Chapter 5

Hybrid Dilated Banyan Networks with Bypasses

In the previous chapter, the throughput of the original and the 2-dilated banyan network are improved by inserting bypasses at various positions in their networks. Since the hybrid dilated banyan network is constructed by the original and the 2-dilated banyan network, the performance of hybrid dilated banyan network should be improved by inserting the bypasses. This chapter applies all-bypass-connection, one-bypass-connection and one-bypass-connection at two neighbored stages to the hybrid dilated banyan network discussed in chapter 3, and analyzes their output rates.

5.1 Hybrid Dilated Banyan Networks with All-Bypass-Connection

The restriction of bypass position for the hybrid dilated banyan network is same as the previous chapter. For an $N \times N$ hybrid dilated banyan network and $n = \log_2 N$, the switching elements from $((m-1)2^{n-d}+1)$-th to $m2^{n-d}$-th columns at the d-th stage ($1 \leq d \leq n-1$) can be connected by bypasses, where $m = 1, \ldots, 2^{d-1}$. Note that there are no bypass connection at the first stage of hybrid dilated banyan network because no blocking occurs in this stage, and no bypass at the n-stage otherwise a cell may be forwarded to the incorrect destination.

Since the output rates for all types of 4×2 re-arrangeable output switching elements with bypasses at different input rate are not obtained in the previous chapter, the introduction of bypasses to these switching elements is postponed until the next section. In order to compare the bypass effect to each kind of switching element, the hybrid dilated banyan networks which are inserted bypasses at the stage of banyan switching elements or 2-dilated banyan switching elements are considered.

Fig. 5.1 and Fig. 5.2 show the one of the 32×32 hybrid dilated banyan network with

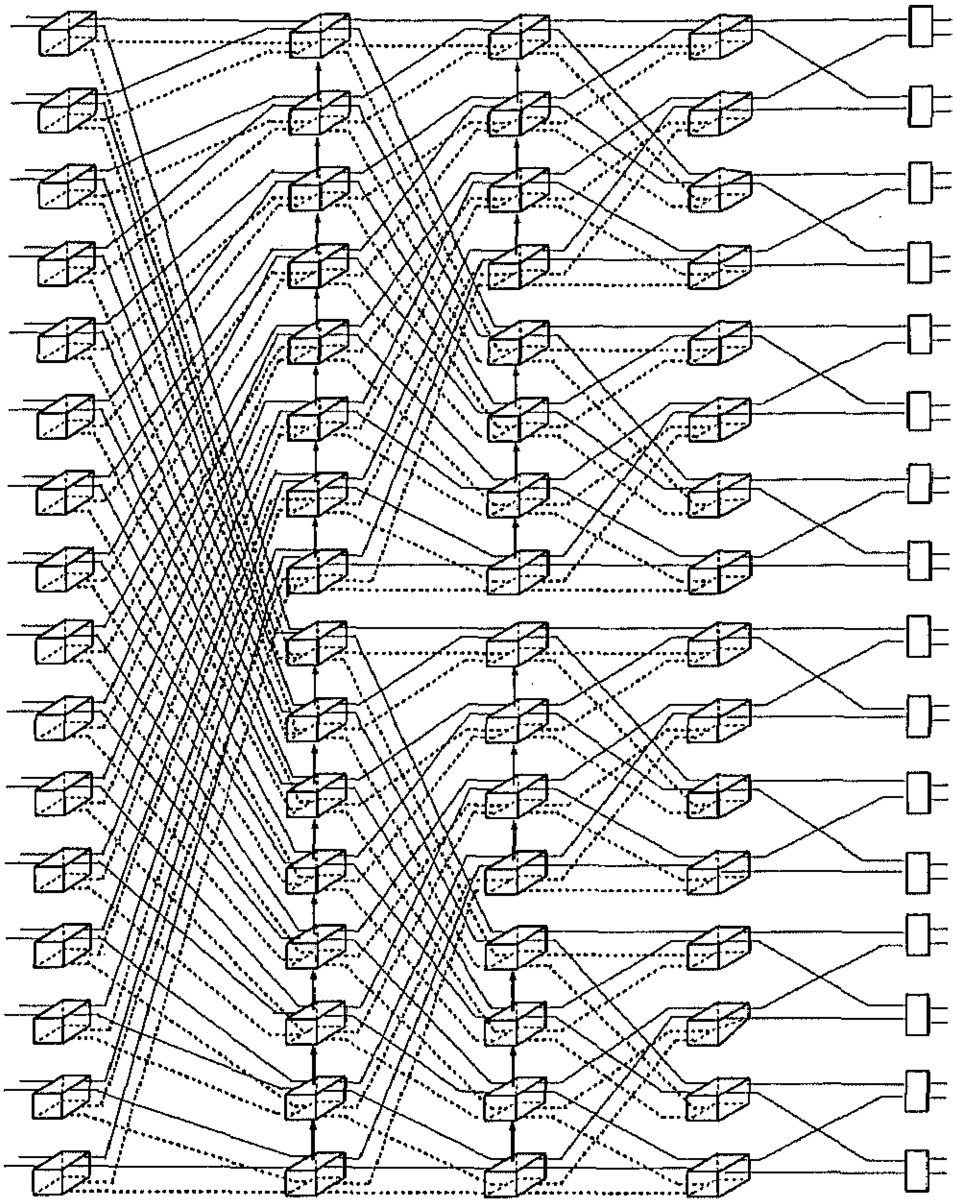

Figure 5.1: The structure of 32×32 hybrid dilated banyan network with bypasses at 2-dilated banyan switching elements.

bypasses of each case. In these figures, the numbers of bypasses in each hybrid dilated banyan network are not equal. From the constructions of each network, all output ports result in the same output rate.

From the equations in the previous chapter, the output rates of each kind of the hybrid dilated banyan networks with all-bypass-connection can be obtained. The result of $2^8 \times 2^8$ network, of which arrival rate (input rate) is 1.0, is shown in Fig. 5.3. In this figure, y axis shows the output rate of the network and x axis shows the number of stages which use 2-dilated banyan switching elements.

In Fig. 5.3,

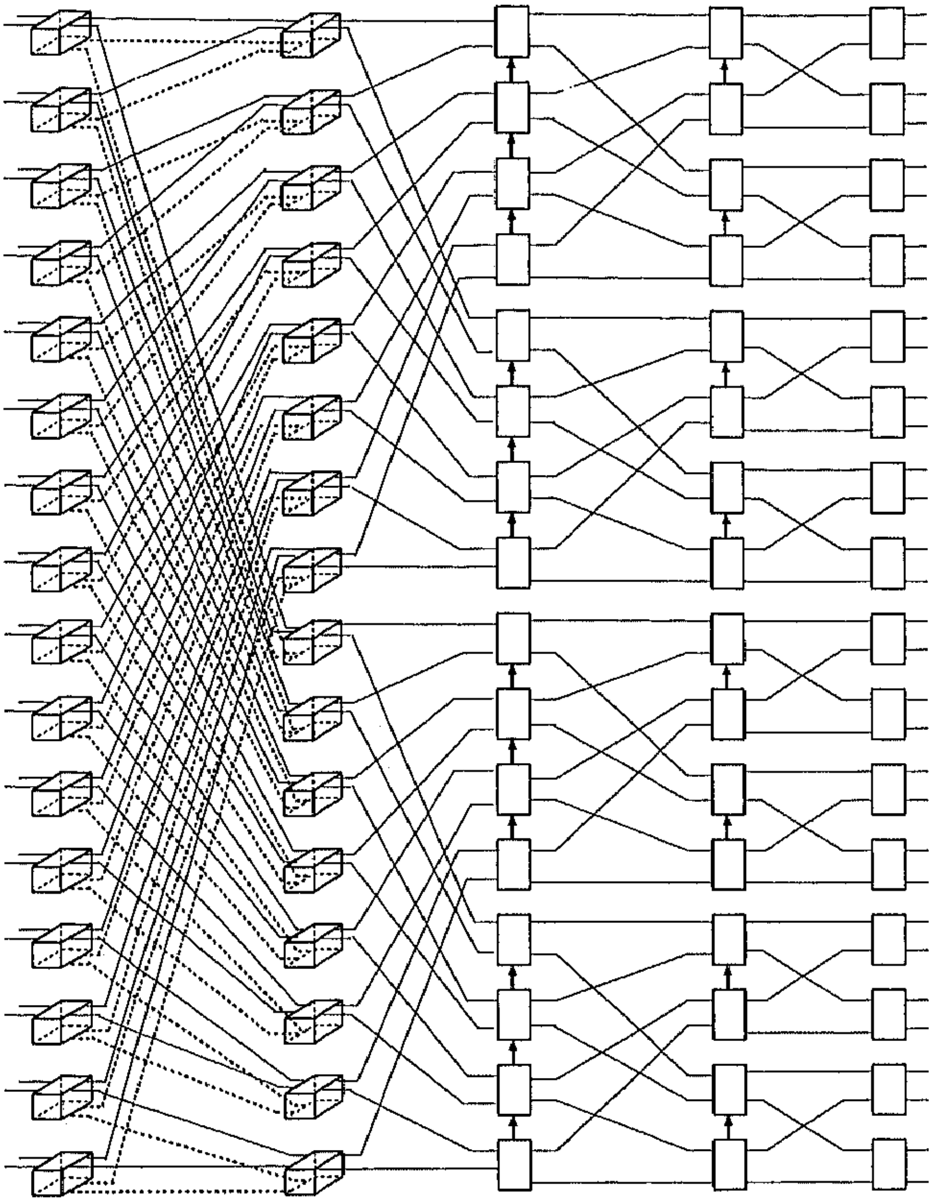

Figure 5.2: The structure of 32 × 32 hybrid dilated banyan network with bypasses at banyan switching elements.

- "2dilated-allby" shows the output rate of the 2-dilated banyan networks with all-bypass-connection,
- "2dilated-noby" shows the output rate of the 2-dilated banyan networks without bypass,
- "hy-allby" shows the output rate of the hybrid dilated banyan networks with all-bypass-connection except for 4 × 2 re-arrangeable output switching elements,
- "hy-by-di" shows the output rate of the hybrid dilated banyan networks with all-bypass-connection at the stages of 2-dilated banyan switching elements,

- "hy-by-ban" shows the output rate of the hybrid dilated banyan networks with all-bypass-connection at the stages of banyan switching elements,
- "hy-noby" shows the output rate of the hybrid dilated banyan networks without bypass,
- "banby" shows the output rate of the banyan network with all-bypass-connection and,
- "banyan" shows the output rate of the original banyan network.

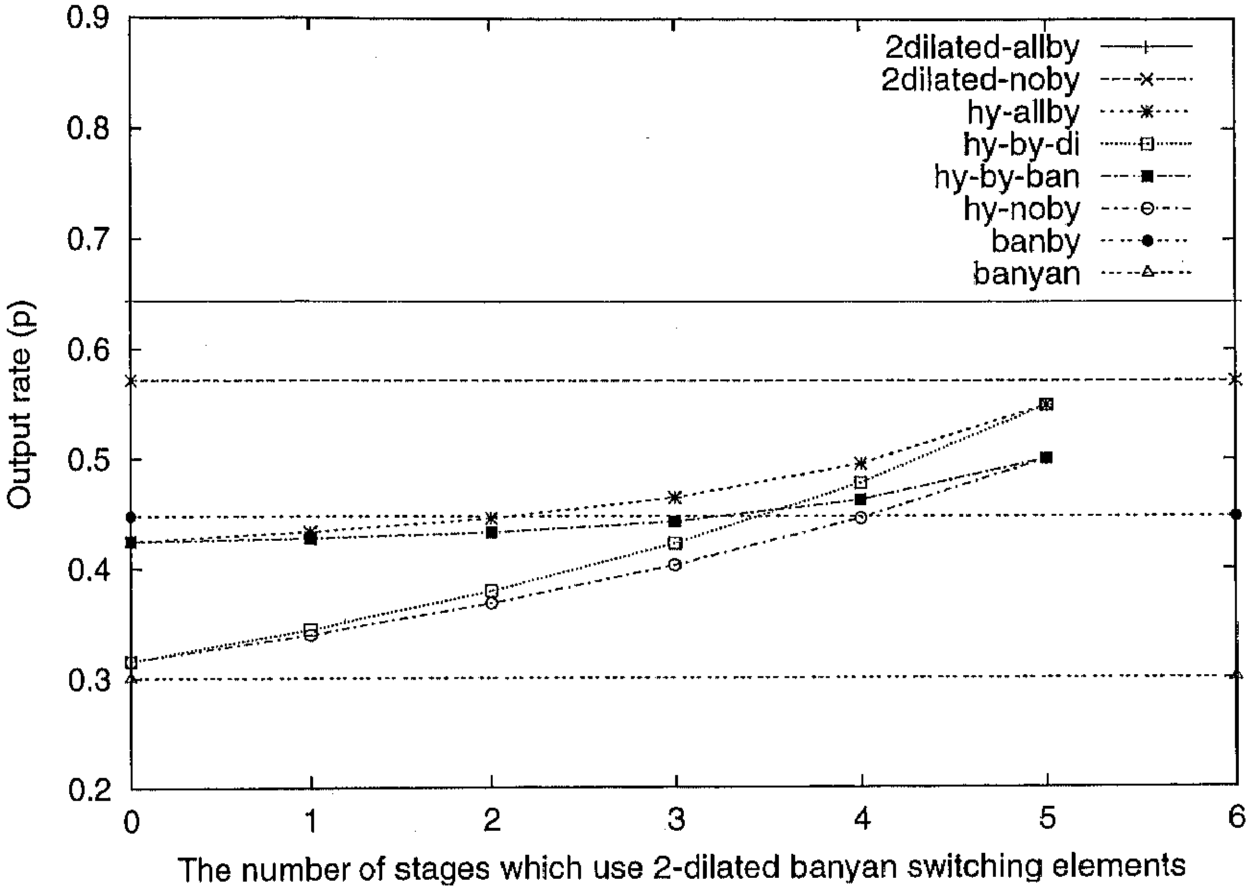

Figure 5.3: The output rate of $2^8 \times 2^8$ hybrid dilated banyan networks at input rate = 1.0.

From Fig. 5.3, the followings are concluded. All output rates of each sort of the hybrid dilated banyan networks is lower than ones of the 2-dilated banyan networks with and without bypasses ("2dilated-allby" and "2dilated-noby"), but greater than the output rate of the original banyan network ("banyan").

"hy-allby" is 34.6 to 9.8 percent, "hy-by-di" is 0 to 9.8 percent, but "hy-by-ban" is just 3.46 to 0 percent higher output rate than "hy-noby". When the number of stages of the 2-dilated banyan switching elements (written by x in below) increases, the number of stages using banyan switching elements $(6 - x)$ decreases. This fact causes that bypasses inserted in "hy-by-ban" decreases and the performance improvement becomes smaller when x is bigger. For example, when $x = 5$, the banyan switching elements exist at only final stage of "hy-by-ban." From the restriction of bypass position, no bypass can be connected in the final stage. Thus "hy-by-ban" equals to the "hy-noby" in this case.

The lines of "hy-by-ban" and "hy-by-di" cross together when x is between 3 and 4. The former is 34.6 to 4.7 percent higher than the latter ($x = 0 \sim 3$), but 3.4 to 9.8 percent lower ($x = 4 \sim 5$). In especial, we can find the difference of the effect to apply bypasses for each kind of switching element. The number of stages connecting bypasses at both "hy-by-ban" and "hy-by-di" is three when $x = 2$ for the former and $x = 3$ for the latter. However the former output rate is higher than the latter one. The same fact is also resulted when the number of bypass stages is zero to two (i.e., $x = 5 \sim 3$ for "hy-by-ban" and $x = 0 \sim 2$ for "hy-by-di"). For four or five bypass stages, this result is reversed. Therefore bypasses of the original banyan switching elements are more effective than those of the 2-dilated banyan switching elements, when they are inserted at less than four stages.

In final, the output rate of "hy-allby" is 34.6 to 3.6 percent higher than "hy-by-di" when $x = 0 \sim 4$, and 1.4 to 9.8 percent higher than "hy-by-ban" when $x = 1 \sim 5$. These facts result that the more bypasses are connected, the higher output rates are obtained.

5.2 Usage of 4×2 Re-Arrangeable Output Switching Element with Bypass in the Hybrid Dilated Banyan Network.

As discussed at the section 4.1, the bypasses at 4×2 re-arrangeable output switching elements improves the output rate of the restricted of hybrid dilated banyan network. This section focuses on inserting bypasses at the stage of 4×2 re-arrangeable output switching elements of any hybrid dilated banyan networks with bypasses in the previous section. More detailed proofs of this section shown in Appendix A.1.3.

5.2.1 Input Rate and Output Rate of 4×2 Re-Arrangeable Output Switching Elements with Bypasses

In subsection 4.1.2, the output rates of 4×2 re-arrangeable output switching elements with an input or an output bypass (called Type 1 and Type 3 in below) under the same input rate are solved. However each input rate of the 4×2 re-arrangeable output switching elements with a bypass differs each other in this section. The new 4×2 re-arrangeable output switching elements Type 2 with input and output bypasses is also needed. Each output rate of these switching elements under the different input rate is calculated as follows.

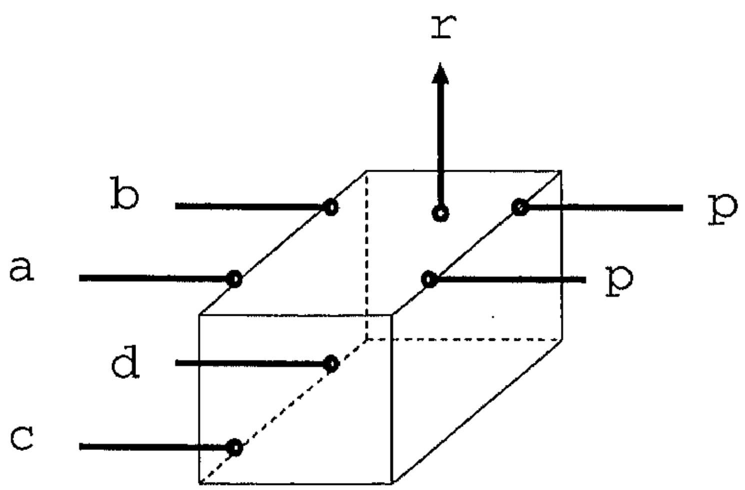

Figure 5.4: 4 × 2 re-arrangeable output switching element Type 1.

The 4 × 2 re-arrangeable output switching element Type 1 is shown in Fig. 5.4. Assume a, b, c and d as the input rate of the switching element, p as the output rate and r as the output rate of the cell forwarded to the bypass which is used as the bypass input rate of switching element Type 2 or type 3. Then p and r are related as follows:

$$\begin{aligned} p &= \frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c + \frac{1}{2}d - \frac{1}{4}ab - \frac{1}{4}ac - \frac{1}{4}bd \\ &\quad -\frac{1}{4}ad - \frac{1}{4}bc - \frac{1}{4}cd + \frac{1}{8}abc + \frac{1}{8}abd \\ &\quad +\frac{1}{8}acd + \frac{1}{8}bcd - \frac{1}{16}abcd, \qquad (5.1) \\ r &= \frac{1}{2}ab + \frac{1}{2}ac + \frac{1}{2}ad + \frac{1}{2}bc + \frac{1}{2}bd + \frac{1}{2}cd \\ &\quad -\frac{1}{2}abc - \frac{1}{2}abd - \frac{1}{2}acd - \frac{1}{2}bcd. \qquad (5.2) \end{aligned}$$

The 4 × 2 re-arrangeable output switching element Type 2 is shown in Fig. 5.5. Assume a, b, c and d as the input rate of the switching element, p as the output rate. q is the bypass input rate from the lower switching element Type 1 or Type 2 and r as the output rate of the cell forwarded to the bypass which is used as the bypass input rate of switching element Type 2 or Type 3. Then p and r are calculated as in below:

$$\begin{aligned} p &= \frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c + \frac{1}{2}d - \frac{1}{4}ab - \frac{1}{4}ac - \frac{1}{4}bd \\ &\quad -\frac{1}{4}ad - \frac{1}{4}bc - \frac{1}{4}cd + \frac{1}{8}abc + \frac{1}{8}abd \end{aligned}$$

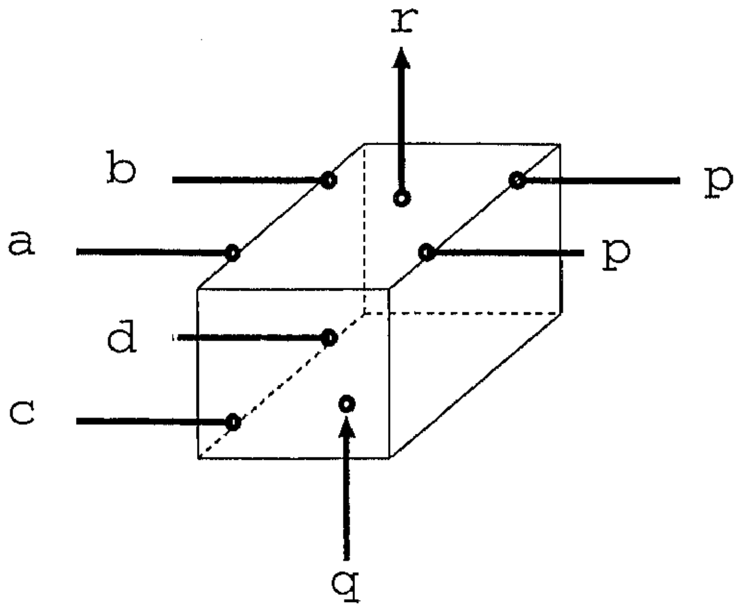

Figure 5.5: 4 × 2 re-arrangeable output switching element Type 2.

$$
\begin{aligned}
& +\frac{1}{8}acd + \frac{1}{8}bcd - \frac{1}{16}abcd \\
& +\frac{1}{2}q - \frac{1}{4}aq - \frac{1}{4}bq - \frac{1}{4}cq - \frac{1}{4}dq \\
& +\frac{1}{8}abq + \frac{1}{8}acq + \frac{1}{8}adq + \frac{1}{8}bcq + \frac{1}{8}bdq + \frac{1}{8}cdq \\
& -\frac{1}{16}abdq - \frac{1}{16}acdq - \frac{1}{16}bcdq - \frac{1}{16}abcq + \frac{1}{32}abcdq, \qquad (5.3) \\
r \;=\; & \frac{1}{2}ab + \frac{1}{2}ac + \frac{1}{2}ad + \frac{1}{2}bc + \frac{1}{2}bd + \frac{1}{2}cd \\
& -\frac{1}{2}abc - \frac{1}{2}abd - \frac{1}{2}acd - \frac{1}{2}bcd \\
& +\frac{1}{2}aq + \frac{1}{2}bq + \frac{1}{2}cq + \frac{1}{2}dq - \frac{1}{2}abq \\
& -\frac{1}{2}acq - \frac{1}{2}adq - \frac{1}{2}bcq - \frac{1}{2}bdq - \frac{1}{2}cdq + abcdq. \qquad (5.4)
\end{aligned}
$$

Note that (5.1) and (5.2) equal to (5.3) and (5.4), respectively, if $q = 0$.

The 4 × 2 re-arrangeable output switching Type 3 is shown in Fig. 5.6. Assume a, b, c and d as the input rate of the switching element, p as the output rate. q is the bypass input rate from the lower switching element Type 1 or Type 2. Then p is calculated as in below:

$$
\begin{aligned}
p \;=\; & \frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c + \frac{1}{2}d - \frac{1}{4}ab - \frac{1}{4}ac - \frac{1}{4}bd \\
& -\frac{1}{4}ad - \frac{1}{4}bc - \frac{1}{4}cd + \frac{1}{8}abc + \frac{1}{8}abd \\
& +\frac{1}{8}acd + \frac{1}{8}bcd - \frac{1}{16}abcd
\end{aligned}
$$

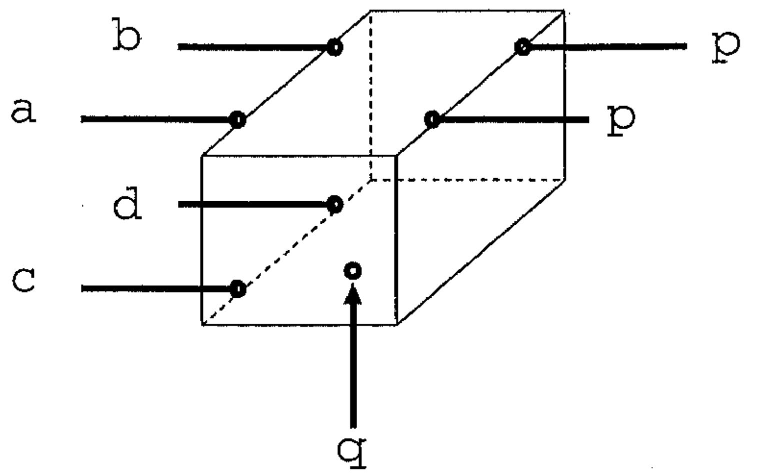

Figure 5.6: 4 × 2 re-arrangeable output switching element Type 3.

$$
\begin{aligned}
&+\frac{1}{2}q - \frac{1}{4}aq - \frac{1}{4}bq - \frac{1}{4}cq - \frac{1}{4}dq \\
&+\frac{1}{8}abq + \frac{1}{8}acq + \frac{1}{8}adq + \frac{1}{8}bcq + \frac{1}{8}bdq + \frac{1}{8}cdq \\
&-\frac{1}{16}abdq - \frac{1}{16}acdq - \frac{1}{16}bcdq - \frac{1}{16}abcq + \frac{1}{32}abcdq. \qquad (5.5)
\end{aligned}
$$

Note that (5.5) coincides with (5.3).

By using all of the above equations, we can analyze all output rates of the hybrid dilated banyan networks with bypasses included at the stage of 4 × 2 re-arrangeable output switching elements.

5.2.2 Comparisons of the Output Rates of Hybrid Dilated Banyan Networks with and without Bypasses at the stage of 4 × 2 Re-Arrangeable Output Switching Elements

The results of three types of $2^8 \times 2^8$ hybrid dilated banyan networks with and without bypasses at 4 × 2 re-arrangeable output switching elements at the arrival rate (input rate) 1.0 are shown in Fig. 5.7 to Fig. 5.9. In these figures, x axis and y axis equal to those of Fig. 5.3, respectively.

The bypasses in the hybrid dilated banyan networks are connected at both 2-dilated and original banyan switching elements in Fig. 5.7. They are also inserted only at the former switching elements in Fig. 5.8 and at the latter in Fig. 5.9. In these figures,

- "hy-allby-42" shows the output rate of the hybrid dilated banyan networks with all-bypass-connection at the whole stages including the stage of 4 × 2 re-arrangeable output switching elements.

- "hy-by-di-42" shows the output rate of the hybrid dilated banyan networks with all-bypass-connection at the stage of 4×2 re-arrangeable output switching elements and the 2-dilated banyan switching elements.

- "hy-by-ban-42" shows the output rate of the hybrid dilated banyan networks with all-bypass-connection at the stage of 4×2 re-arrangeable output switching elements and the banyan switching elements. Note that the meaning of other groups have already identified in Fig. 5.2.

From Fig. 5.7, "hy-allby-42" has 42.0 to 14.6 percent higher output rate than "hy-noban", but 4.5 to 6.8 percent higher than "hy-allby" by the bypasses at the stage of 4×2 re-arrangeable output switching elements. This results because insertion of bypasses into the stages of the 2-dilated banyan switching elements has less effective than the one into the stages of the banyan switching elements, as the previous chapter mentioned.

From Fig. 5.8, "hy-by-di-42" is 5.9 to 14.6 percent higher than "hy-noby" and 4.4 to 7.4 percent higher than "hy-by-di". From Fig. 5.9, "hy-by-ban-42" is 42.0 to 5.1 percent higher than "hy-noby" and 5.1 to 7.8 percent higher than "hy-by-ban". Fig. 5.7 to Fig. 5.9 show that the bypass-connection at the stage of 4×2 re-arrangeable output switching elements contribute to improve the output rate of the networks.

Fig. 5.10 indicates the comparison of "hy-allby-42", "hy-by-di-42" and "hy-by-ban-42". "hy-allby-42" is 34.0 to 0 percent higher output rate than "hy-by-di-42" and 0 to 9.2 percent higher than "hy-by-ban-42". When the number of stages of the 2-dilated banyan switching elements $x = 0$ to 3, "hy-by-ban-42" is 43 to 5.1 percent higher than "hy-by-di-42". However, when $x = 4$ or 5, the former output rate is 2.9 to 9.2 percent lower output rate than the latter. This fact also comes from the difference of the bypass effect at the original and the 2-dilated banyan switching elements, as in the one of "hy-allby", "hy-by-di" and "hy-by-ban" as shown in Fig. 5.3.

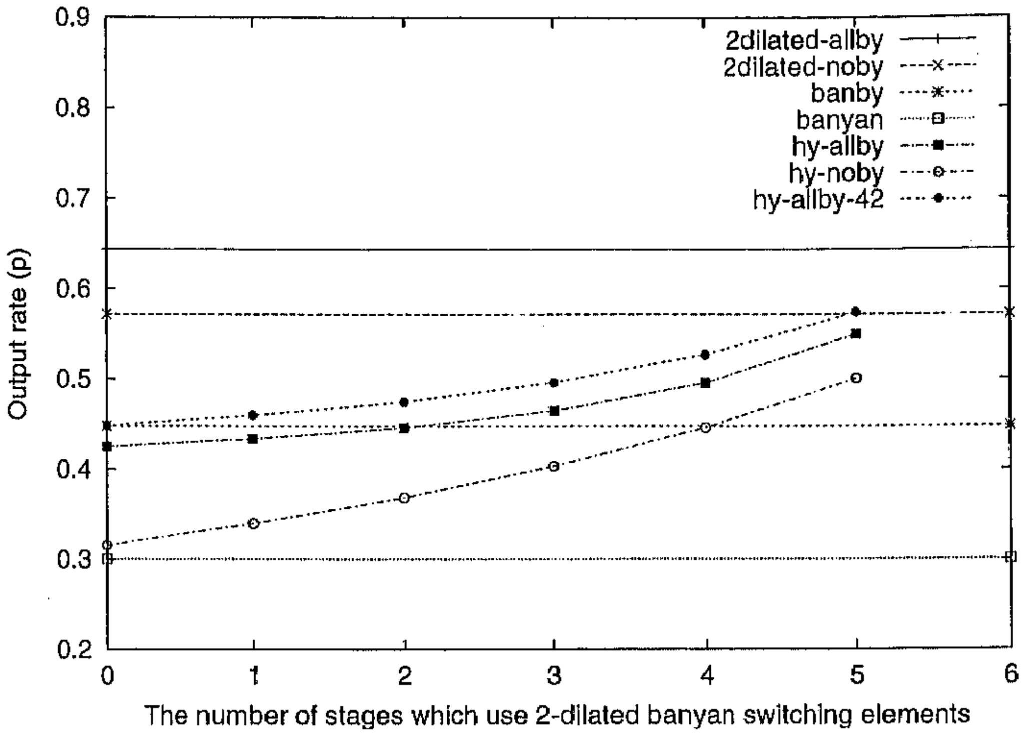

Figure 5.7: The output rate of hybrid dilated banyan networks with bypasses at the whole stages including the stage of 4×2 re-arrangeable output switching elements.

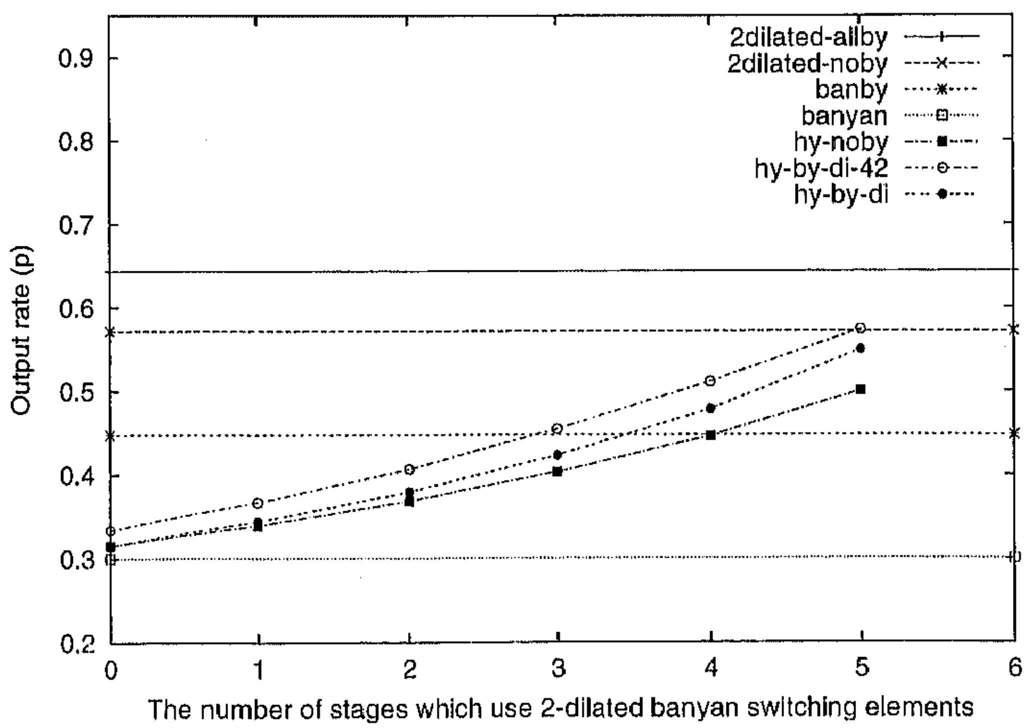

Figure 5.8: The output rate of hybrid dilated banyan networks with bypasses and at the stages of 2-dilated banyan switching elements and 4×2 re-arrangeable output switching elements.

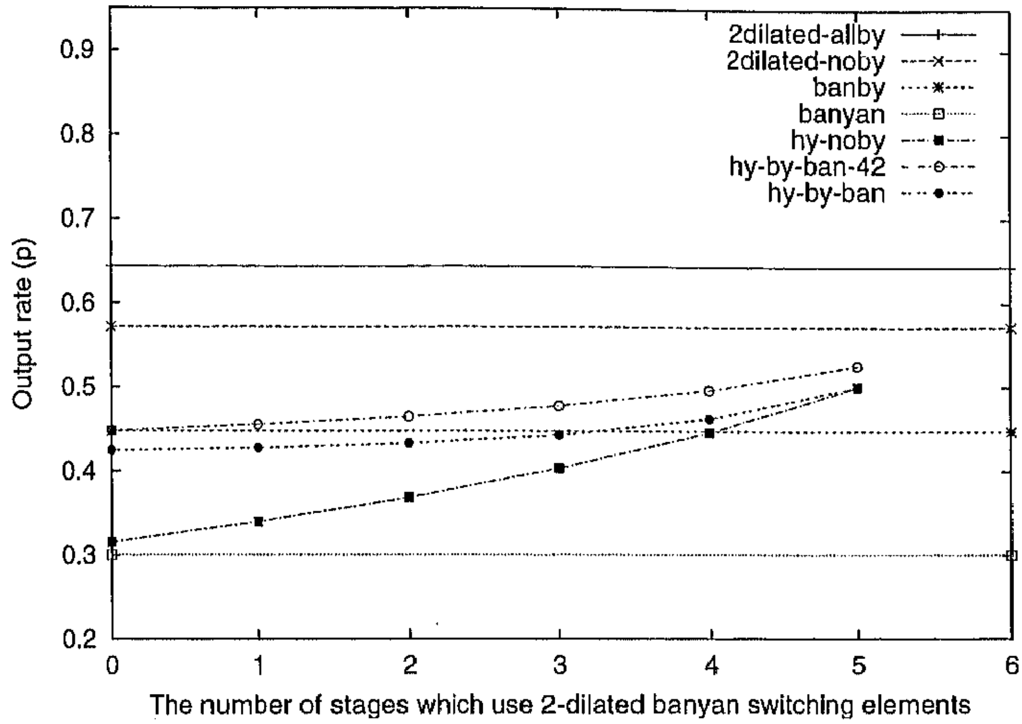

Figure 5.9: The output rate of the networks with bypasses at the stages of banyan switching elements and 4×2 re-arrangeable output switching elements.

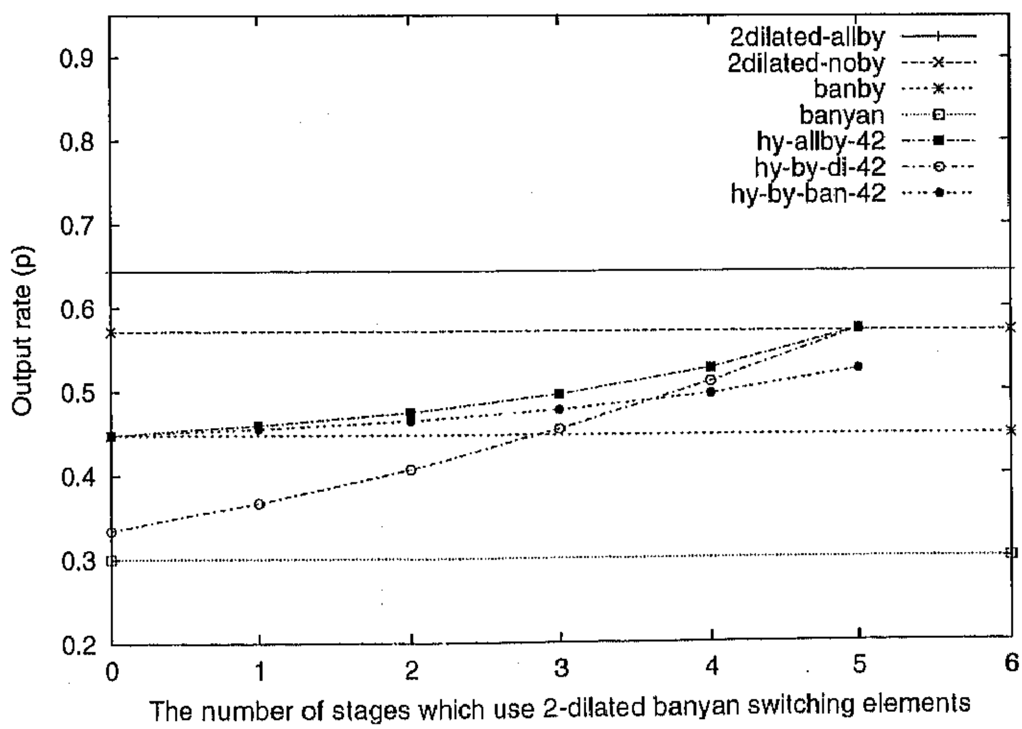

Figure 5.10: The output rate of the networks with all-bypass-connection at the stage of 4×2 re-arrangeable output switching elements.

5.3 Hybrid Dilated Banyan Networks with One-Bypass-Connection

To reduce delay of switching time at all-bypass-connection in the previous section, hybrid dilated banyan networks with one-bypass-connection is also introduced. As in section 4.4, each bypass is connected between switching elements at $(2m-1)$-th and $2m$-th columns in all stages except the first and the final ones so as to maximize the number of bypasses in the $N \times N$ network, where $m = 1, ..., \frac{N}{2}$. Fig. 5.11 illustrates a 32×32 hybrid dilated banyan network with one-bypass-connection.

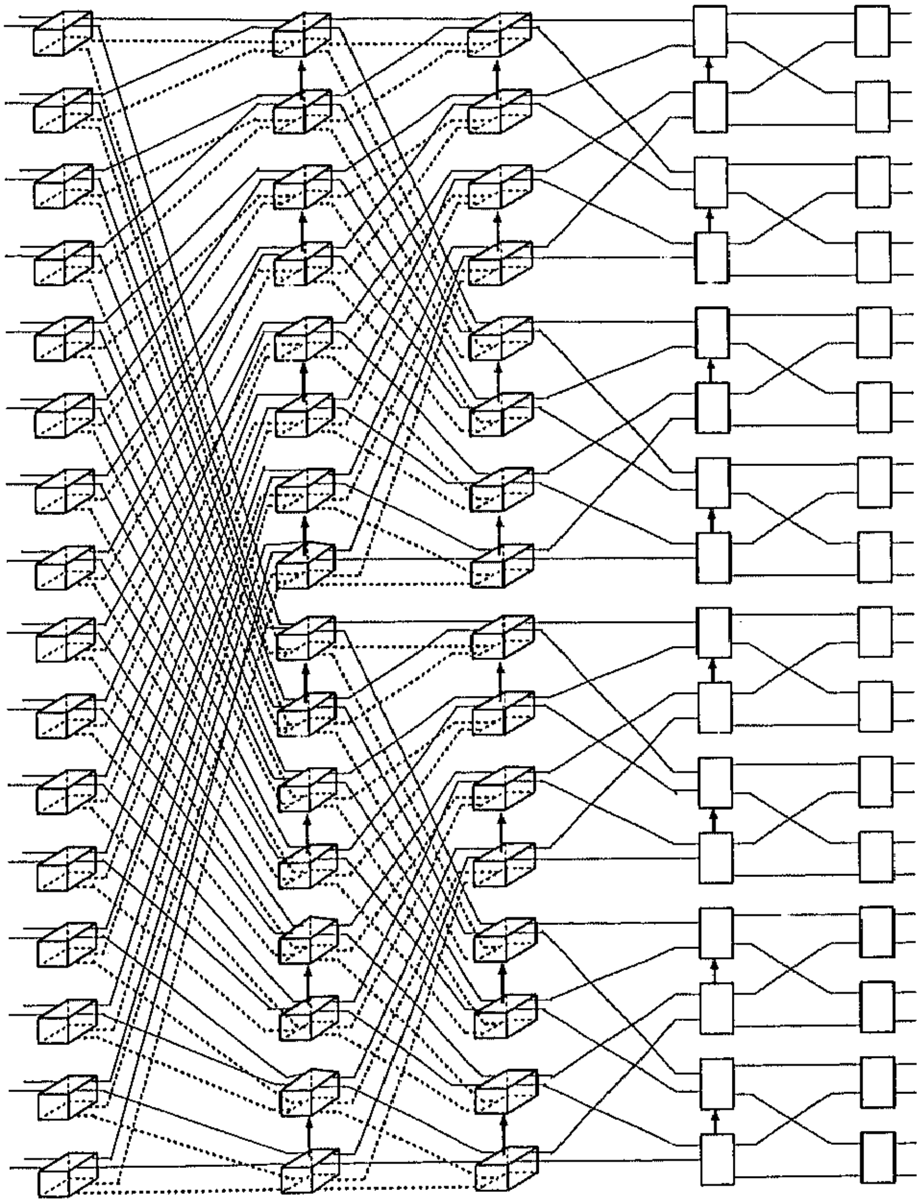

Figure 5.11: 32×32 hybrid dilated banyan network with one-bypass-connection.

Fig. 5.12 to Fig. 5.14 show the output rate of the $2^8 \times 2^8$ hybrid dilated banyan networks with one-bypass-connection at an input rate 1.0. In these figures,

- "hy-one" shows the output rate of the hybrid dilated banyan networks with one-bypass-connection when no bypass is inserted at the stage of 4×2 re-arrangeable output switching elements.

- "hy-one-42" shows the output rate of the hybrid dilated banyan networks with one-bypass-connection when the bypasses are also connected at the stage of 4×2 re-arrangeable output switching elements.

From Fig. 5.12, "hy-one" is 14.9 to 4.3 percent higher than "hy-noby", but 17.2 to 3.9 percent lower than "hy-allby".

From Fig. 5.13, "hy-one-42" is also 17.5 to 9.5 percent higher than "hy-noby", but 20.8 to 4.7 percent lower than "hy-allby-42". As mentioned in section 4.4, the one-bypass-connection occurs decrement of output rates compared to all-bypass-connection, but essentially decreases the switching delay of each network. Thus this connection method provides a useful choice to network designers.

From Fig. 5.14, "hy-one-42" is 2.3 to 4.9 percent higher than "hy-one". This result also shows the effect of the bypasses in the stage of 4×2 re-arrangeable output switching elements.

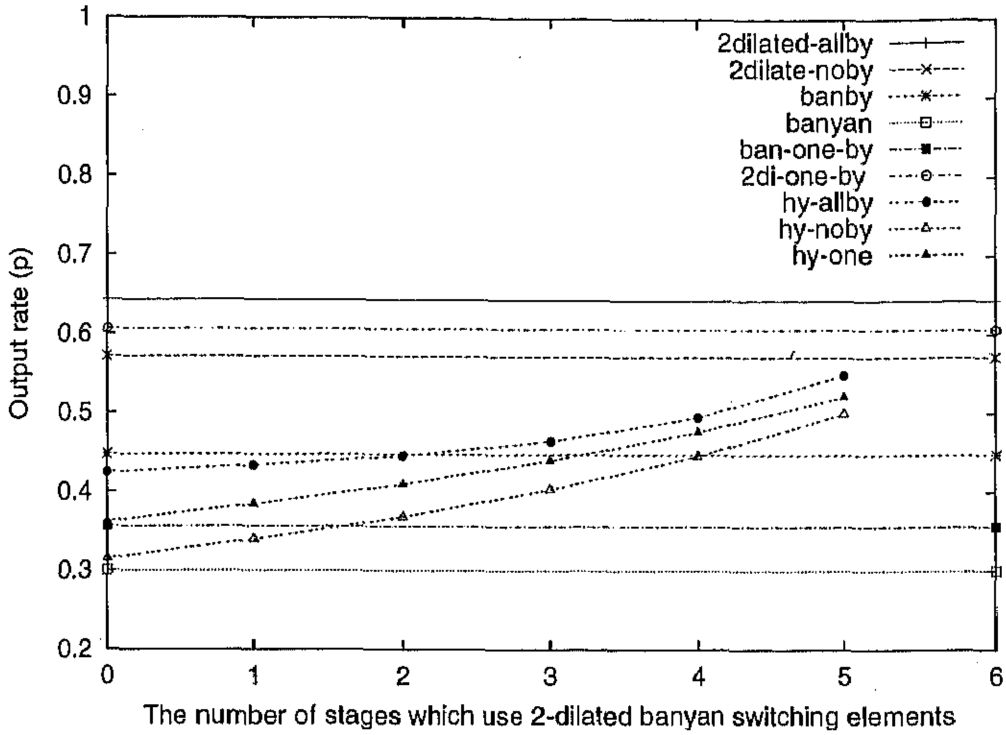

Figure 5.12: The output rate of hybrid dilated banyan networks with one-bypass-connection under the condition of 4×2 re-arrangeable output switching elements have no bypass.

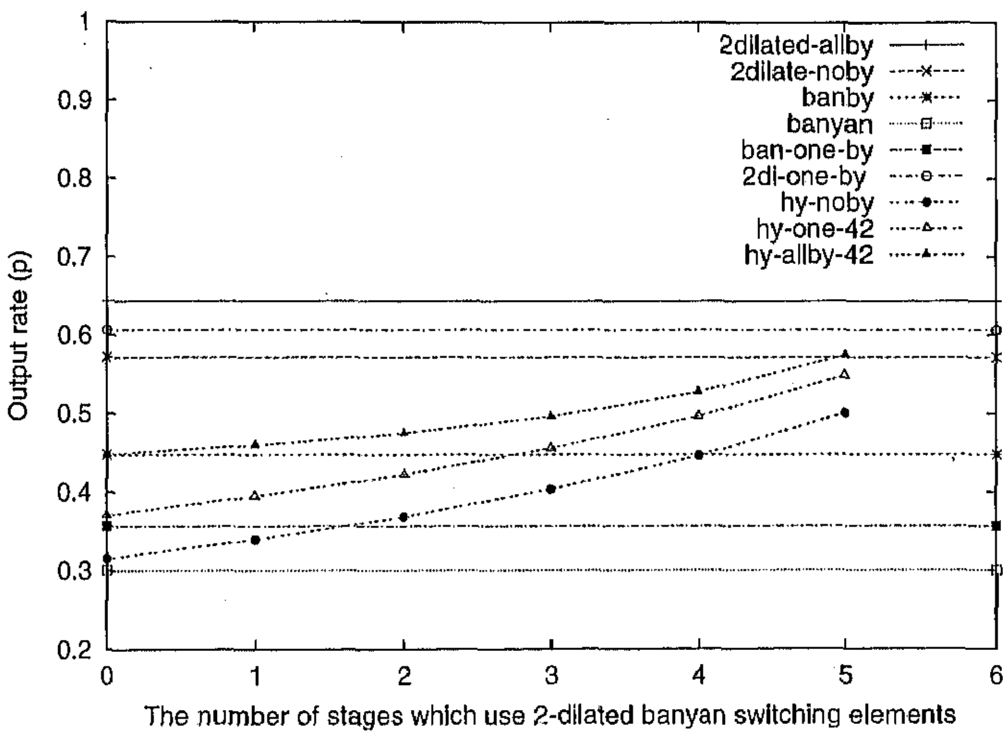

Figure 5.13: The output rate of hybrid dilated banyan networks with all-bypass-connection or one-bypass-connection under the condition of whether 4×2 re-arrangeable output switching elements have bypasses or not.

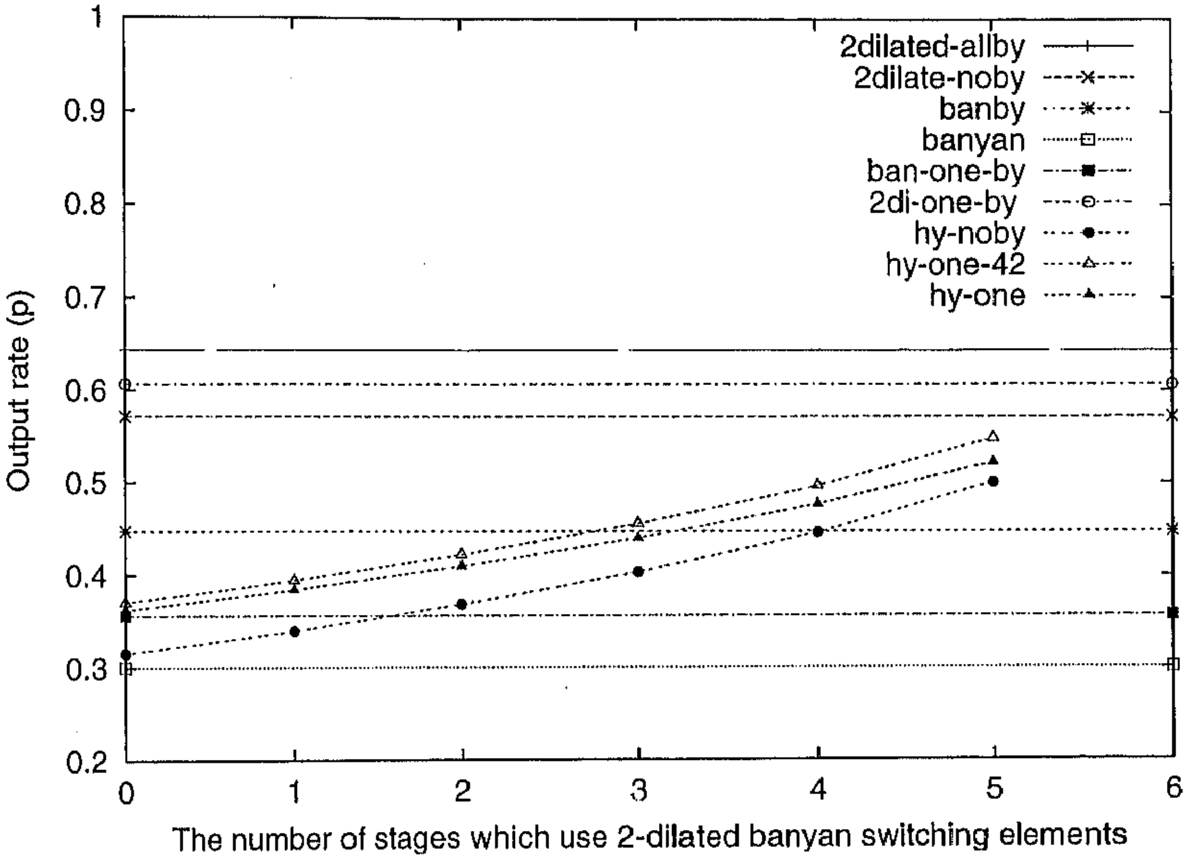

Figure 5.14: The output rate of hybrid dilated banyan networks with one-bypass-connection or without bypass under the condition of whether 4×2 re-arrangeable output switching elements have bypasses or not.

5.4 Hybrid Dilated Banyan Networks with One-Bypass-Connection at Two Neighbored Stages

In this section, one-bypass-connection at two neighbored stages is introduced in the hybrid dilated banyan networks to compare the effective of output rates by changing a position of one-bypass-connection. As in the case of the 2-dilated banyan networks described in section 4.5, the insertion position of one-bypass-connection are the second and third stages, the fourth and fifth stages, and the sixth and seventh stages. At each inserted stage, every $(2m-1)$-th switching element and $2m$-th switching element is connected by a bypass, where $m = 1, \ldots, \frac{N}{2}$ for the $N \times N$ network. Note that no blocking occurs at the first stage of hybrid dilated banyan network.

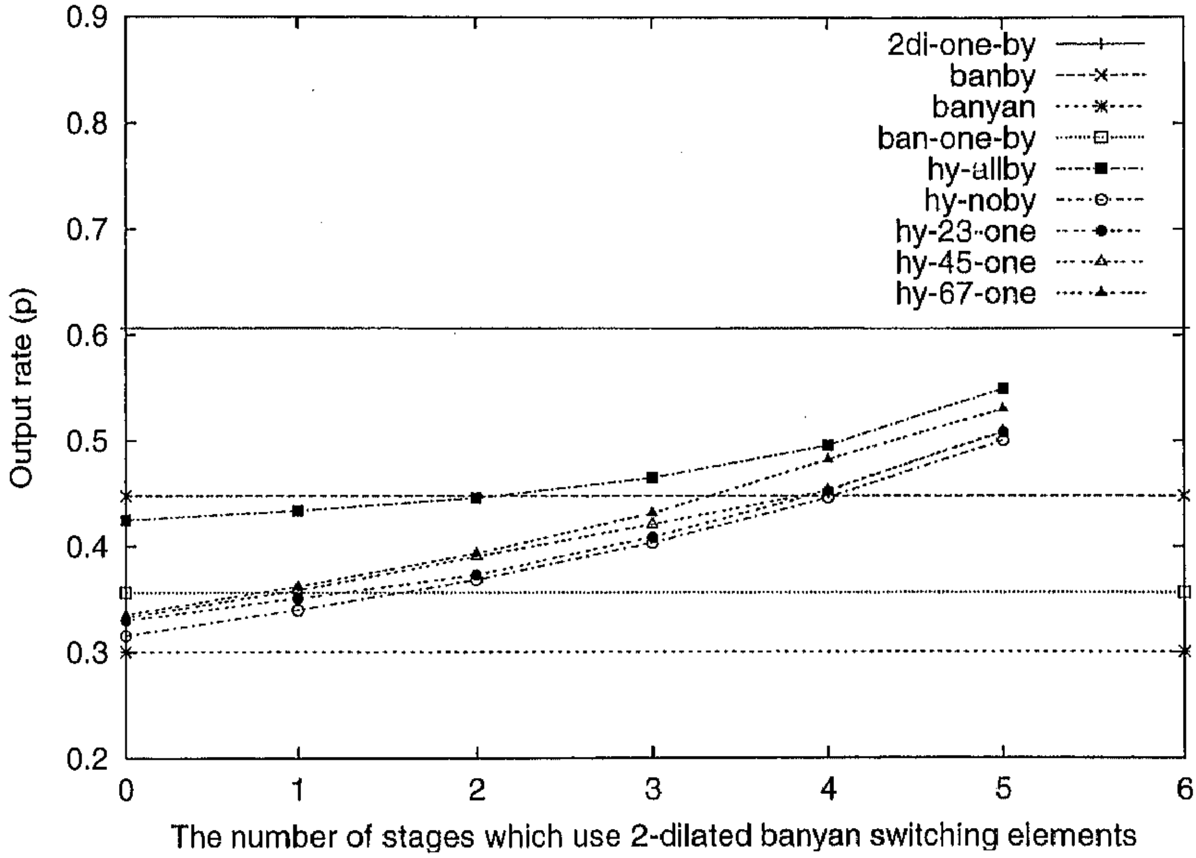

Figure 5.15: The output rate of hybrid dilated banyan networks with one-bypass-connection at two neighbored stages.

At the input rate 1.0, Fig. 5.15 shows the output rate of the $2^8 \times 2^8$ hybrid dilated banyan networks with one-bypass-connection at two neighbored stages. All plots in this figure are introduced until the previous section except the followings.

- "hy-23-one" shows the output rate of the hybrid dilated banyan networks with one-bypass-connection at the second and third stages.

- "hy-45-one" shows the output rate of the hybrid dilated banyan networks with one-bypass-connection at the fourth and fifth stages.

- "hy-67-one" shows the output rate of the hybrid dilated banyan networks with one-bypass-connection at the sixth and seventh stages.

This section focuses on the difference output rates in one-bypass-connection at two neighbored stages. From Fig. 5.15, "hy-23-one", "hy-45-one" and "hy-67-one" are 4.5 to 1.2 percent, 5.5 to 1.5 percent and 8.1 to 5.9 percent higher output rate than "hy-noby", respectively. The stages with bypasses in "hy-23-one" have 2-dilated banyan switching elements when the number of stages (x) with 2-dilated banyan switching elements is 2 to 5. However, ones in "hy-67-one" have the original banyan switching elements when x is 0 to 3. Thus, the difference of output rates by changing the bypass position is also related to the one of the sort of switching elements connected by a bypass. As we have found in this thesis, the bypass between banyan switching elements is more effective than the bypass between 2-dilated banyan switching elements. From this fact, "hy-67-one" has the highest output rate among the three types of the one bypass connecting at two neighbored stages.