Chapter 4

Networks with Bypasses

In the previous chapter, the construction of a hybrid dilated banyan network and its performance analysis are presented. From the result of this analysis, we found that the probability of cell loss at the 4×2 re-arrangeable output switching element is extremely larger than the one of other switching elements. This chapter introduces bypasses to the 4×2 re-arrangeable output switching element to improve the output rate of the hybrid dilated banyan network[23]. Since the bypasses are very effective to increase the output rate of other networks, they are introduced to all switching elements of the original and the 2-dilated banyan networks[24, 25]. However we can notice that the bypasses also increase delay for switching. To solve this problem, the one-bypass-connection method is proposed and its performance analysis is provided[25]. In this chapter, the size of each network is assumed $N \times N$ and $n = \log_2 N$ when they are not specially mentioned.

4.1 4×2 Re-Arrangeable Output Switching Element with Bypasses

The purpose of a bypass is to forward a blocked cell to the upper switching element of the same stage to decrease the internal blocking in the network. But all switching element in the same stages of the network can not be connected with bypasses because a wrong bypass may forward cells to the incorrect destinations. In the case of introducing bypasses at the d-th stage ($1 \leq d \leq n-1$), the switching elements from $((m-1)2^{n-d}+1)$-th to $m2^{n-d}$-th columns can be sequentially connected by bypasses, where $m = 1, \ldots, 2^{d-1}$. Note that no bypass at n-th stage is allowed because a cell may be forwarded to the incorrect output port.

In this thesis, each bypass is connected two switching elements at i-th and $(i+1)$-th columns, and directed from i-th lower and $(i+1)$-th upper one, where $(m-1)2^{n-d}+1 \leq i \leq m2^{n-d}-1$. In other words, a blocked cell is forwarded from the lower to the upper switching element through a bypass. If there is a bypass from $((m-1)2^{n-d}+1)$-th to $m2^{n-d}$-th switching elements, it makes a bypass loop. This loop may decrease cell loss rate of its network, however, cell transfer delay of the network expands about twice and each switching element is more complicated to prevent any cells rounding on the loop. For two switching elements connected by one or more bypasses each other, the lower one must finish processing cells before the upper one begins, because of the bypass direction. Since a sequence control of cells is not concerned in ATM, all cells in the same stage must be forwarded to the next stage after cells at the top switching element have been processed. In the next section, we will discuss about cell transfer delay according to this fact.

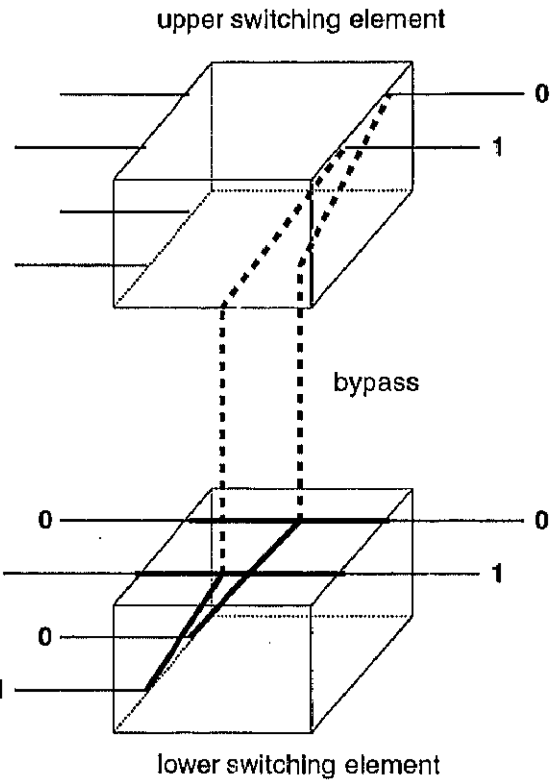

Figure 4.1: 4 × 2 re-arrangeable upper switching element and lower output switching element with two bypasses.

This section focuses on the effectiveness of bypasses at the 4 × 2 re-arrangeable output switching element in an $N \times N$ hybrid dilated banyan network. For simple analysis, the construction of this network is fixed as follows: the first $(n-1)$ stages are a 2-dilated banyan network and only final stage is a banyan network. Therefore, the 4 × 2 re-arrangeable output switching elements are inserted at the $(n-1)$-th stage. This construction provides the maximum output rate among the same size of hybrid dilated banyan networks. From the restriction of the bypass position, at most two switching elements are allowed to be connected bypasses as shown in Fig. 4.1.

4.1.1 Usage of One or Two Bypasses at the Stage of 4×2 Re-Arrangeable Output Switching Elements

In 4×2 re-arrangeable output switching element, up to three cells can be blocked. If such three blocked cells are forwarded, two of them must be blocked. Therefore, more than three bypasses are not useful in the case of Fig. 4.1. In this figure, four cells are assumed to be entered in the lower switching element. Two of them require the output port 0 and the other two request the output port 1. But each output port blocked cells. If both blocked cells are forwarded to the upper switching element and no cell arrives from both input ports at this switching element, they can be successfully rerouted through the bypasses.

From the above reason, this section limits the number of bypasses between two 4×2 re-arrangeable output switching elements to two.

Fig. 4.2 and Fig. 4.3 depict 16×16 hybrid dilated banyan network with one or two bypasses at the stage of 4×2 re-arrangeable output switching elements, respectively. At these figures, 4×2 re-arrangeable output switching elements are set at the stage 3.

4.1.2 Input Rate and Output Rate of Hybrid Dilated Banyan Network with One or Two Bypasses at the Stage of 4×2 Re-Arrangeable Output Switching Elements

Under the conditions in the section 2.3, the output rates of the 4×2 re-arrangeable output switching element with one or two bypasses are analyzed for the following two cases. For more details, see Appendix A.1.3 and A.1.4.

1. For the lower 4×2 re-arrangeable output switching element with one or two bypasses, the output rate p_{out} is calculated as follows:

$$p_{out} = 2p_{in} - \frac{3}{2}{p_{in}}^2 + \frac{1}{2}{p_{in}}^3 - \frac{1}{16}{p_{in}}^4, \tag{4.1}$$

   where p_{in} is the input rate at the input port of this switching element. Note that the above output rate is equal to (2.4), since the number of input bypass in the lower switching element is zero.

2. For the upper 4×2 re-arrangeable output switching element with a bypass, the output rate p_{out} is shown in below:

$$p_{out} = 2p_{in} - \frac{7}{2}p_{in}^3 + \frac{67}{16}p_{in}^4 - \frac{9}{4}p_{in}^5 + \frac{19}{32}p_{in}^6 - \frac{1}{16}p_{in}^7, \tag{4.2}$$

   and in the case of two bypasses, the output rate p_{out} is:

$$p_{out} = 2p_{in} - 3p_{in}^3 + \frac{21}{8}p_{in}^4 - \frac{21}{16}p_{in}^6 + \frac{15}{16}p_{in}^7 - \frac{81}{256}p_{in}^8 + \frac{7}{128}p_{in}^9 - \frac{1}{256}p_{in}^{10}, \tag{4.3}$$

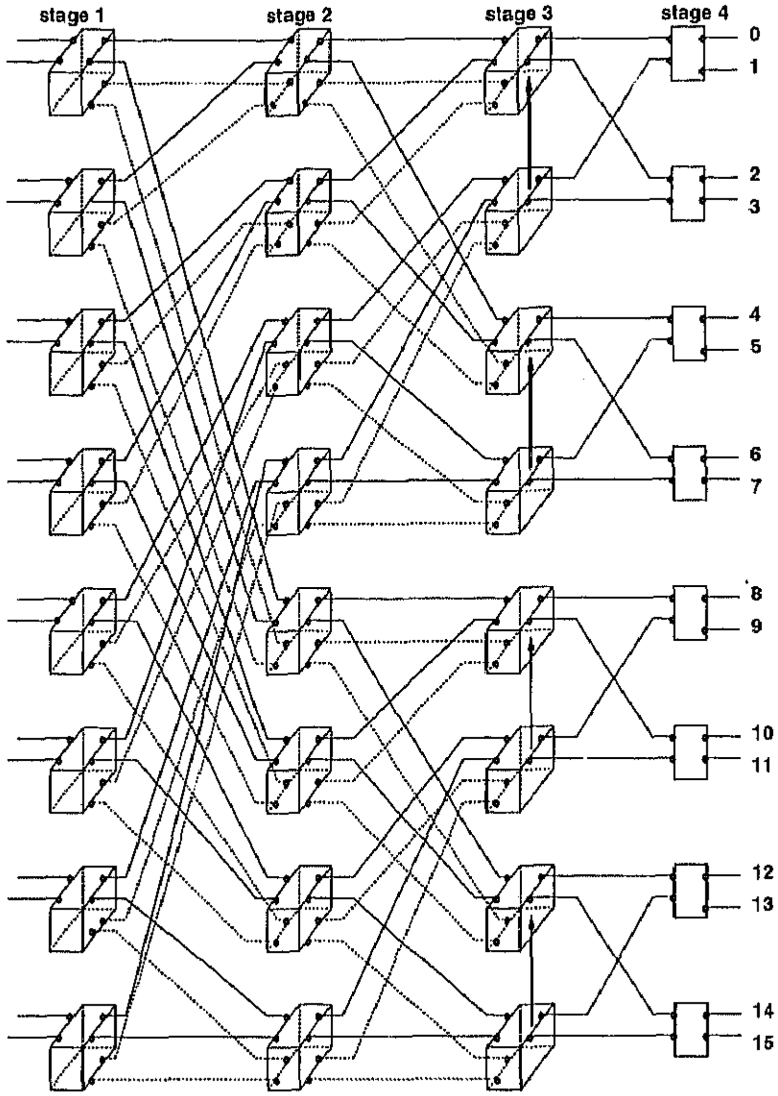

Figure 4.2: 16 × 16 hybrid dilated banyan network with a bypass at the stage of 4 × 2 re-arrangeable output switching element.

where p_{in} is the input rate at the input port of each upper switching element.

From the above equations, let p_{low} and p_{up} be the output rates p_{out} at the output ports of the lower and upper 4 × 2 re-arrangeable output switching element with one or two bypasses, respectively. As the final stage of Fig. 4.2 or Fig. 4.3 is illustrated, the original banyan network switching element is connected with both lower and upper 4×2 re-arrangeable output switching elements. Since $p_{low} \neq p_{up}$, both input rates at this switching element differ each other. Therefore, the output rate p_{out} is not obtained from (2.1). From Appendix A.1.1, p_{out} is calculated as below:

$$p_{out} = \frac{1}{2}p_{low} + \frac{1}{2}p_{up} - \frac{1}{4}p_{low}p_{up}. \tag{4.4}$$

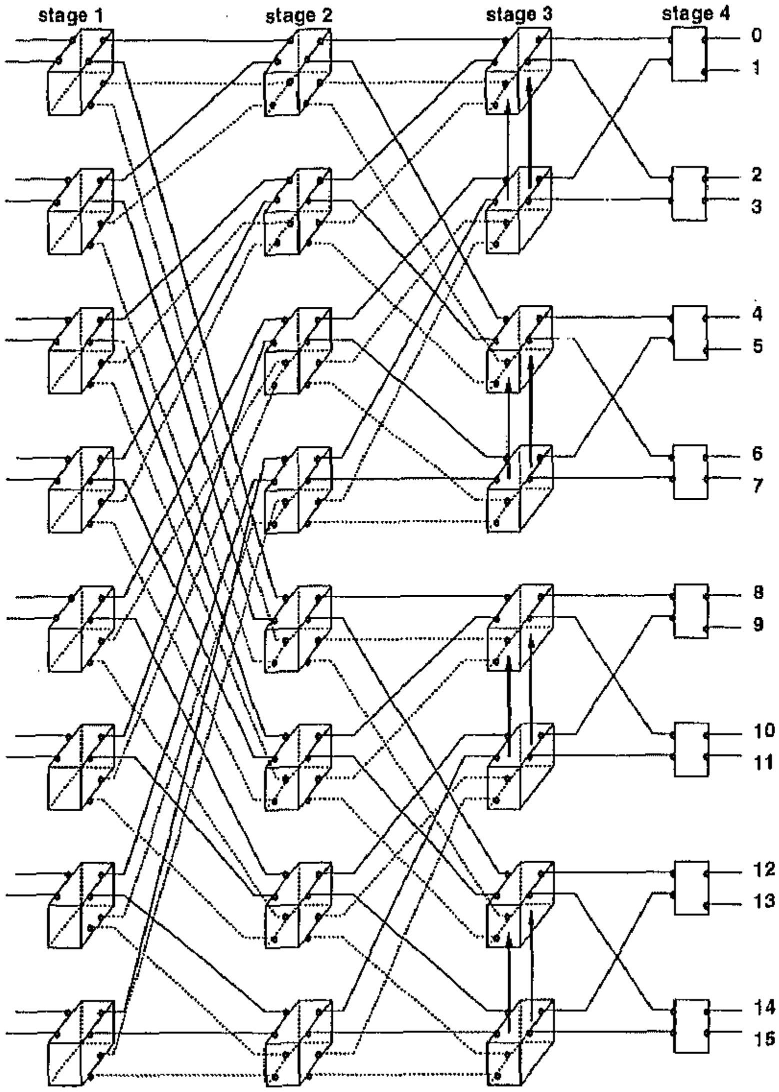

Figure 4.3: 16 × 16 hybrid dilated banyan network with two bypasses at the stage of 4 × 2 re-arrangeable output switching element.

By using all presented equations, Fig. 4.4 shows the comparisons among three types of hybrid dilated banyan network: hybrid dilated banyan network without bypass, hybrid dilated banyan network with a bypass at the stage of 4 × 2 re-arrangeable output switching element and hybrid dilated banyan network with two bypasses at the stage of 4 × 2 re-arrangeable output switching element with the arrival rate (input rate) 1.0. The x and y axles in this figure mean the output rate and the number of the stages, respectively.

The plots of "i" ($i = 3 \sim 9$) and "i-j" ($j = 1, 2$) at i stages are the output rates of the $2^i \times 2^i$ hybrid dilated banyan network, where "i" has no bypass and "i-j" has j bypasses at the stage of 2 × 4 re-arrangeable output switching element. At the $(i-1)$ and the $(i-2)$ stages, these plots shows the output rates at the lower 2 × 4 re-arrangeable output switching element and at the 2-dilated banyan switching element in the $(i-2)$-th stage, respectively.

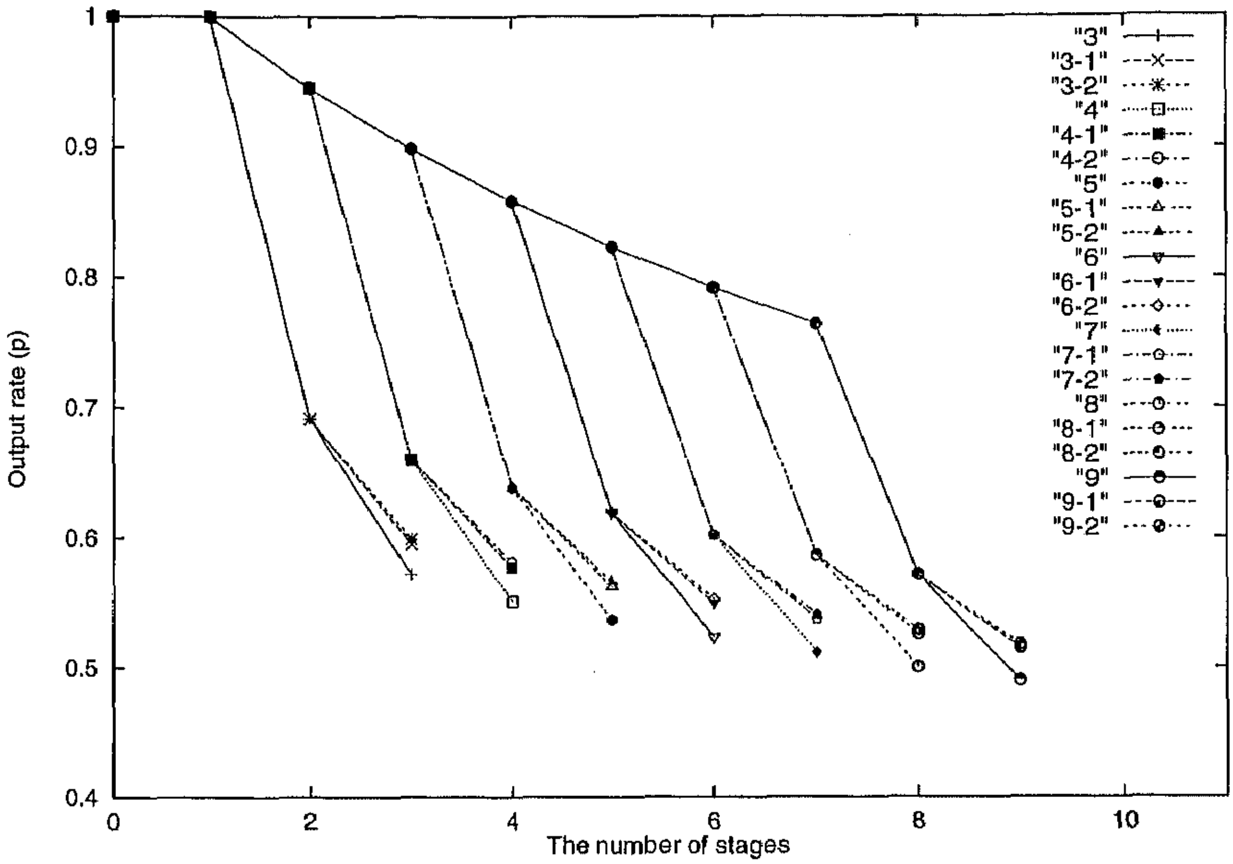

Figure 4.4: The output rate of the hybrid dilated banyan networks with and without bypass at the stage of 4×2 re-arrangeable output switching element.

From Fig. 4.4, the comparisons are mentioned as follows.

1. A hybrid dilated banyan network which the bypass is used at the stage of 4×2 re-arrangeable output switches has a higher output rate than a hybrid dilated banyan network without bypass. The output rate of networks with bypass facility is approximately 5 percent higher than the output rate of networks without bypass. The throughput is improved from 0.5 to 0.525 at the 9 stages case with two bypasses. The maximum ATM transfer speed defined by ATM-Forum is 622 Mbps. Since a cell has header information (5 bytes) and payload (48 bytes), whose total bits are 424, the cell transfer speed is 1.46 Mcells/s. Therefore, 5 percent performance improvement corresponds to 0.0365 Mcells/s. This means the band capacity equals to more 9.35 MPEG1 (1.5Mbps) channels or 3.5 MPEG2 (NTSC, 4Mbps) channels is improved than the hybrid dilated banyan network without bypass.

2. Comparing with the usage of a bypass and two bypasses at the stage of 4×2 re-arrangeable output switching elements, the one bypass networks almost have the same output rate

as the two bypasses networks. This mean that the usage of a bypass have better cost performance than the usage of two bypasses if the connection cost per a bypass is same. From this result, the usage of two bypasses is not adopted in the rest of this thesis.

This section discusses about bypasses at the specific stage and obtains better improvements. When bypasses are introduced to all stages, the output rate of the network should be increased. The next section introduces bypasses to the original and the 2-dilated banyan networks.

4.2 Original and 2-Dilated Banyan Networks with Bypasses

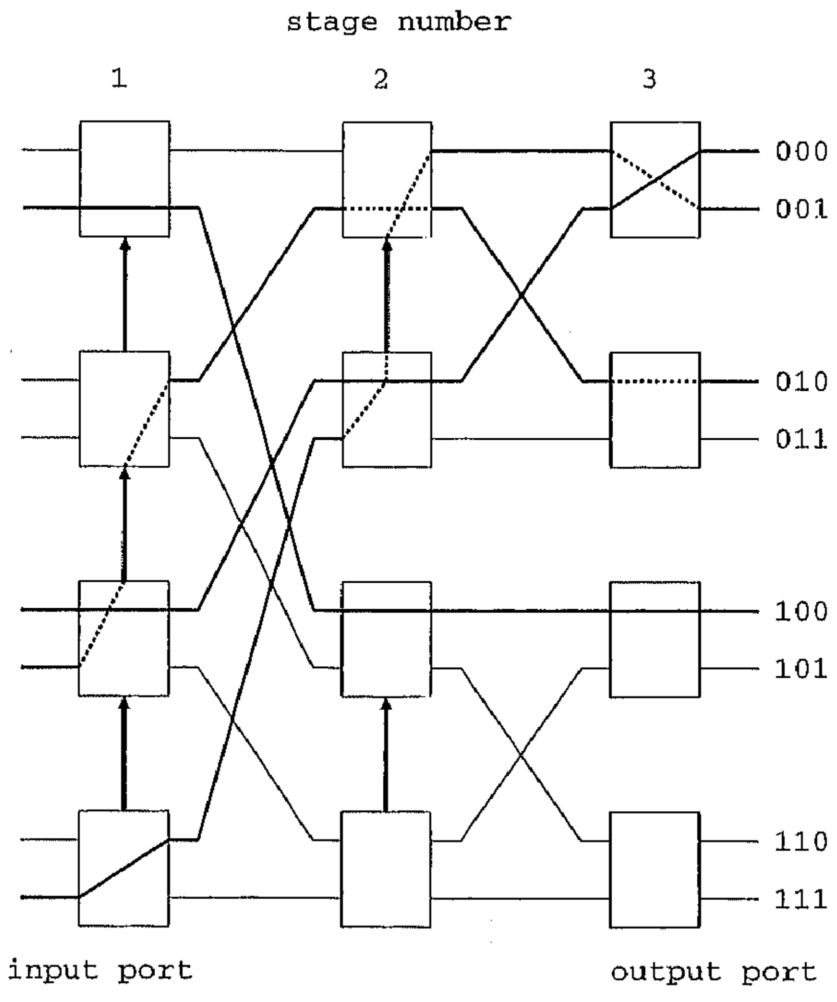

Figure 4.5: 8 × 8 banyan network with bypasses.

This section introduces the bypasses to all switching elements of the original and the 2-dilated banyan networks. The 8 × 8 banyan network with bypasses is shown in Fig. 4.5. At this figure, the two blocked cells in the first and second stages are forwarded to the correct destination by using the bypasses.

In the case of a 2-dilated banyan network, the bypasses are inserted during switching elements as the banyan network. But no blocking occurs in the first stage of 2-dilated banyan network, then the bypasses are not needed in this stage. The 16×16 2-dilated banyan network with bypasses is shown in Fig. 4.6, where the blocked cell in the second stage is forwarded by the bypass.

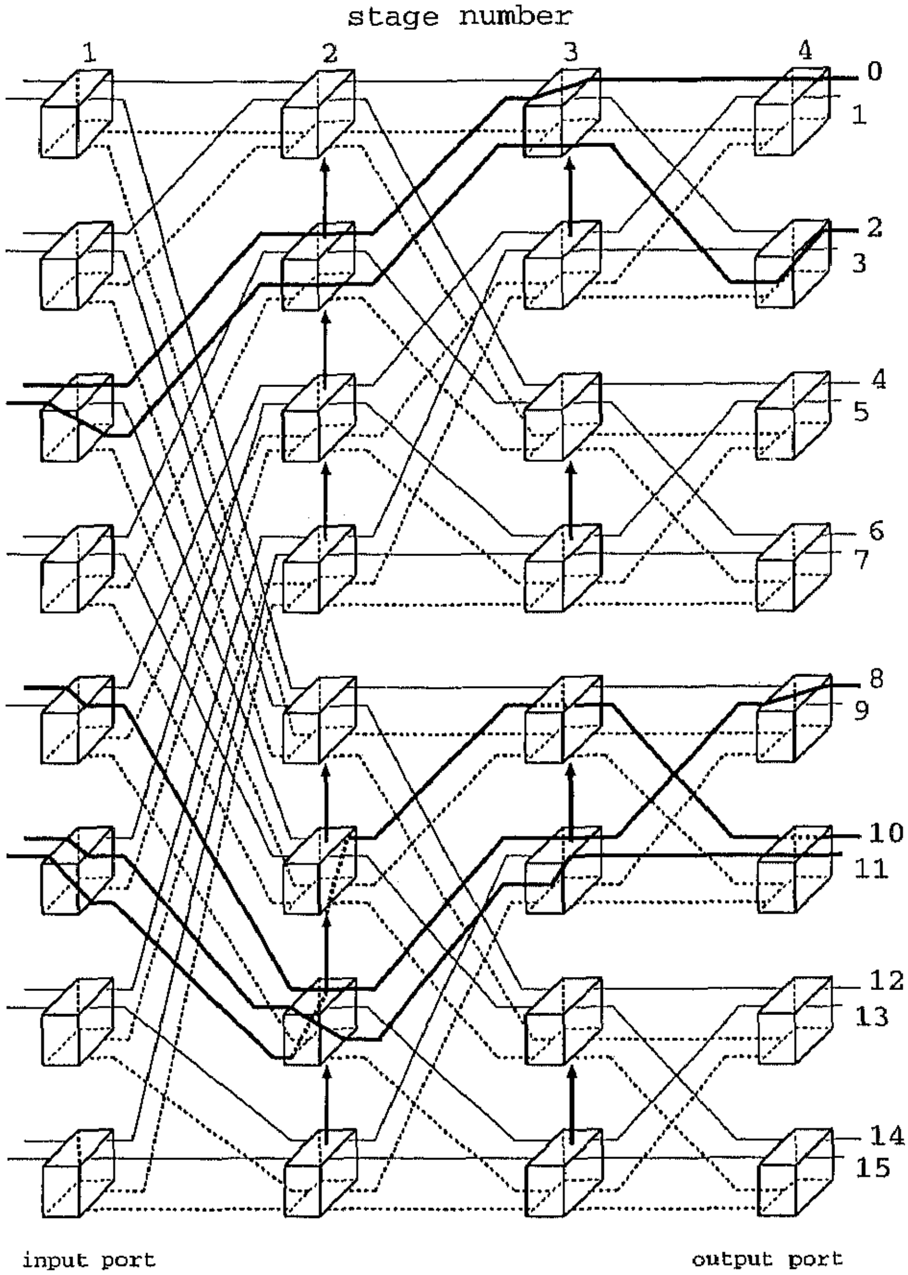

Figure 4.6: 16×16 2-dilated banyan network with bypasses.

In the original and the 2-dilated banyan networks, it supposes that every input rate of the same stage is equal. But such an assumption is not satisfied in the case of networks with bypasses. Thus, the output rates are rather complicated as in the next subsection.

4.2.1 Input Rate and Output Rate of Banyan Network with Bypasses

There are four kinds of switching elements in banyan network with bypasses. The output rates in each kind of switching elements are calculated at different input rates and used the results of each switching element as the input rates to calculate the output rates of the next switching elements. More precise proofs are introduced in Appendix A.1.1. The first switching element is Type 1 shown in Fig. 4.7.

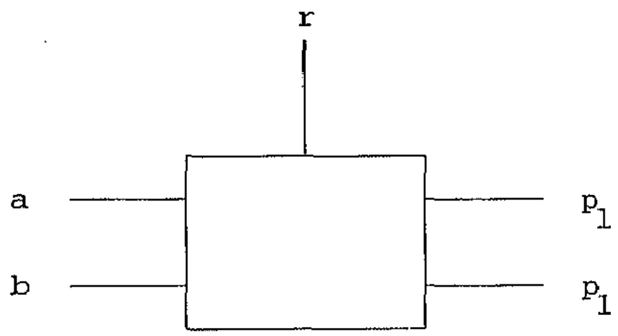

Figure 4.7: Banyan switching element Type 1.

Assume a and b as the input rates of the switching elements, p_l as the output rate and r as the output rate of the cell forwarded to the bypass and used as the bypass input rate for Type 2 or Type 3 switching element in below. From the routing algorithm of the switching elements, p_l and q are:

$$p_l = \frac{1}{2}a + \frac{1}{2}b - \frac{1}{4}ab, \tag{4.5}$$

$$r = \frac{1}{2}ab. \tag{4.6}$$

Switching element Type 2 in Fig. 4.8 has both of input and output bypasses. Assume a and b as the input rates of the switching elements, q as the input rate of the the input bypass from the lower switching element, p_m as the output rate and r as the output rate of the cell forwarded to the output bypass. Then p_m and r are satisfied in below:

$$p_m = \frac{1}{2}a + \frac{1}{2}b - \frac{1}{4}ab + \frac{1}{2}q - \frac{1}{4}aq - \frac{1}{4}bq + \frac{1}{8}abq, \tag{4.7}$$

$$r = \frac{1}{2}ab + \frac{1}{2}aq + \frac{1}{2}bq - \frac{1}{2}abq. \tag{4.8}$$

Note that (4.7) and (4.8) are equal to (4.5) and (4.6), respectively, if $q = 0$.

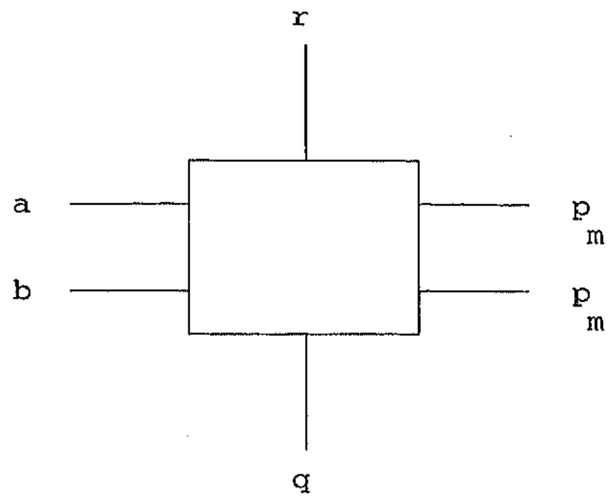

Figure 4.8: Banyan switching element Type 2.

Switching element Type 3 in Fig. 4.9 has only an input bypass. Assume a and b as the input rates of the switching elements, q as the input rate of the input bypass from the lower switching element and p_u as the output rate. Then p_u is the same result of (4.7), as described in below:

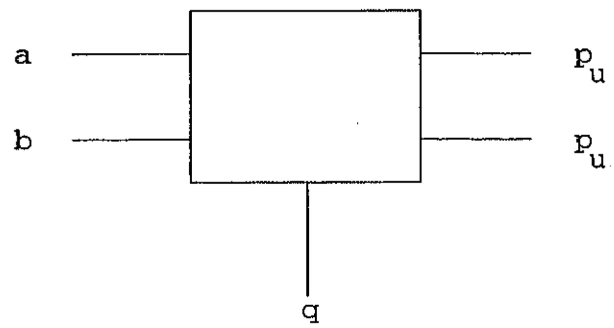

Figure 4.9: Banyan switching element Type 3.

$$p_u = \frac{1}{2}a + \frac{1}{2}b - \frac{1}{4}ab + \frac{1}{2}q - \frac{1}{4}aq - \frac{1}{4}bq + \frac{1}{8}abq. \tag{4.9}$$

Switching element Type 4 in Fig. 4.10 has no bypass, i.e., it is just the switching element of ordinary banyan network. Assume a and b as the input rates of the switching elements, p_{out} as the output rate. Then p_{out} is calculated as follows:

$$p_{out} = \frac{1}{2}a + \frac{1}{2}b - \frac{1}{4}ab. \tag{4.10}$$

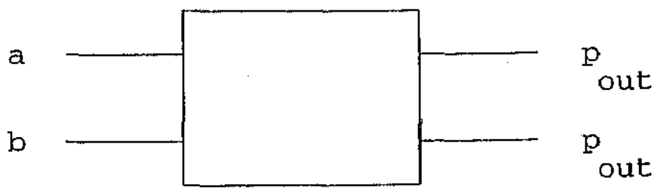

Figure 4.10: Banyan switching element Type 4.

Note that if $a = b$, then (4.10) coincides with (2.1).

From the construction of the banyan network with bypasses, all output rates in the final stage are equal. For throughput analysis, it is enough to calculate only one output rate of the network.

4.2.2 Input Rate and Output Rate of 2-Dilated Banyan Network with Bypasses

As in the previous subsection, the output rates of five kinds of switching elements in the 2-dilated banyan network are solved at difference input rates in this subsection, where their details are in Appendix A.1.2. The results of output rates in each stage will be use as input rates to the next stage. In a 2-dilated banyan network, 2×4 re-arrangeable input switching elements without bypass are used at the first stage. When the output rate of this network is p, the output rate p_1 at the first stage is $\frac{1}{2}p$ which is the same as (2.2). From the second stage, three switching elements, called Type 1, Type 2 and Type 3, which correspond to the ones of the banyan with bypasses respectively, are constructed. At the last stage of the network, 4×2 re-arrangeable output switching elements Type 4, whose the input rates are not equal, are used.

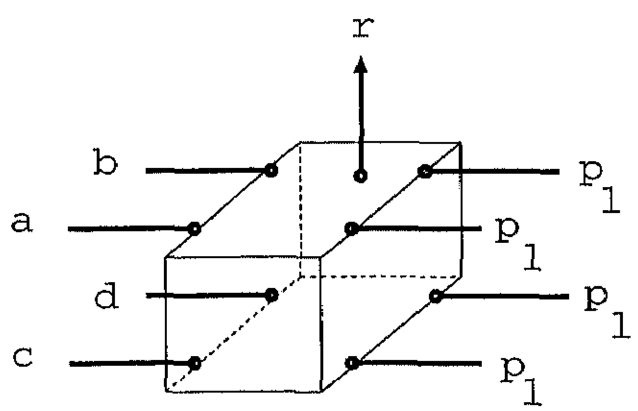

Figure 4.11: 2-dilated banyan switching element Type 1.

At switching element Type 1 in Fig. 4.11, assume a, b, c and d as the input rates of the switching element, p_l as the output rate and r as the output rate forwarded to the bypass and used as the bypass input rate of Type 2 or Type 3 switching element. Then p_l and q are as follows:

$$\begin{aligned} p_l &= \frac{1}{4}a + \frac{1}{4}b + \frac{1}{4}c + \frac{1}{4}d - \frac{1}{16}abc - \frac{1}{16}abd \\ &\quad - \frac{1}{16}acd - \frac{1}{16}bcd + \frac{1}{16}abcd, \qquad (4.11) \\ r &= \frac{1}{4}abc + \frac{1}{4}acd + \frac{1}{4}abd + \frac{1}{4}bcd - \frac{3}{8}abcd. \qquad (4.12) \end{aligned}$$

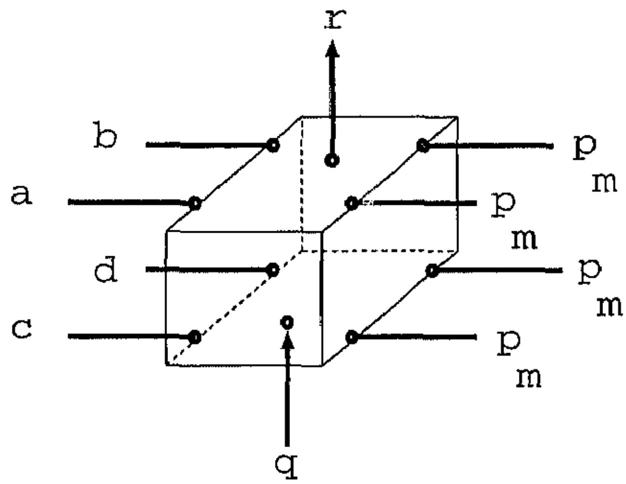

Figure 4.12: 2-dilated banyan switching element Type 2.

At switching element Type 2 in Fig. 4.12, assume a, b, c, d and q as in the Type 1, and p_m as the output rate and r as the output rate forwarded to the bypass. Then p_m and r are calculated as Type 1, and described in below. Note that (4.13) and (4.14) are identical to (4.11) and (4.12) respectively, if $q = 0$.

$$\begin{aligned} p_m &= \frac{1}{4}a + \frac{1}{4}b + \frac{1}{4}c + \frac{1}{4}d - \frac{1}{16}abc - \frac{1}{16}abd - \frac{1}{16}acd \\ &\quad - \frac{1}{16}bcd + \frac{1}{16}abcd + \frac{1}{4}q - \frac{1}{16}abq - \frac{1}{16}acq \\ &\quad - \frac{1}{16}adq - \frac{1}{16}bcq - \frac{1}{16}cdq - \frac{1}{16}bdq + \frac{1}{16}abdq \\ &\quad + \frac{1}{16}acdq + \frac{1}{16}bcdq + \frac{1}{16}abcq - \frac{3}{64}abcdq, \qquad (4.13) \\ r &= \frac{1}{4}abc + \frac{1}{4}acd + \frac{1}{4}abd + \frac{1}{4}bcd - \frac{3}{8}abcd - \frac{3}{8}abcq \end{aligned}$$

$$-\frac{3}{8}acdq - \frac{3}{8}abdq - \frac{3}{8}bcdq + \frac{3}{8}abcdq + \frac{1}{4}abq$$
$$+\frac{1}{4}acq + \frac{1}{4}adq + \frac{1}{4}bcq + \frac{1}{4}bdq + \frac{1}{4}cdq. \quad (4.14)$$

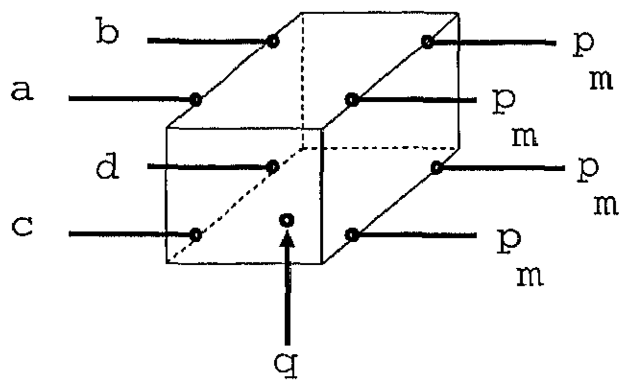

Figure 4.13: 2-dilated banyan switching element Type 3.

At switching element Type 3 in Fig. 4.13, assume a, b, c, d and q as in the Type 1, and p_u as the output rate. Then p_u is the same result of (4.13), as in below:

$$\begin{aligned} p_u &= \frac{1}{4}a + \frac{1}{4}b + \frac{1}{4}c + \frac{1}{4}d - \frac{1}{16}abc - \frac{1}{16}abd - \frac{1}{16}acd \\ &-\frac{1}{16}bcd + \frac{1}{16}abcd + \frac{1}{4}q - \frac{1}{16}abq - \frac{1}{16}acq \\ &-\frac{1}{16}adq - \frac{1}{16}bcq - \frac{1}{16}cdq - \frac{1}{16}bdq + \frac{1}{16}abdq \\ &+\frac{1}{16}acdq + \frac{1}{16}bcdq + \frac{1}{16}abcq - \frac{3}{64}abcdq. \end{aligned} \quad (4.15)$$

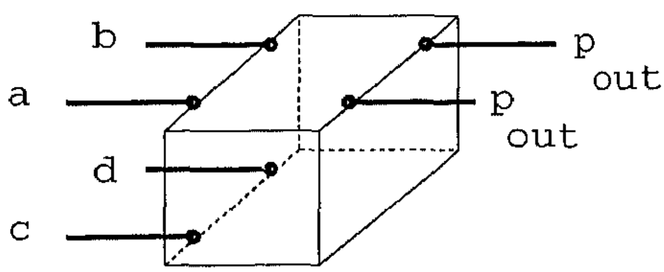

Figure 4.14: 2-dilated banyan switching element Type 4.

At the last stage of the network in Fig. 4.14, assume a, b, c and d as in Type 1, p_{out} as the output rate. Then p_{out} is calculated as follows:

$$p_{out} = \frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c + \frac{1}{2}d - \frac{1}{4}ab - \frac{1}{4}ac$$

$$-\frac{1}{4}ad - \frac{1}{4}bc - \frac{1}{4}cd + \frac{1}{8}abc + \frac{1}{8}abd$$
$$+\frac{1}{8}acd + \frac{1}{8}bcd - \frac{1}{16}abcd. \quad (4.16)$$

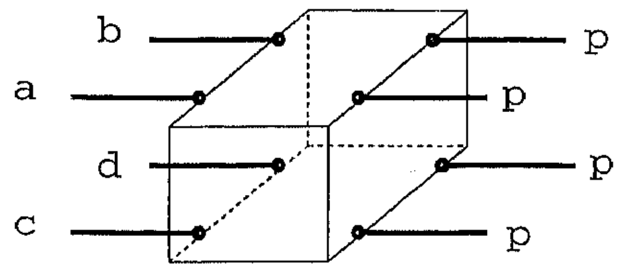

Figure 4.15: 2-dilated banyan switching element Type 5.

Switching element Type 5 presented in Fig. 4.15 will be used at section 4.4. It is a switching element without input and output bypass. Assume a, b, c and d as the input rate, and p as the output rate. Then p is as in below:

$$p = \frac{1}{4}a + \frac{1}{4}b + \frac{1}{4}c + \frac{1}{4}d - \frac{1}{16}abc - \frac{1}{16}abd$$
$$-\frac{1}{16}acd - \frac{1}{16}bcd + \frac{1}{16}abcd. \quad (4.17)$$

From the construction of the switch, all output rates at the final stage of 2-dilated banyan network are also equal.

4.3 Comparison of Output Rates of Networks with and without Bypass

From the equations presented at the previous section, the results of output rates for the $2^1 \times 2^1$ to $2^8 \times 2^8$ banyan networks with and without bypass, whose arrival rates (input rates) are 1.0 and 0.5, are indicated in Fig. 4.16 and Fig. 4.17. The results for the 2-dilated banyan networks with and without bypass are also indicated in Fig. 4.18 and Fig. 4.19. In these figures, x axis shows the number of stages and y axis denotes the output rates of each network.

In Fig. 4.16 and Fig. 4.17,

- "banyan" shows the output rate of the original banyan networks and,
- "bypass" shows the output rate of the banyan networks with bypasses.

From Fig. 4.16, the banyan networks with bypasses have approximately 3 to 49 percent higher output rates than the original banyan networks. For example at the 7 stages case in Fig. 4.16, its throughput becomes from 0.359 to 0.473, which is about 31.8 percent improvement. As the same discussion in subsection 4.1.2, for the maximum ATM transfer speed 622 Mbps, 31.8 percent performance improvement corresponds to the additional capacity of 42.8 MPEG1 (1.5 Mbps) or 16.0 MPEG2 (NTSC, 4 Mbps) channels.

When the network load is not so heavy, i.e., its input rate is 0.5, Fig. 4.17 shows that the output rates with usage of the bypasses are increased from 3 to 49 percent to the original ones. This fact concludes that the bypasses for the banyan networks are also much effective even if their network loads are low.

In Fig. 4.18 and Fig. 4.19,

- "dilate" shows the output rate of the 2-dilated banyan networks and,
- "dilateby" shows the output rate of the 2-dilated banyan networks with bypasses.

The 2-dilated banyan network is needed to be constructed at least 2×4 re-arrangeable input switching elements and 4×2 re-arrangeable output switching elements. Therefore, its switch size must be greater than $2^2 \times 2^2$. Indeed, when the number of stages in Fig. 4.18 is 1, the output rate is meaningless, since its result is observed from a 2×4 re-arrangeable input switching element.

For the cases from 2 to 8 stages in Fig. 4.18, the 2-dilated banyan networks with bypasses have approximately 1 to 12.5 percent higher than the ones without bypass. For example, the throughput at the 7 stages case is improved from 0.5862 to 0.6461, which corresponds that more 22.4 MPEG1 (1.5 Mbps) or 8.4 MPEG2 (NTSC, 4 Mbps) channels are useful than the original 2-dilated one.

In Fig. 4.19, the output rates increase 0.1 to 5.3 percent, which are less improved than the previous cases. This means that the bypass strongly effects on the throughput of the 2-dilated banyan network at the high input rate.

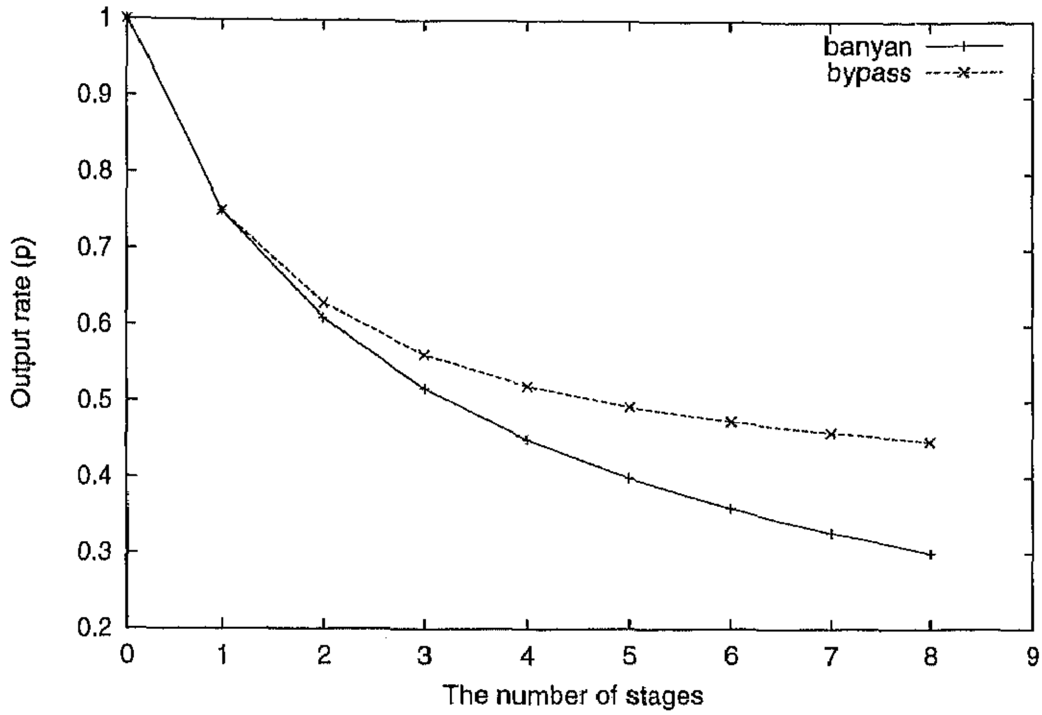

Figure 4.16: The output rates of banyan networks with and without bypass at input rate = 1.0.

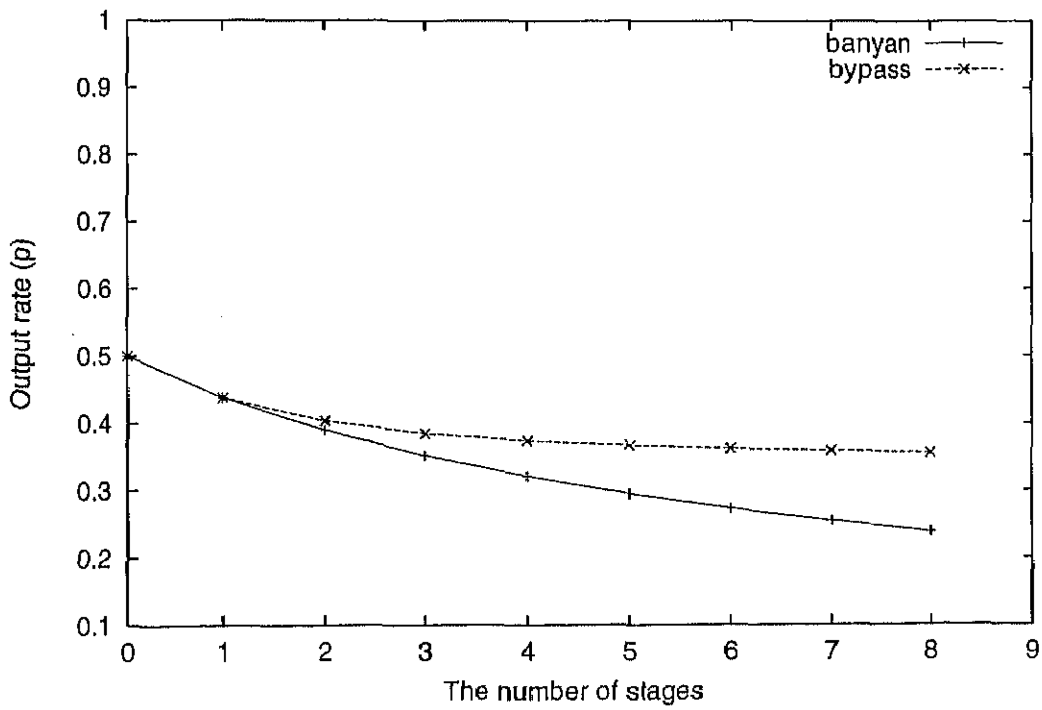

Figure 4.17: The output rates of banyan networks with and without bypass at input rate = 0.5.

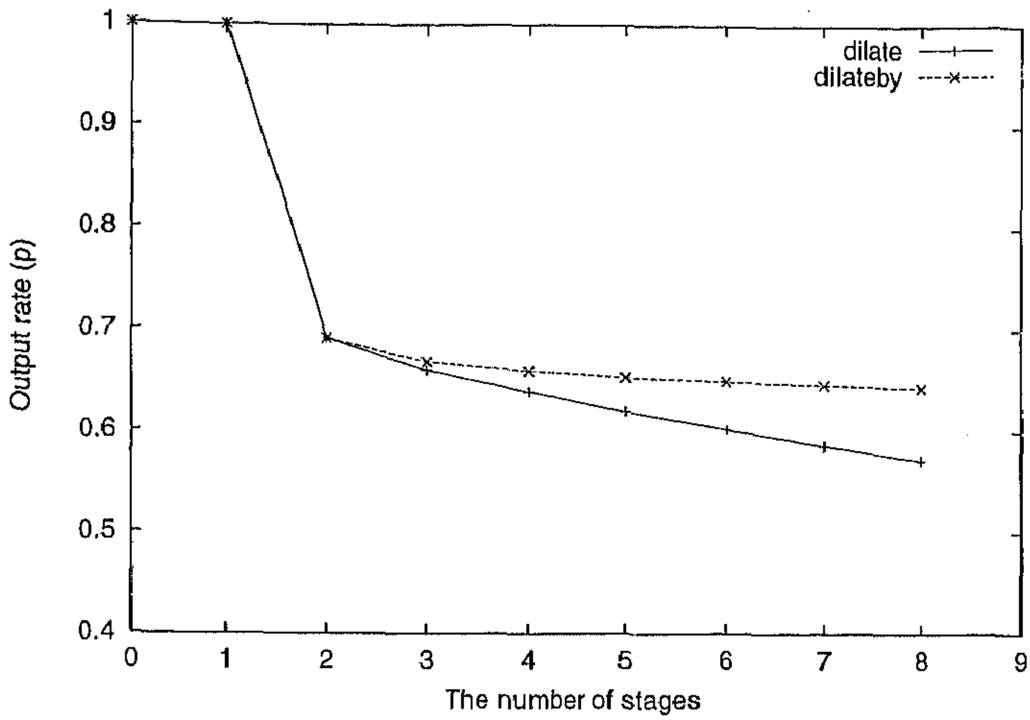

Figure 4.18: The output rates of 2-dilated banyan networks with and without bypass at input rate = 1.0.

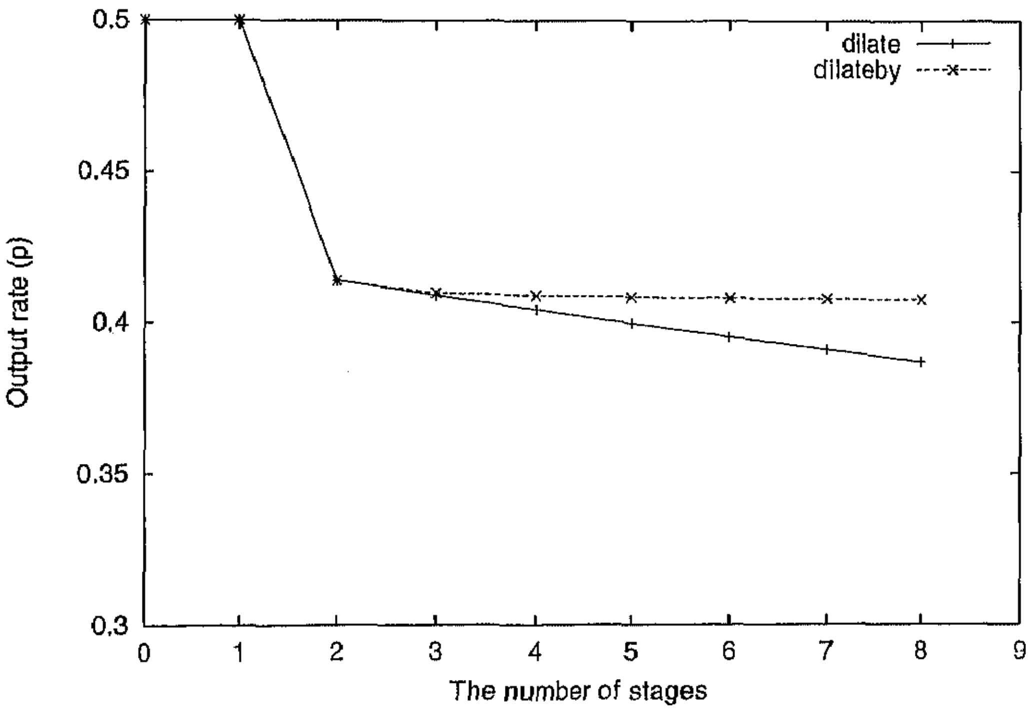

Figure 4.19: The output rates of 2-dilated banyan networks with and without bypass at input rate = 0.5 .

4.4 Original and 2-Dilated Banyan Networks with One-Bypass-Connection

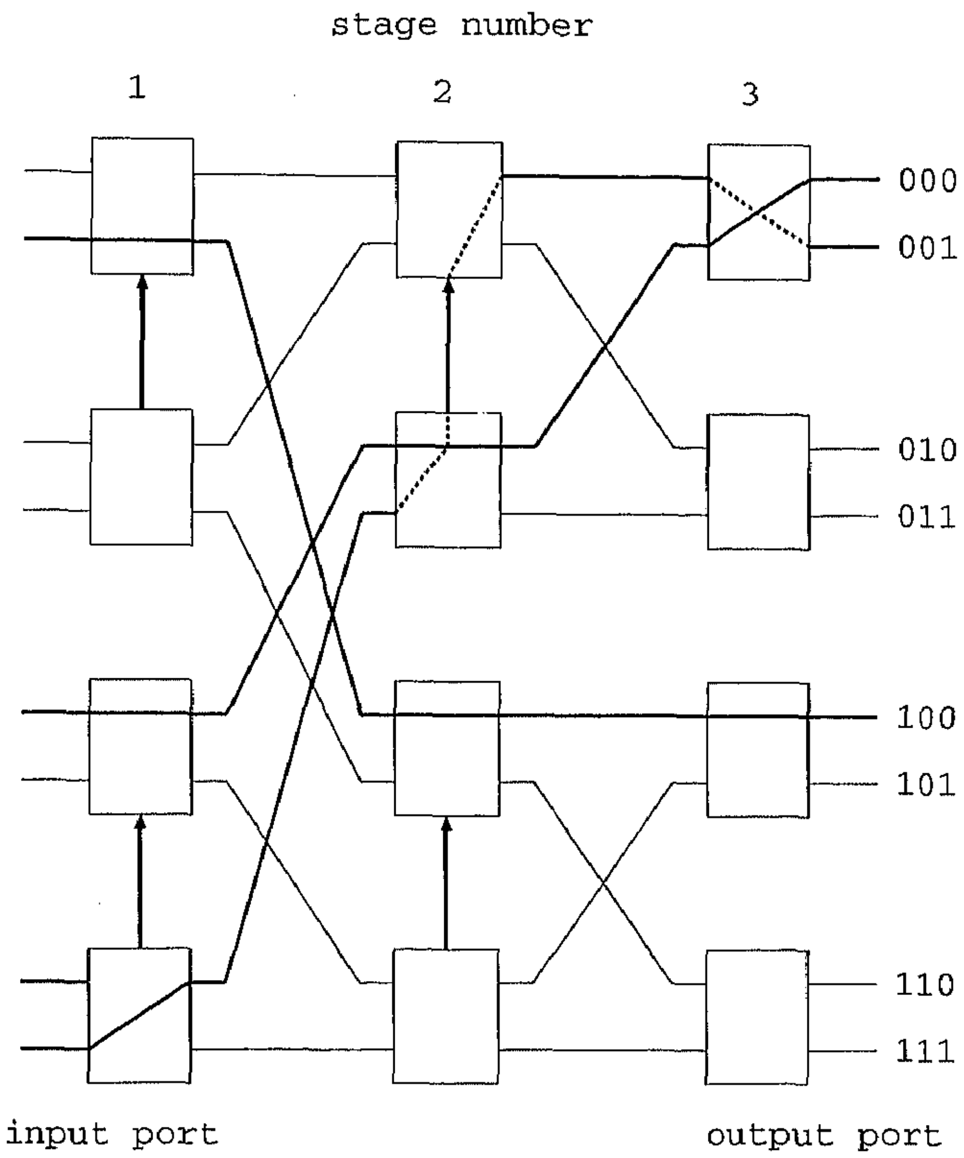

Figure 4.20: 8×8 banyan network with one-bypass-connection.

In the previous section, the throughput of the original and the 2-dilated banyan network are improved by inserting bypasses between the switching elements in the same stage of the networks. However a switching element with a bypass must wait for the switching process until the lower switching element decides whether it forwards a cell to the bypass or not.

For example, the top switching element of the first stage at the 8×8 banyan network with bypasses in Fig. 4.5 can process its input cells, after all switching elements at the fourth to the second columns in the same stage have done. Therefore, its cell transfer delay may be four times greater than the one of the original banyan network, and the total switching times of the

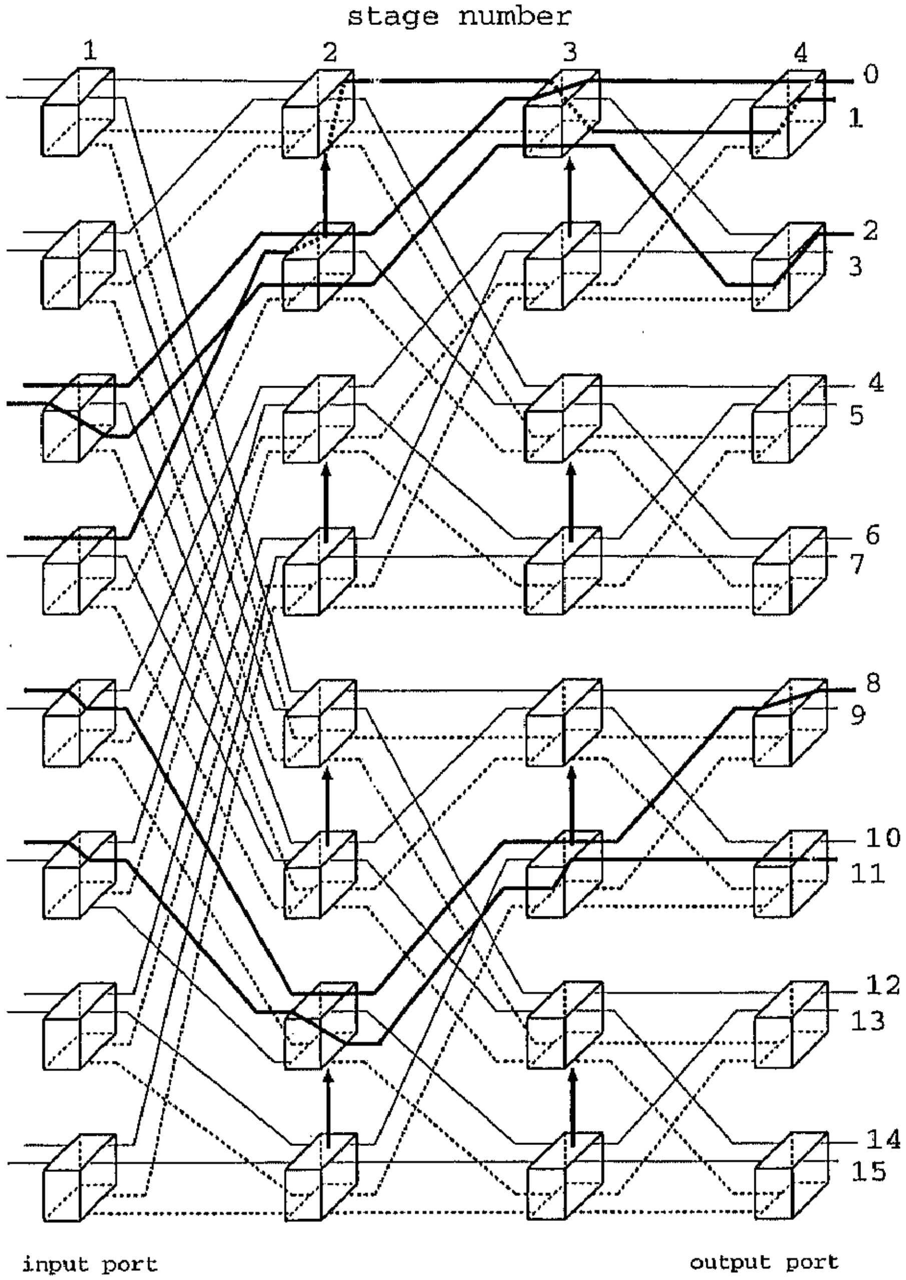

Figure 4.21: 16 × 16 2-dilated banyan network with one-bypass-connection.

original and the 2-dilated banyan network are increased. In order to minimize the increment of cell transfer delay, this section constructs these networks with no Type 2 switching element and limits the maximum length of switching elements sequence connected by bypasses to at most two. Then, their cell transfer delay may be estimated twice as long as the original ones.

There are various ways of designs to place bypasses for the above restriction. When the highest output rate is required, switching elements at every $(2m-1)$-th and $2m$-th columns in all stages except the final one need to be connected by bypasses so as to maximize the number of bypasses in the $N \times N$ network, where $m = 1, \ldots, \frac{N}{2}$. As the section 4.2 was mentioned, bypasses in the first stage of 2-dilated banyan network are also omitted. These maximized connection method is called one-bypass-connection. To distinguish from the one-bypass-connection, the connection method for the original and the 2-dilated banyan networks with bypasses in the previous section are also called all-bypass-connection. Fig. 4.20 and Fig. 4.21 illustrate the 8×8 banyan network with one-bypass-connection and the 16×16 2-dilated banyan network with one-bypass-connection.

From the results in section 4.2, the output rates of any design of networks with one-bypass-connection can also be obtained. Fig. 4.22 and Fig. 4.23 show the output rates of three kinds of banyan network and 2-dilated banyan networks, respectively, where their input rates are 1.0. In these figures,

- "nobypass" means the output rate of the original network,
- "onebypass" is the network with one-bypass-connection, and
- "allbypass" is the original or the 2-dilated banyan network with all-bypass-connection.

In Fig. 4.22, from 3 to 8 stages cases, banyan networks with one-bypass-connection have approximately 6.36 to 18.67 percent higher output rate than the original banyan network but 2.18 to 25.59 percent lower output rates than the banyan networks with all-bypass-connection.

In Fig. 4.23, from 4 to 8 stages cases, 2-dilated banyan networks with one-bypass-connection have approximately 2.52 to 6.22 percent higher output rate than the original 2-dilated banyan network but 0.65 to 5.95 percent lower output rates than the 2-dilated banyan network with all-bypass-connection.

As the number of stages are increased, the network with all-bypass-connection has larger output rate but greater cell transfer delay than the one with one-bypass-connection. For example, the former may have 64 times delay than the latter at the 8 stages case. Though the output rate of the one-bypass-connection is decreased, this connection method gives worthful selections for network designers.

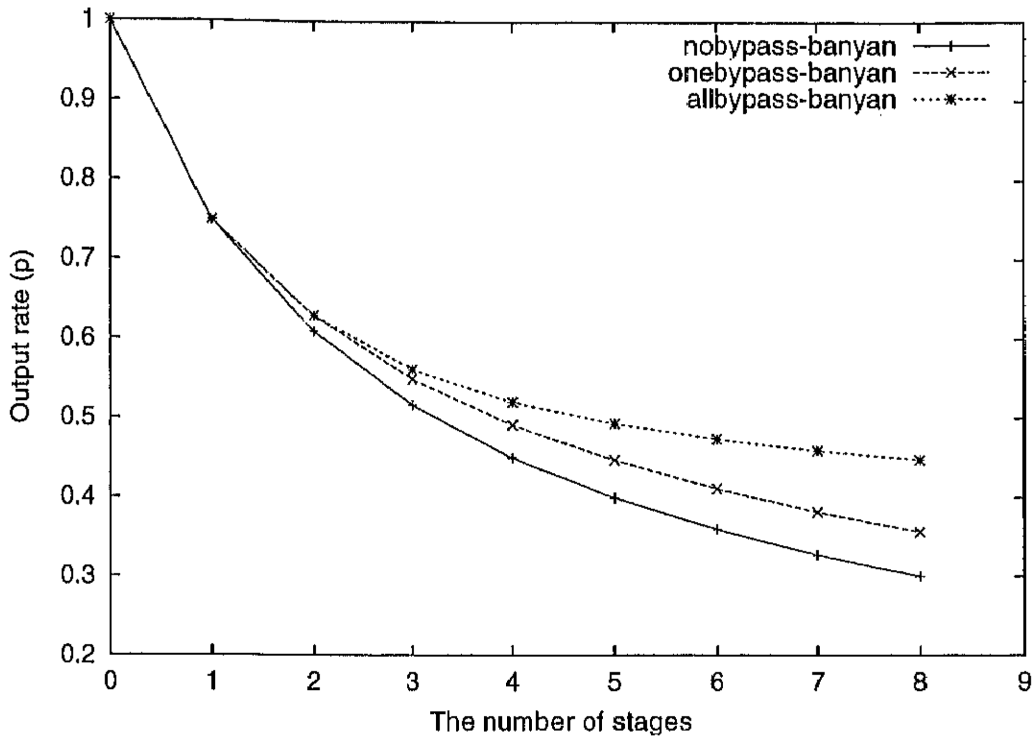

Figure 4.22: The output rates of banyan networks with one-bypass-connection, all-bypass-connection and no bypass at input rate = 1.0 .

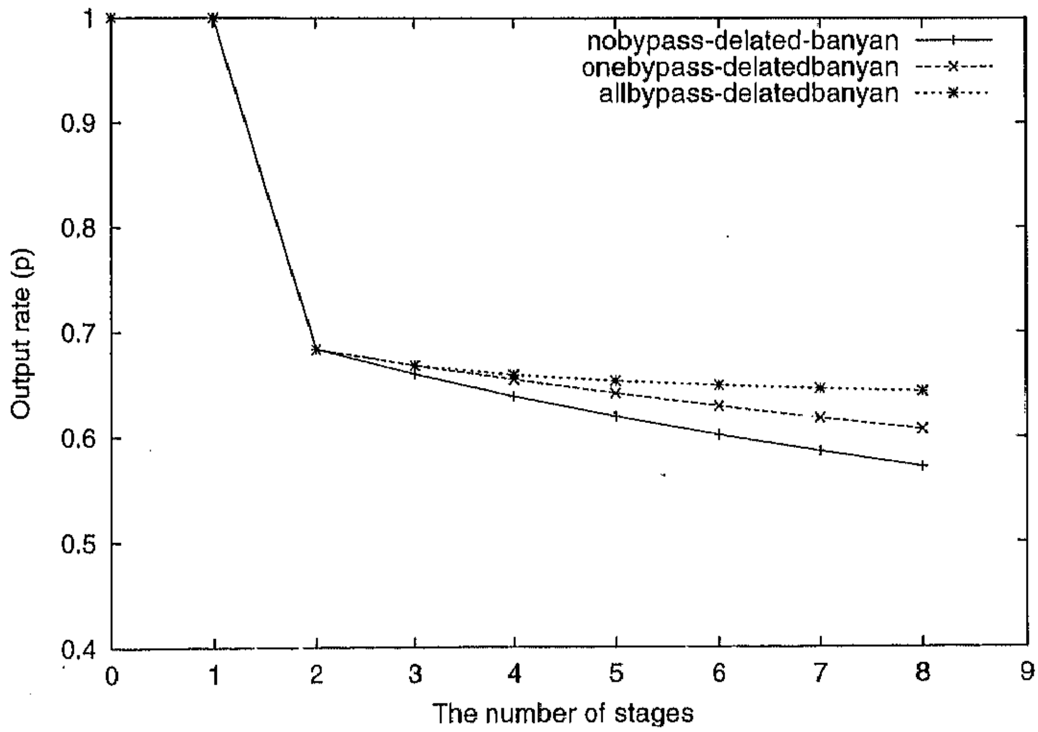

Figure 4.23: The output rates of 2-dilated banyan networks with one-bypass-connection, all-bypass-connection and no bypass at input rate = 1.0 .

4.5 One-Bypass-Connection at Two Neighbored Stages

In the previous section, one-bypass-connection are inserted at the all possible stages. This section investigates the effectiveness of one bypass at two neighbored stages for output rate. One bypass at two neighbored stages is to set up one bypass facility at a pair of neighbored stages. For this purpose, bypasses are inserted to only the first and second stages or, the third and fourth stages, the fifth and sixth stages at one-bypass-connection banyan networks, respectively. At each inserted stage, every $(2m-1)$-th and $2m$-th switching elements are connected by a bypass, where $m = 1,\ldots,\frac{N}{2}$ for the $N \times N$ network. This new connection method is called one-bypass-connection at two neighbored stages.

For the 2-dilated banyan network, bypasses are not needed at the first stage. Thus, the new one-bypass-connection at two neighbored stage method for 2-dilated banyan network is constructed as the same manner except that bypasses insertion begins from the second stage, i.e., inserting the second and third stages, and so on. Fig. 4.24 shows output rates of the banyan network with one-bypass-connection at two neighbored stages and the whole stages, all-bypass-connection and no bypass at input rate = 1.0. In these figures,

- "12" indicates the output rate of banyan network with one-bypass-connection at the first and second stages.
- "34" indicates the output rate of banyan network with one-bypass-connection at the third and fourth stages.
- "56" indicates the output rate of banyan network with one-bypass-connection at the fifth and sixth stages. The other plot marks are same as in Fig. 4.22 and Fig. 4.23.

Fig. 4.25 shows output rates of the 2-dilated banyan network with one-bypass-connection at two neighbored stages and the whole stages, all-bypass-connection and no bypass at input rate = 1.0. In these figures,

- "23" shows the output rate of 2-dilated banyan network with one-bypass-connection at the second and third stages.
- "45" shows the output rate of 2-dilated banyan network with one-bypass-connection at the fourth and fifth stages.
- "67" shows the output rate of 2-dilated banyan network with one-bypass-connection at the sixth and seventh stages.

We focus our discussion to the difference of the output rates in one-bypass-connection at two neighbored stages. From Fig. 4.24, the networks with bypasses at the second and third stages, the fourth and fifth stages and the sixth and seventh stages give 3.3, 4.9 and 5.8 percent a higher output rate than the network without bypass. Thus the bypasses inserted at two neighbored stages near the output ports gives higher output rates than the one near the input ports. Though the maximum difference of their output rates is less than 1 percent, the bypasses insertion stage influences on the performance improvement of banyan network.

Fig. 4.25 also shows the same results as in Fig. 4.24. However, this figure indicates the output rates in one-bypass-connection at two neighbored stages almost equal each other. They give 2 percent higher output rates than the network without bypass. Since the probability of cell blocking in the 2-dilated banyan network is much smaller than the one of the banyan network, the bypasses insertion position has less influence on the output rate than that of the original banyan networks.

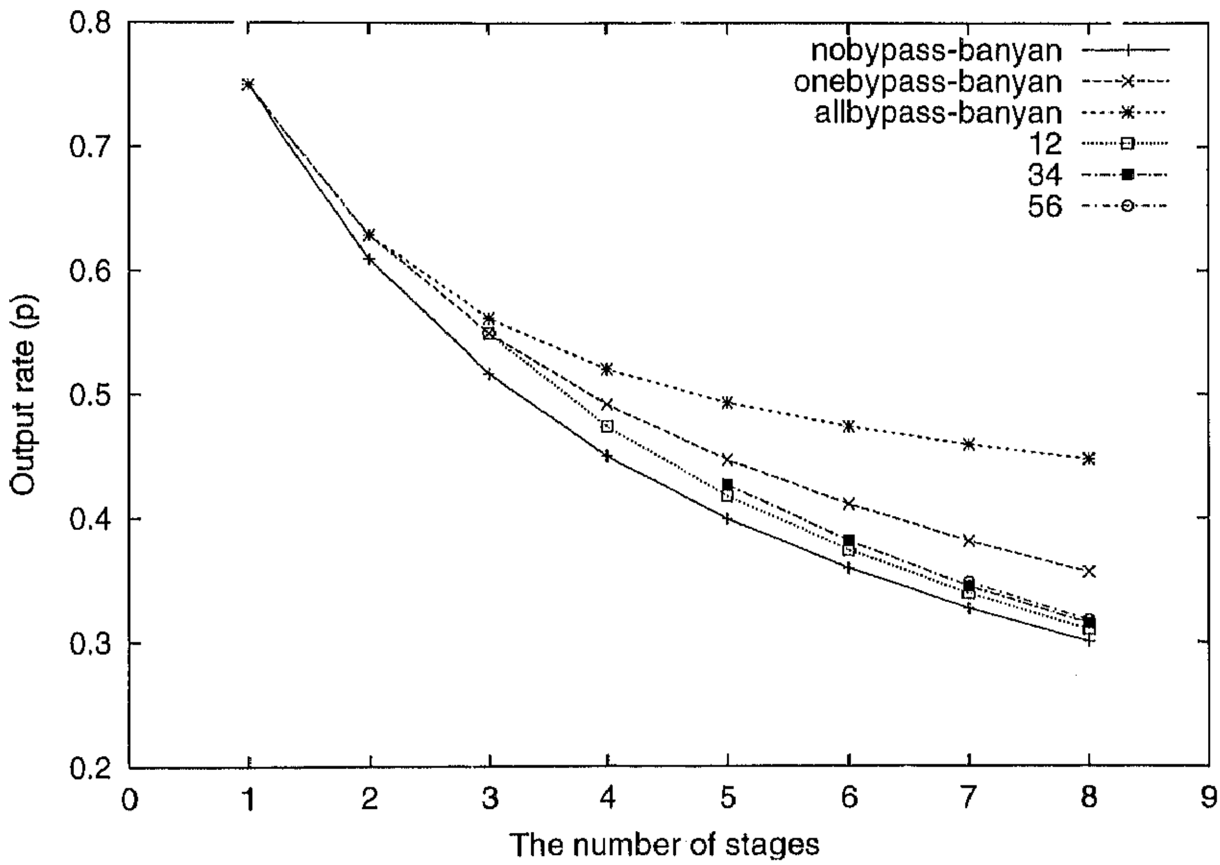

Figure 4.24: The output rates of banyan networks with one-bypass-connection at two neighbored stages and the whole stages, all-bypass-connection and no bypass at input rate = 1.0

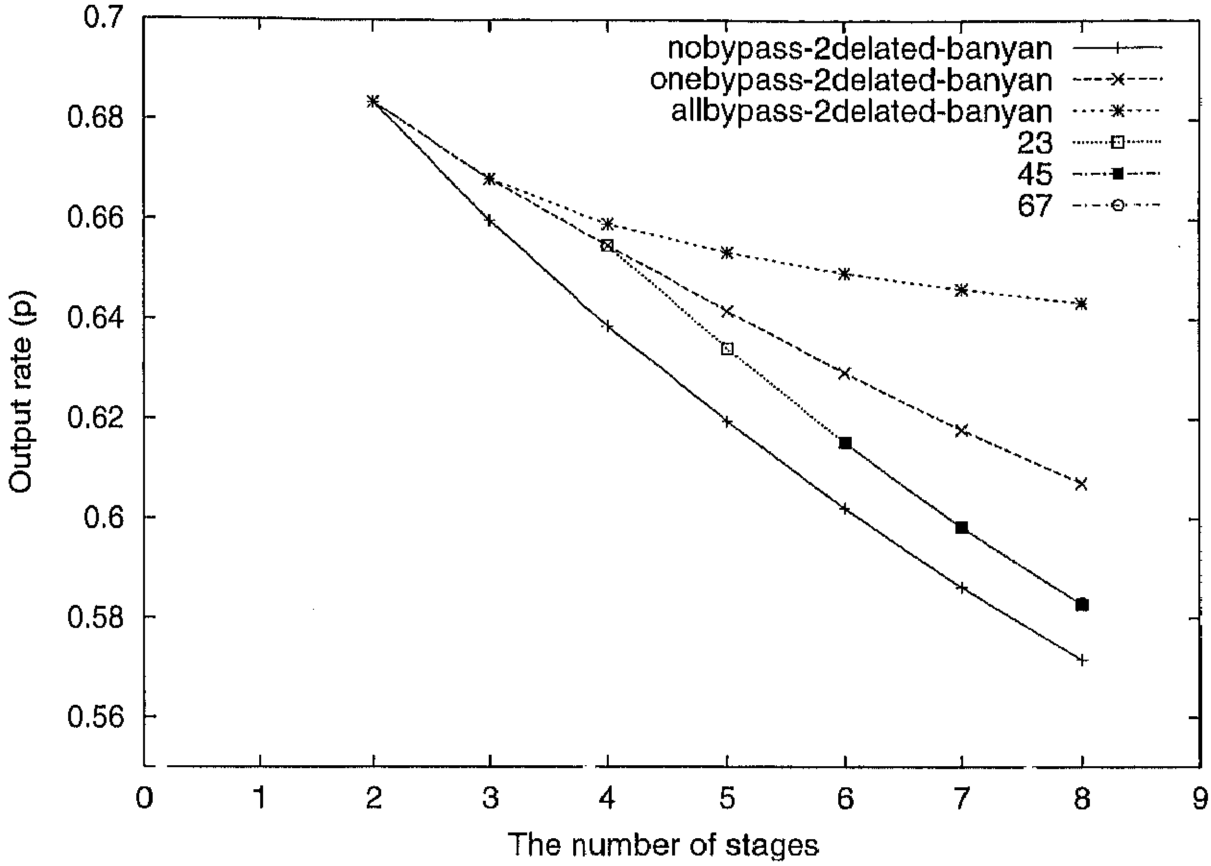

Figure 4.25: The output rates of 2-dilated banyan networks with one-bypass-connection at two neighbored stages and the whole stages, all-bypass-connection and no bypass at input rate $= 1.0$.

4.6 Adaptation of Bypasses to Other Networks

The bypass methods in this chapter can be applied to the networks using the banyan networks or the 2-dilated banyan networks as their basic structure. In such networks, the tandem banyan network and the hybrid dilated banyan network are included.

As the subsection 3.2.1 described, the $N \times N$ tandem banyan network structure of K banyan networks connected in series, where each banyan network has N input ports and N output ports. Each output port of a banyan network is connected to both the input of the subsequent banyan network and the output port concentrator of the tandem banyan network. When a cell is blocked at any stage in the n-th banyan network, the cell is transfered to a wrong output and forwarded to the connected input port of the $(n+1)$-th banyan network if $n < K$, or eliminated otherwise. If bypasses is inserted at this stage, the cell may arrive at correct output port of the n-th banyan network. From this fact, the performance of the tandem banyan network is expected to improve.

The $N \times N$ hybrid dilated banyan network in chapter 3 is constructed from the 2-dilated banyan networks at first x stages and the original banyan networks at the rest $(n - x)$ stages, when $n = \log_2 N$. Bypasses can be inserted at any stages expect the first and final stages and can result in a good performance of the network. Indeed, in section 4.1 it has been confirmed to improve the performance by inserting bypasses at the x-th stage constructed by 4×2 rearrangeable output switching element, when $x = n - 1$. The next chapter analyzes the hybrid dilated banyan networks with bypasses.