propagation, the fingers will curl in the direction of rotation of the instantaneous electric field $\mathcal{E}$. If it rotates clockwise, it is left-hand polarized. Right- and left-hand circularly polarized waves are shown in Figs. A.2(c) and A.2(d). A helical antenna produces circularly polarized waves and the sense of rotation of the wave is the same as the sense of the helix windings, for example, a right-hand wound helix produces a right-hand circularly polarized wave. Finally, a wave may be elliptically polarized, with either right- or left-hand sense of rotation, as shown in Figs. A.2(e) and A.2(f).

In the most general case of elliptical polarization the polarization ellipse

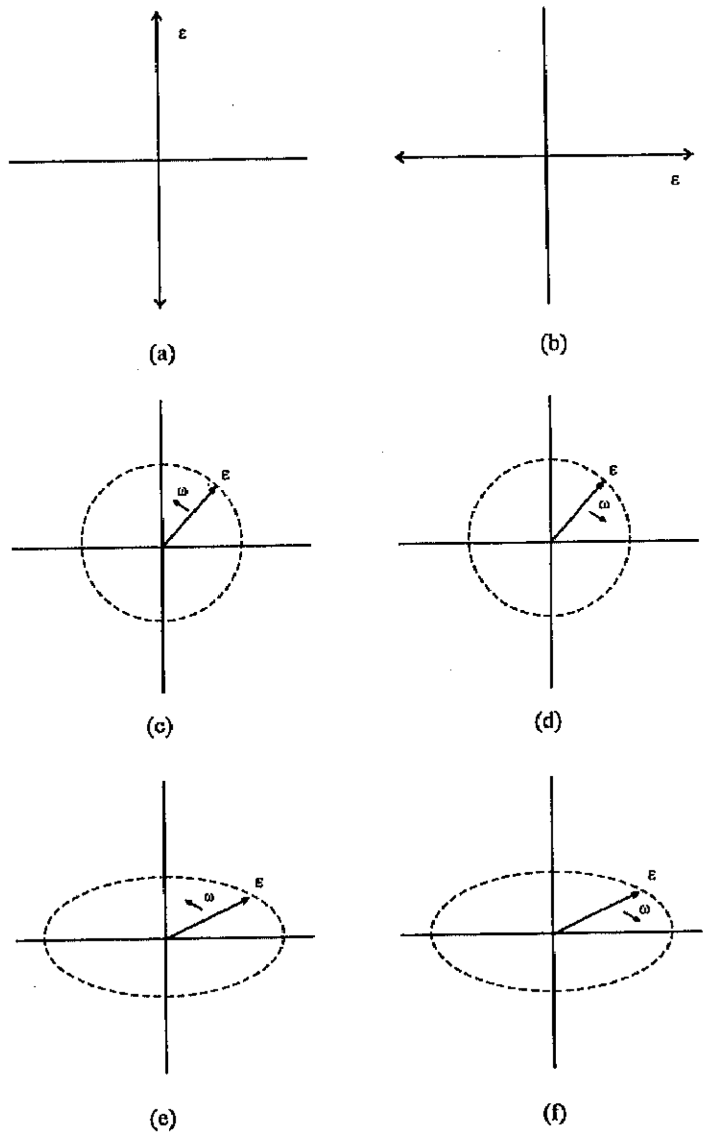

Fig. A.2: Some wave polarization states. The wave is approaching. (a) Linear (vertical) polarization. (b) Linear (horizontal) polarization. (c) Right-hand circular polarization. (d) Left-hand circular polarization. (e) Right-hand elliptical polarization. (f) Left-hand elliptical polarization.

may have any orientation. This elliptically polarized wave may be expressed in terms of two linearly polarized components, one in the x direction and the other in the y direction. Thus, if the wave is travelling in the positive z direction (out of the page), the electric field can be expressed as

$$\vec{E} = (\hat{x}A + \hat{y}B)e^{-jkz} \tag{A.8}$$

The instantaneous total vector field corresponding to the complex expression is

$$\begin{aligned} \vec{E}(z,t) &= \mathrm{Re}[(\hat{x}A + \hat{y}B)e^{j(\omega t - kz)}] \\ &= \hat{x}a\cos(\omega t - kz + \alpha) + \hat{y}b\cos(\omega t - kz + \beta) \end{aligned} \tag{A.9}$$

where

$$A = ae^{j\alpha} \qquad (a = |A|) \tag{A.10}$$

$$B = be^{j\beta} \qquad (b = |B|) \tag{A.11}$$

The electric field components in the x and y directions are

$$\vec{E}_x(z,t) = a\cos(\omega t - kz)\cos\alpha - a\sin(\omega t - kz)\sin\alpha \tag{A.12}$$

$$\vec{E}_y(z,t) = b\cos(\omega t - kz)\cos\beta - b\sin(\omega t - kz)\sin\beta \tag{A.13}$$

Combining A.12 and A.13, eliminating $(\omega t - kz)$, and rearranging the equation, we obtain

$$\left(\frac{E_y}{a}\right)^2 - 2\cos(\alpha - \beta)\left(\frac{E_y}{a}\right)\left(\frac{E_x}{b}\right) + \left(\frac{E_x}{b}\right)^2 = \sin^2(\alpha - \beta) \tag{A.14}$$

Equation A.14 describes a (polarization) ellipse.

For $\alpha - \beta = 0$, the wave is linearly polarized in the direction determined by $\frac{E_x}{b} + \frac{E_y}{a} = 0$; For $\alpha - \beta = \pm\pi/2$, the wave is elliptically polarized, and is determined by $\left(\frac{E_x}{b}\right)^2 + \left(\frac{E_y}{a}\right)^2 = 1$. When $\alpha - \beta = \pi/2$, the wave is right-hand elliptically polarized, and when $\alpha - \beta = -\pi/2$, the wave is left-hand elliptically polarized.

Considering the following circularly polarized waves

$$\vec{R} = (\hat{x} - j\hat{y})e^{-jkz} \tag{A.15}$$

$$\vec{L} = (\hat{x} + j\hat{y})e^{-jkz} \tag{A.16}$$

where $\vec{R}$ represents a right-hand circularly polarized wave, and $\vec{L}$ represents a left-hand circularly polarized wave.

Equation A.8 can be reexpressed as

$$\vec{E} = (\hat{x}A + \hat{y}B)e^{-jkz} = r\vec{R} + l\vec{L} \tag{A.17}$$

where

$$r = \frac{1}{2}(A + jB) \tag{A.18}$$

$$l = \frac{1}{2}(A - jB) \tag{A.19}$$

That means the elliptically polarized wave can be expressed as the combination of the right-hand circularly polarized wave and left-hand circularly polarized wave (See Fig. A.3, where r and l are plotted). When $(\omega t - kz)$ increases, the end point of the electric field of the right-hand circularly polarized wave rotates in the direction ① → ② → ③ → ④, meanwhile, the end point of the electric field of the left-hand circularly polarized wave rotates in the direction ①′ → ②′ → ③′ → ④′. When the end points of the two instantaneous vector fields are at (① , ①′) or (③ , ③′), the two vectors are in the opposite directions and the composition of the two vectors becomes minimum. When the end points of the two electric fields are at (② , ②′) or (④ , ④′), the two vectors are in the same direction and the composition of the two vectors becomes maximum.

Therefore, the axial ratio (AR) can be defined as

$$\mathrm{AR} = \frac{|r| + |l|}{||r| - |l||} \tag{A.20}$$

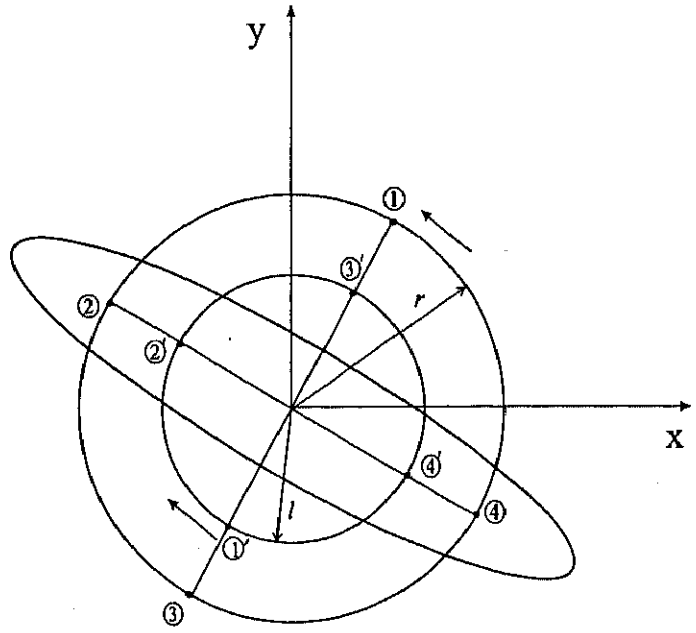

Fig. A.3: The elliptically polarized wave consists of two circularly polarized waves.

or in decibels (dB)

$$\mathrm{AR} = 20\log_{10}\left(\frac{|r|+|l|}{||r|-|l||}\right) \qquad \mathrm{dB} \tag{A.21}$$

The polarization of an antenna is the polarization of the wave radiated by the antenna in a given direction. Therefore, all of the discussions on wave polarization apply directly to antenna polarization. Usually the polarization characteristics of an antenna remain relatively constant over its main beam. However, the radiation in some side lobe directions may differ greatly in polarization from that of the main beam. When the radiation from an antenna is measured, both E_θ and E_ϕ should be measured to be complete. The principal plane patterns of a linearly polarized antenna, such as a line source located on the z-axis, are completely specified when a linearly polarized probe antenna is oriented to respond to E_θ.

The polarization of an antenna is determined by the wave radiated from the antenna, which must, of course, be in the far field where local plane wave behavior exists. Therefore, the plane polarized wave discussions given earlier apply. Furthermore, since the pattern (i.e., the radiation field) is reciprocal, the polarization of an antenna is reciprocal. In other words, an antenna responds best (gives maximum output) for an incident wave of given intensity when the polarization ellipse of the incident electric field has the same axial ratio, the same sense of polarization, and the same spatial orientation of the major axis as that of the receiving antenna for that direction. For example, a right-hand circularly polarized receiving antenna is polarization matched to a right circularly polarized wave. As a mechanical analogy, let a right-hand threaded rod represent a right-hand circularly polarized (RHCP) wave and a right-hand tapped hole represent a RHCP antenna. The rod and hole are matched when screwed either in or out, corresponding to reception or transmission.

Circular polarization gives a steady power flow, and there is no change of power densities with time or space, analogous to power transmission in a two-phase system.

A.3 Convergence Characteristics of the Calculation

For a cavity-backed slot antenna, two factors affect the convergence characteristics of the calculation. One is the number of terms N_{GF} for the cavity Green's function (see Fig. 2.16), the other is the segment length ΔZ (the number of expansion functions) (see Fig. 2.18).

In this section, a rectangular-cavity-backed single square loop slot antenna which has been analyzed in Section 3.1 is used as an example to illustrate the convergence characteristics of the calculation in this dissertation.

The input impedance and axial ratio convergence characteristics versus the term number N_{GF} of the cavity Green's function are shown in Figs. A.4 and A.5, respectively. We can see from the two figures that both axial ratio and input impedance are sensitive to the N_{GF}. By comparing the data in Figs. A.4 and A.5 with the experimental data in Figs. 3.14 and 3.15, it is found that when N_{GF}=100, the results become convergent.

The input impedance and axial ratio convergence characteristics versus the segment length ΔZ are shown in Figs. A.6 and A.7, respectively. We can see from the two figures that the axial ratio is more sensitive to the segment length ΔZ than that of the input impedance. The results become convergent when $\Delta Z=0.05\lambda_0$.

For the calculation of the rectangular-cavity-backed slot antenna, the following conclusions can be made:

The axial ratio is more sensitive to the segment length ΔZ than that of the input impedance; Both the axial ratio and the input impedance are sensitive to the number of terms N_{GF} for the cavity Green's function.

The convergence conditions are as follows:

1. the number of terms N_{GF} for the cavity Green's function = 100.
2. the segment length $\Delta Z = 0.05\lambda_0$.

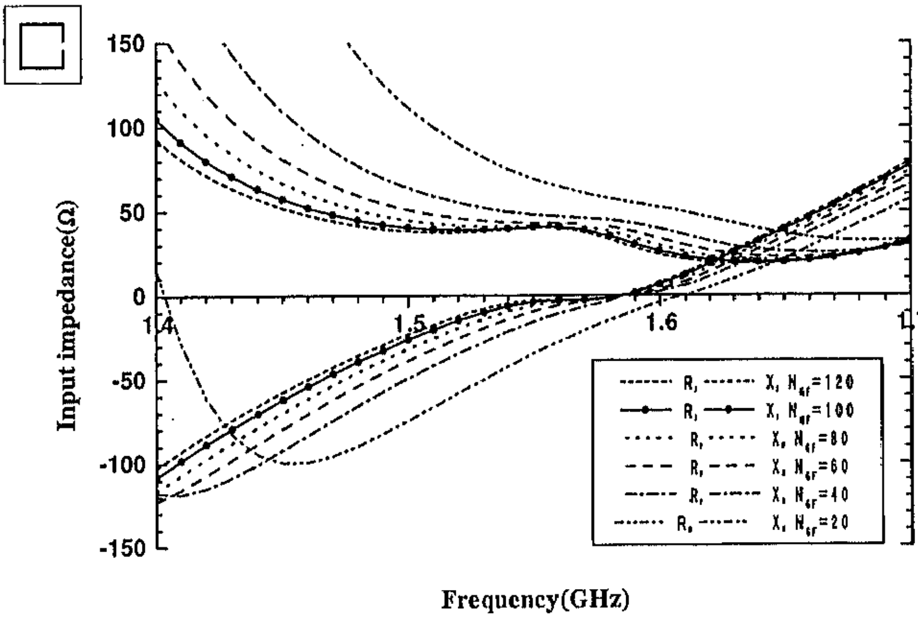

Fig. A.4: Input impedance convergence characteristics versus N_{GF}, where Xc=Yc=145mm, Zc=13mm, Xs=Ys=81mm, W=3mm, Ls=81mm, $\Delta Z=0.05\lambda_0$, λ_0=200mm.

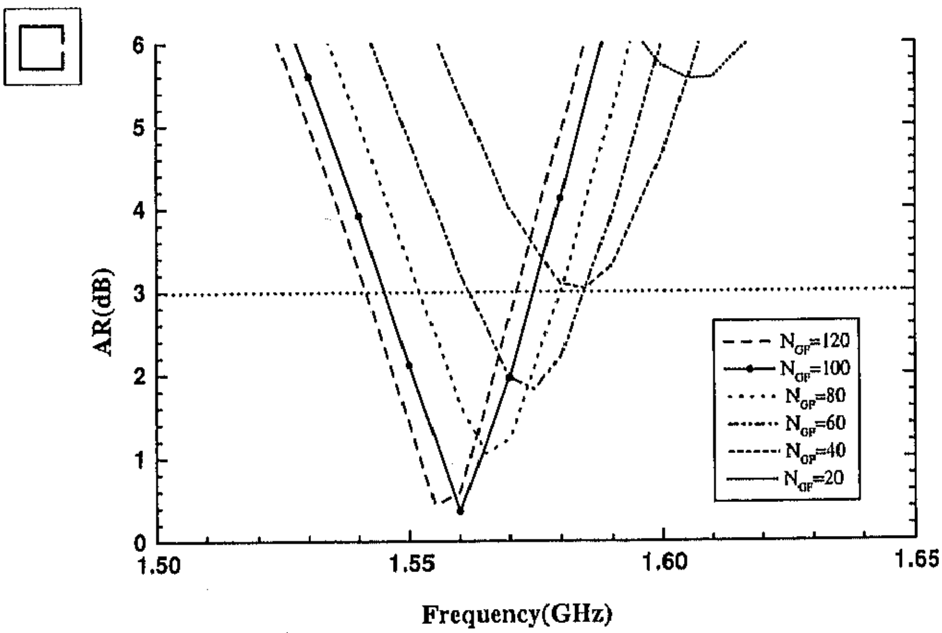

Fig. A.5: Axial ratio convergence characteristics versus N_{GF}, where Xc=Yc=145mm, Zc=13mm, Xs=Ys=81mm, W=3mm, Ls=81mm, $\Delta Z=0.05\lambda_0$, λ_0=200mm.

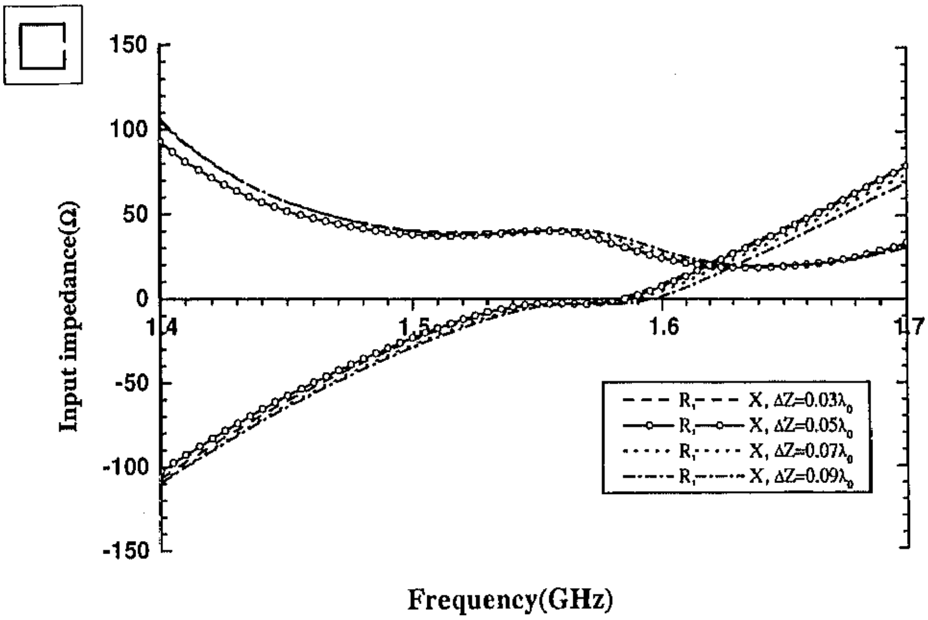

Fig. A.6: Input impedance convergence characteristics versus ΔZ, where Xc=Yc=145mm, Zc=13mm, Xs=Ys=81mm, W=3mm, Ls=81mm, N_{GF}=100, λ_0=200mm.

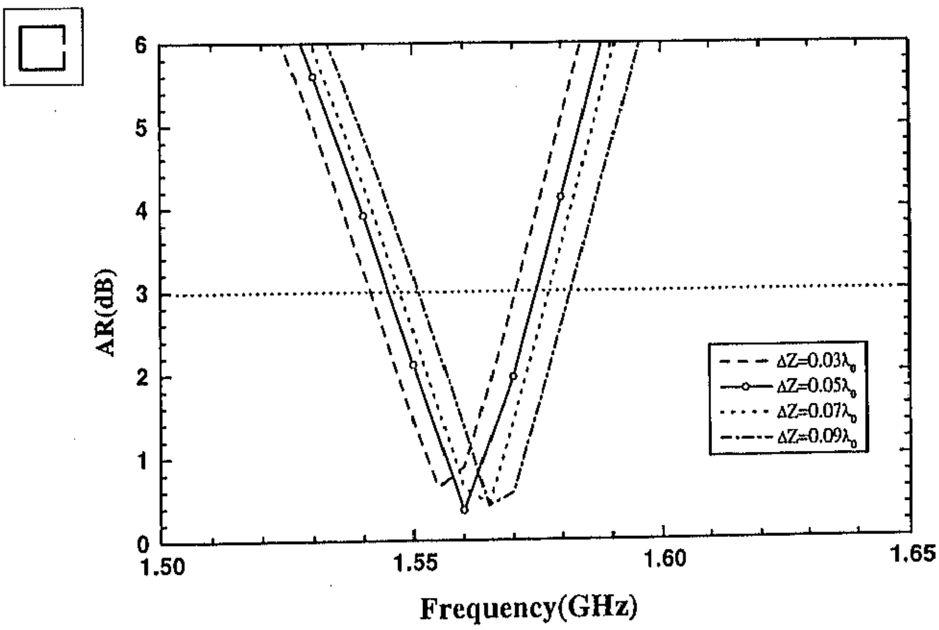

Fig. A.7: Axial ratio convergence characteristics versus ΔZ, where Xc=Yc=145mm, Zc=13mm, Xs=Ys=81mm, W=3mm, Ls=81mm, N_{GF}=100, λ_0=200mm.

A.4 Circular Polarization for Spacecraft Communication

For spacecraft communication, it is important to discuss the antenna's polarization and why it plays an important role in satellite-antenna design. An antenna is defined by the polarization of the electromagnetic (EM) energy that it radiates. It is important to measure this polarization in the far zone of the antenna, that is, at distances sufficiently far from the antenna so that a further increase in this distance will not change the measured polarization. A distance $R = 2D^2/\lambda$ is customarily chosen as adequate for measuring the antenna's polarization, where D is the antenna-aperture size and λ is the operating wavelength. The electric field direction defines the polarization of the EM energy.

Although essentially all polarization properties of EM waves play a role in satellite-antenna design, let us review those that are most important. For example, a linearly polarized (LP) antenna such as a dipole, oriented with its axis vertical (with respect to the earth's surface), will radiate and receive vertically polarized signals. Conversely, it will neither radiate nor receive horizontally polarized signals. This phenomenon is commonly referred to by stating that an antenna will not radiate or receive cross-polarized signals, or that orthogonally polarized signals are rejected. This statement is not limited to LP antennas; circular and elliptically polarized EM waves and antennas have co-polarized and cross-polarized properties identical to those of LP waves and antennas [41]. Circularly polarized (CP) waves have a right-hand sense (i.e., RHCP) if the electric field vector rotates in a clockwise sense as the wave is propagating away from the observer. The electric field vector of a left-hand circularly polarized (LHCP) wave rotates in a counterclockwise sense for receding waves. Changing both the direction of propagation (i.e., receding to approaching) and the sense of rotation (i.e., clockwise to counterclockwise) does not alter the polarization. The important point is that co-polarized antennas couple well to one another and cross-polarized antennas tend to reject one another's signals. Now consider earth-satellite signal links when the frame of reference (i.e., vertical and horizontal) of the earth station will not, in general, coincide with the frame of reference (i.e., north and south) of the satellite. Since the satellite usually serves many users simultaneously and its antenna can assume only one polarization at any instant of time, it follows that when LP antennas are used, the earth station must adjust its frame of reference to coincide with the satellite's frame of reference. Although this is possible, it is far simpler to use CP satellite and earth-terminal antennas and remove the need to align them in order to maximize coupling between them. Consequently, it is not surprising that most satellite antennas are circularly polarized.

When an LP satellite antenna is used, the orientation of the associated EM waves is altered as they propagate through the earth's ionosphere [42]-[43], a phenomenon often referred to as the Faraday rotation effect. This rotation of LP waves is usually negligible (less than a few degrees) at frequencies above 1 GHz. However, at frequencies below 1 GHz Faraday rotation effects can rotate the wave polarization more than 360°. Fortunately, the polarization of a CP wave is not altered by the Faraday rotation effect. Change in polarization due to transverse "static" magnetic fields along the propagation path is much smaller, therefore, circular polarization is preferred because CP waves propagate through the ionosphere with no essential change in polarization. Most spacecraft and earth-terminal antennas are shared by the associated transmitter and receiver. The use of antennas that are orthogonally polarized for transmitted and received radiation enhances the isolation between the transmitter and the receiver. For this and the foregoing reasons it is customary for satellite antennas to be opposite-sense circularly polarized for simultaneous transmitting and receiving functions.