Chapter 3

Hybrid Dilated Banyan Networks

As the previous chapter mentioned, the 2-dilated banyan network improves the output rate of the original one, however the hardware size becomes much greater. This chapter proposes new network, called hybrid dilated banyan network. This network is the hybrid of the above two networks to obtain both advantages.

3.1 The Structure of Hybrid Dilated Banyan Networks

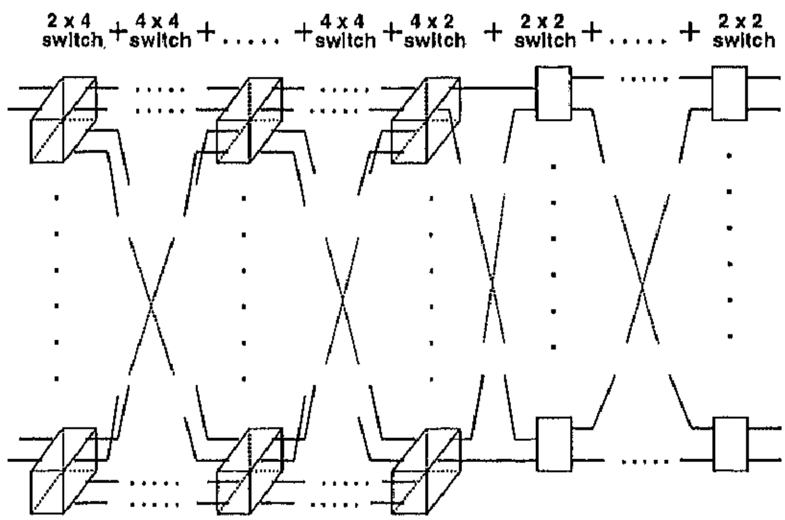

Figure 3.1: The structure of hybrid dilated banyan network.

The hybrid dilated banyan networks are constructed from two parts as in Fig. 3.1. First part is the 2-dilated banyan network. More precisely, there are 2×4 re-arrangeable input switching elements at the first stage, the 2-dilated banyan switching elements in the next $(n-2)$ stages and 4×2 re-arrangeable output switching elements at the n-th stage. Second part is the original banyan network, i.e., the rest of stages is constructed by the banyan switching elements. A 16×16 hybrid dilated banyan network is shown in Fig. 3.2.

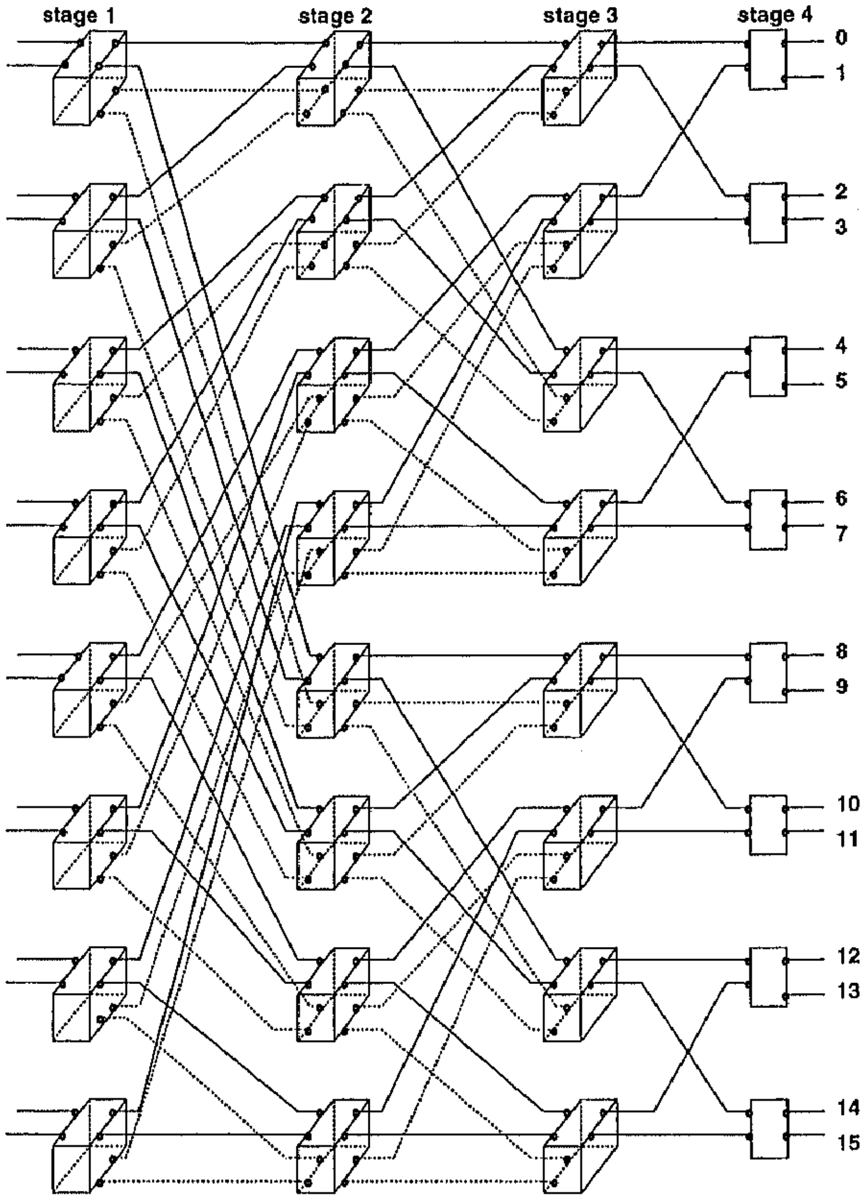

Figure 3.2: 16 × 16 hybrid dilated banyan network.

A routing algorithm of the each part in the hybrid dilated banyan networks is equal to the ones of the original banyan and the 2-dilated banyan network, respectively. The output rate of the hybrid dilated banyan network is calculated from (2.1) to (2.4) under the conditions in section 2.3.

Fig. 3.3, Fig. 3.4, Fig. 3.5 and Fig. 3.6 show the output rates of the hybrid dilated banyan networks compared to the original banyan network with the input rates of $1.0, 0.75, 0.5$ and 0.25. In these figures,

- b shows the output rate of a banyan network.
- $s2$ shows the output rate of a hybrid dilated banyan network using 2×4 re-arrangeable

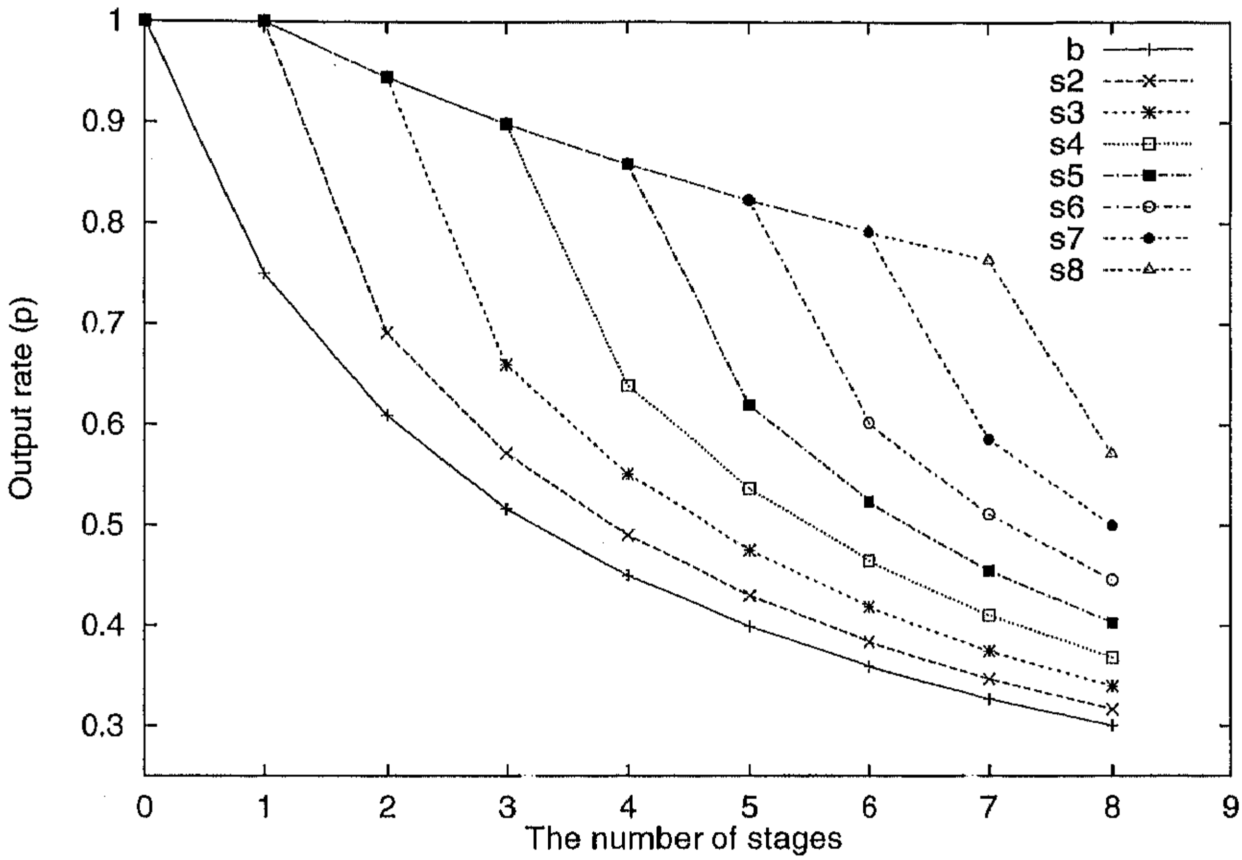

Figure 3.3: Output rate vs. the number of stages at input rate = 1.

input switching elements at the first stage, 4×2 re-arrangeable output switching elements at the second stage and banyan switching elements from the third stage to the final one.

- si ($i = 3 \sim 8$) shows the output rate of a hybrid dilated banyan network with 2×4 re-arrangeable input switching elements at the first stage, $(i-2)$ stages of 2-dilated banyan switching elements from the second stage to the $(i-1)$-th stage and 4×2 re-arrangeable output switching elements being set at the i-th stage. The rest of the hybrid dilated network from the $(i+1)$-th stage to the final stage consists of banyan switching elements. In special, when the number of stages is i, si means the output rate of $2^i \times 2^i$ 2-dilated banyan network.

At each si ($i = 2 \sim 8$), there is a plot in the $(i-1)$-stage. The plot means the output rate at the $(i-1)$-th stage, i.e., before the 4×2 re-arrangeable output switching element, in the $2^i \times 2^i$ 2-dilated banyan network. These plots notify that the 4×2 re-arrangeable switching element reduces much output rate. For example, more than 20 percent of output rate are decreased at the $(i-1)$ stages for each si. This result suggests that the output rate should be effectively increased if the output rate reduction at the 4×2 re-arrangeable switching element

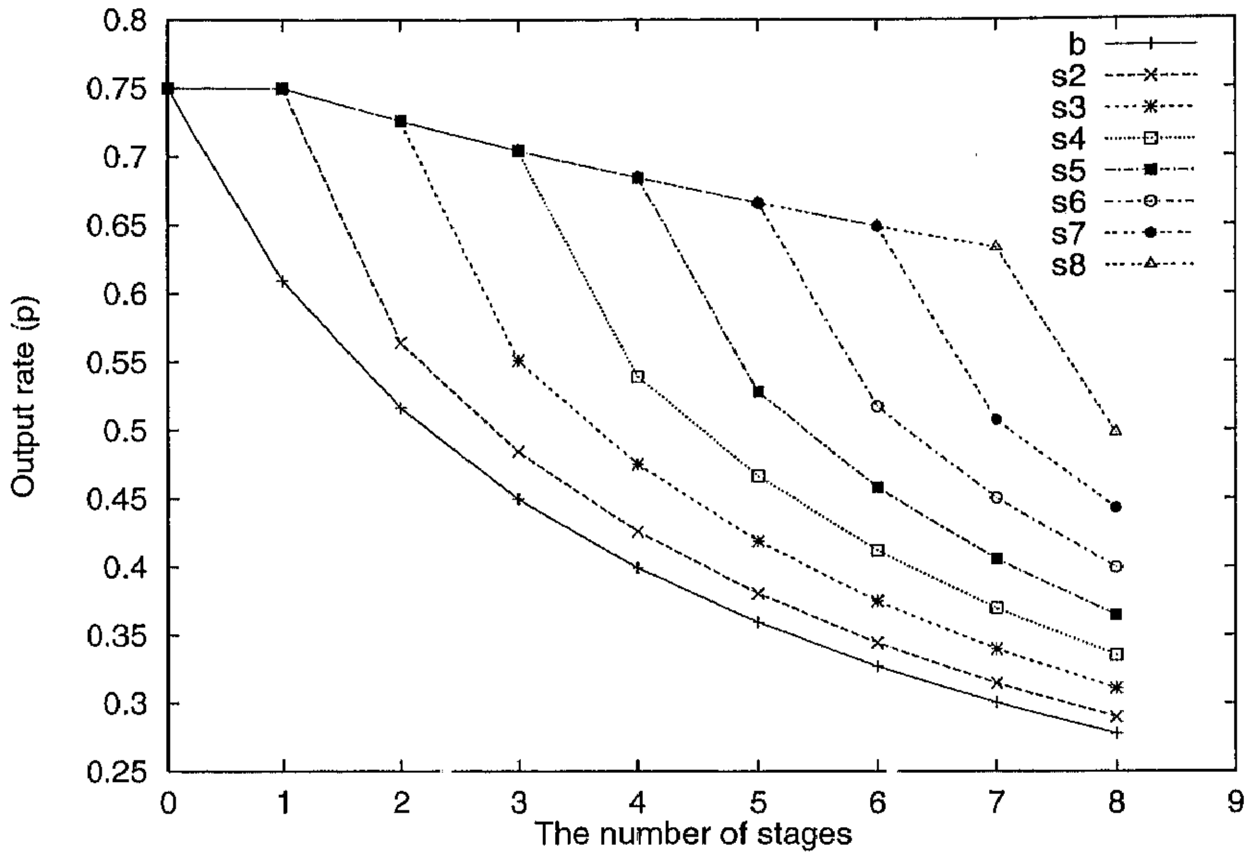

Figure 3.4: Output rate vs. the number of stages at input rate = 0.75.

becomes much small. This improvement will be discussed in the next chapter by a bypass method.

From Fig. 3.3 to Fig. 3.6, the followings are concluded. For each input rate and network size in each figure, the 2-dilated and the original banyan networks have the maximum and the minimum output rate, respectively. The output rates of the hybrid dilated banyan networks at each network size stay between the outputs of the 2-dilated and the original banyan networks at the same network size, and their output rates become bigger when the number of 2-dilated switching elements are increased.

When each network does not have much traffic (the input rate is 0.25 or 0.5), the blocking probability is also low in each stage of the hybrid dilated banyan network and the original banyan network.

When the network traffic is heavy (the input rate is 0.75 or 1.0), however, the blocking probability in each stage becomes also high. Since the hybrid dilated banyan network can reduce more blocking than that of the original banyan network, the performance of the former network becomes much larger than the one of the latter. The hybrid dilated banyan network

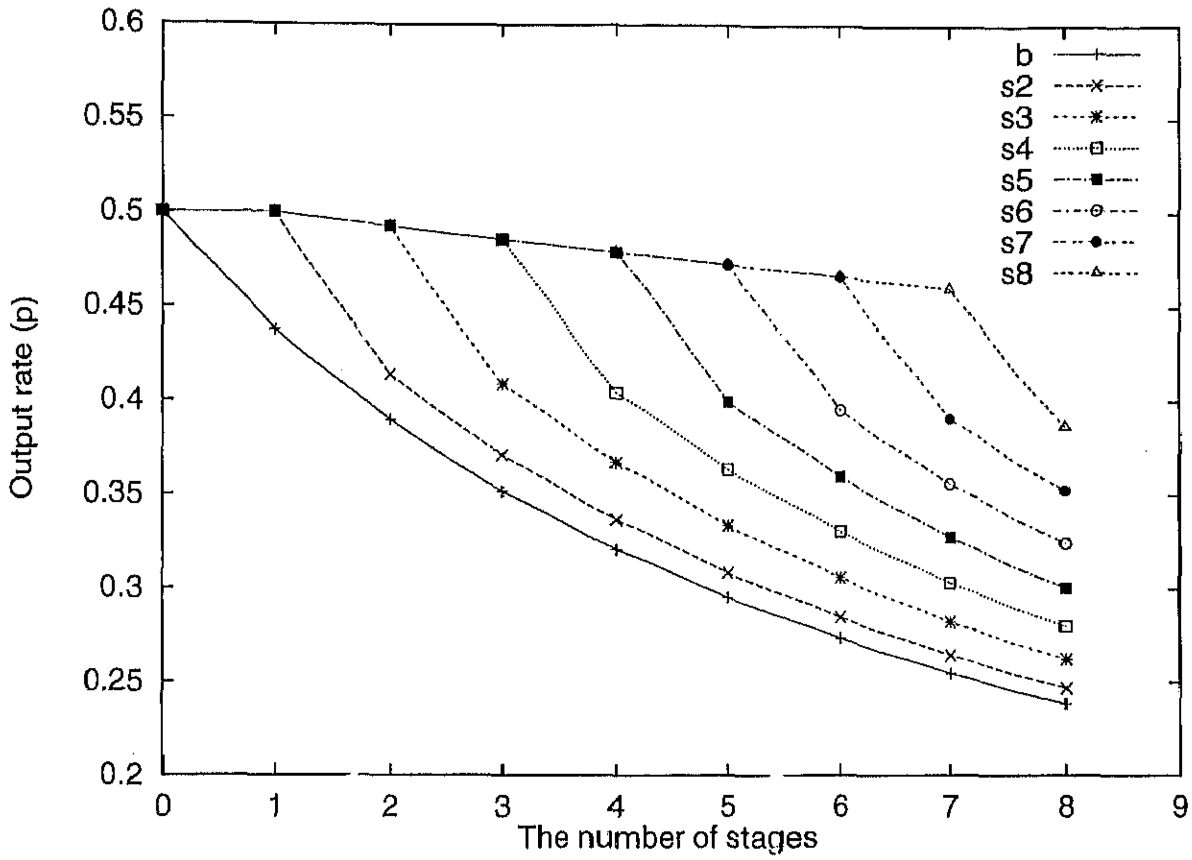

Figure 3.5: Output rate vs. the number of stages at input rate = 0.50.

is more suitable for the heavy network traffic.

From Appendix A.2.2, the 2-dilated banyan switching element has twice more hardware size than the original one. Therefore, the hybrid dilated banyan network should be less costed than the 2-dilated banyan network. Though the former network has a less output rate than the latter, the former gives network designer a flexible choice to construct an ATM switch. For example, if the design of $2^5 \times 2^5$ ATM switch with input rate 1.0 and more than output rate 0.4 (throughput) are required, the original $2^5 \times 2^5$ banyan network can not achieve such an output rate (throughput). The hybrid dilated banyan network and the 2-dilated banyan network provide the $s2 \sim s5$ type of networks as shown in Fig. 3.3 and the network designer can select the most suitable network within the required cost.

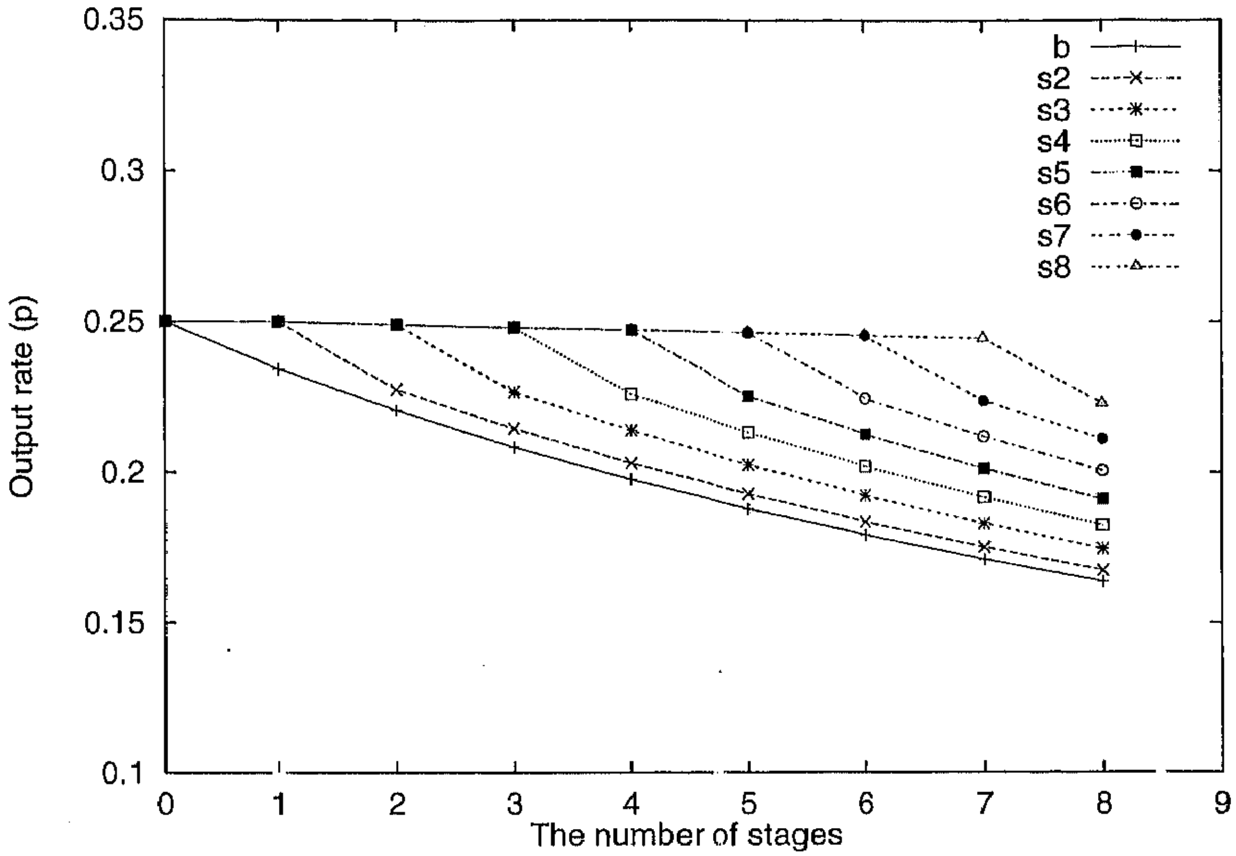

Figure 3.6: Output rate vs. the number of stages at input rate = 0.25.

3.2 Comparison of a Hybrid Dilated Banyan Network with Delta, Rerouting Banyan and Tandem Networks

In the previous section, the structure of a hybrid dilated banyan network and its performance analysis are described. This section compares the hybrid dilated banyan network with delta, rerouting banyan and tandem network[21, 22].

3.2.1 Routing Process and Hardware Size

At first, the routing process and hardware size of each network are estimated. A delta network is an $a^n \times b^n$ switching network with n stages, consisting of $a \times b$ crossbar modules (more cost-effective if b is a power of 2)[27, 31]. In this section, the following two cases are considered, i.e., a $2^n \times 2^n$ delta network being constructed from the 2×2 banyan switching element, and $4^n \times 4^n$ delta network being constructed from 4×4 switching element. The 4×4 switching element has 4 input ports and 4 output ports, and uses the same routing process as a 2×2

banyan switching element. The hardware size of a 4×4 switching element is nearly 4 times as large as a 2×2 banyan switching element. The routing process of both networks is shown as the original banyan network in section 2.1. Let N be 2^n. Then the $N \times N$ delta network constructed from the 2×2 banyan switching elements has n stages. When it is constructed from the 4×4 switching elements, it has $\frac{n}{2}$ stages, since $N = 4^{\frac{n}{2}}$. The hardware size of the $2^n \times 2^n$ delta network is $\frac{nN}{2}$ 2×2 banyan switching elements and the one of the $4^{\frac{n}{2}} \times 4^{\frac{n}{2}}$ delta network is $\frac{nN}{8}$ 4×4 switching elements. Because the 4×4 switching element hardware size is nearly 4 times as large as the 2×2 switching element, the $4^{\frac{n}{2}} \times 4^{\frac{n}{2}}$ delta network conclusively have the same hardware size as the $2^n \times 2^n$ delta network.

The $N \times N$ rerouting banyan network[17] has N input ports, N output ports and the rerouting links without buffer. Each stage of the rerouting banyan network is constructed by $\frac{N}{2}$ 2×2 switching elements, which transmit cells as the 2×2 banyan switching element of the original banyan network if cell blocking does not occurred. When two cells are blocking in a switching element of this network, the other is routed by using the other link and start a new routing at the next stage. Therefore, the rerouting banyan network needs specific switching elements and extra stages for rerouting the blocked cells[32]-[34]. To estimate the hardware size of this network, let $n = \log_2 N$ and m the number of extra stages of the specific switching elements. Then, the total number of the specific switching elements is $\frac{(n+m)N}{2}$. Since the specific switching element has just two extra links more than the original banyan switching element, the hardware size of the former is nearly same as the one of the latter.

The $N \times N$ tandem banyan network structure consists of K banyan networks connected in series, and each banyan network has N input ports and N output ports. It does not have any buffers. In each switching element of the banyan networks, however, every blocked cell is forced to be transmitted to the wrong output port, instead of eliminating it. Since any blocked cells arrive at incorrect destination output port of the banyan networks, such cells are rerouted again at the subsequent banyan network. For example, suppose a cell arrives at the i-th output port in the k-th banyan network. If its correct output port is i, it is transmitted to the i-th concentrator (statistical multiplexer)[18, 19]. Otherwise, it is entered at the i-th input port in the $(k+1)$-th banyan network if $k < K$, or else eliminated. Thus, each cell can attempt up to K times to reach its destination output. Consequently, at least K cells can arrive at each concentrator, but only one of them is transmitted to the output port of the tandem network. To achieve a low cell loss rate, the tandem banyan network may require many banyan networks. i.e., $K > 1$. For the estimation of the hardware size of this network, The size of the switching element in the tandem network is assumed to nearly equal to the original banyan network. Then the tandem network consists of $\frac{nKN}{2}$ switching elements, where n is the number of switching elements in each banyan network and $n = \log_2 N$.

The $N \times N$ hybrid dilated banyan network does not require buffers and its routing process is shown in section 3.1. To estimate the hardware size of this network, let n be the number of stages in the network and let x be the number of stages which use the 2-dilated banyan switching elements. From Appendix A.2.2, the 2-dilated banyan switching element has the size of nearly four switching elements of the original banyan network, two demultiplexers and two multiplexers. The 2×4 re-arrangeable input switching element has two demultiplexers. The 4×2 re-arrangeable output switching element has two switching elements of the original banyan networks and two multiplexers. Then, this network consists of $\frac{(3x+n)N}{2}$ switching elements, $(2x+1)N$ demultiplexers and $(2x+1)N$ multiplexers. Since the switch elements of the original banyan network consists of two demultiplexers and two multiplexers, the hardware size of the 2-dilated banyan network is nearly $\frac{(5x+n+1)N}{2}$.

3.2.2 Delay

This subsection discusses about delay time of each network. A regular $N \times N$ banyan network is constructed from $n = \log_2 N$ stages, which each stage has $\frac{N}{2}$ 2×2 banyan switching elements. Let D_b refer to the delay to process a cell in each switching element at each stage. Therefore, the switching delay time within one banyan network is nD_b.

The $N \times N$ delta network constructed from 2×2 banyan switching element needs the same delay as the banyan network. Therefore, the delay of this delta network results in nD_b, where $n = \log_2 N$. The $N \times N$ delta network constructed from 4×4 switching element has $\frac{n}{2}$ stages. The delay of the 4×4 switching element is almost twice as much as that of 2×2 switching element. Therefore, the delta network is required the same delay with the $2^n \times 2^n$ delta network.

The $N \times N$ rerouting banyan network needs the specific switching elements and some extra stages to reroute blocked cells. Although each specific switching element takes more delay than the D_b for the propagation time of its through link, this subsection assumes such propagation time can be ignored for a simple calculation. When m is the number of extra stages to reroute and $n = \log_2 N$, the delay of the $N \times N$ rerouting banyan network is $(n+m)D_b$.

The $N \times N$ tandem banyan network is constructed from K banyan networks connected in series. It requires the concentrator at each output port. The maximum switching time should be $nKD_b + \alpha$, where $n = \log_2 N$ and α is delay of the concentrator .

In the $N \times N$ hybrid dilated banyan network, no blocked cells are through a 2×4 re-arrangeable input switching element, x 2-dilated banyan networks, a 4×2 re-arrangeable output switching element and $(n - x - 2)$ banyan network, where $n = \log_2 N$ and $x + 2 \leq n$. Assume the delay of the demultiplexer is t_d and the multiplexer is t_m. The delay of each 2×2 switching element within the 2-dilated banyan switching element is also supposed to be D_s. From the construction of each network in Appendix A.2, the delay of the $N \times N$ hybrid dilated banyan network is $(x+1)t_d + xD_s + (x+1)t_m + (n-x-1)D_b$. Assume the delay of the 2×2 banyan switching elements nearly equals to the one of the 2×2 switching element in the 2-dilated banyan switching element, i.e., $D_s \approx D_b$, then the delay time is $(x+1)t_d + (x+1)t_m + (n-1)D_b$. From the construction of the 2×2 switching element, the total delay of a demultiplexer and a multiplexer nearly equals to the delay of a 2×2 switching element ($t_d + t_m \approx D_b$). Then the delay of the $N \times N$ hybrid dilated banyan network is approximate as $(n + x)D_b \geq nD_b$.

3.2.3 Switching Efficiency

This subsection discusses about switching efficiency (output rate) of four kinds of networks with the same numbers of input and output ports. The output rate of the delta network is changed depending on a switching element, but for any cases it is lower than the ones of the other three networks. By adding sufficient many numbers of the banyan networks, the tandem network can achieve the highest output rate. The hybrid dilated banyan network has higher output performance than the rerouting banyan network in some cases. But when a number of the extra stages in the rerouting banyan network are sufficiently used, the rerouting banyan network can realize higher output rate than the hybrid dilated banyan network.

For example, to compare the output performance of the four networks of the $2^8 \times 2^8$ ATM switch, let each input rate be 1.0. The output rates of the $2^8 \times 2^8$ and the $4^4 \times 4^4$ delta networks are 0.3003 and 0.3669, respectively. Although the $4^{\frac{n}{2}} \times 4^{\frac{n}{2}}$ delta network gets better performance than the 2×2 delta network, the $4^{\frac{n}{2}} \times 4^{\frac{n}{2}}$ delta network cannot be constructed if n is an odd number. The output rate of the tandem banyan network is 0.9930. The output rates of the rerouting banyan networks for 1 to 6 extra stages are from 0.3188 to 0.4540, and ones for 7 to 27 extra stages are from 0.4903 to 0.6431 which is the maximum of the $2^8 \times 2^8$ rerouting banyan network without buffer. The output rate of the hybrid dilated banyan network for 0 to 6 stages of 2-dilated banyan switching elements is from 0.3168 to 0.5716.

3.2.4 Flexibility for Network Designs

Delta network provides the selections of its design by modifying the size of its switching element. As mentioned in the previous subsection, a larger size of switching elements improves a little output rate, but restricts the available network size of the delta network. Thus, this network has less flexible for network designs.

Rerouting banyan network can be added a number of extra stages to increase its output rate but also generates its delay. Therefore, this network can be designed to satisfy the required performance.

Tandem network is constructed from series of banyan networks. By increasing the number of the banyan networks, the tandem network provides the required output rate, but its delay and hardware size are also increased linearly. Therefore, it may be difficult to design the system suit for the required performance.

Hybrid dilated banyan network is constructed from four kinds of switching elements, however, a network designer is easy and flexible to design the required networks by using the analytic results of output rate in Fig. 3.3.

3.2.5 The Results of Comparison

The hybrid dilated banyan network was compared with delta, rerouting banyan and tandem network as shown in Table 3.1. The hybrid dilated banyan network has better switching efficiency and flexibility than the delta network. It has also shorter delay and better switching efficiency than the rerouting banyan network in some kinds of switching structure, and has smaller delay, smaller hardware size and better flexibility than the tandem network.

	Delta banyan	Rerouting banyan	Tandem banyan	Hybrid dilated banyan
Switching efficiency ($2^8 \times 2^8$ switch with input rate 1.0)	0.3003 ($2^8 \times 2^8$ delta) and 0.3669 ($4^4 \times 4^4$ delta)	0.3188 to 0.6431	0.3003 to 0.9930	0.3168 to 0.5716
Hardware (SE. = banyan switching elements)	$nN/2$ SE.	$(n+m)N/2$ SE.	$nKN/2$ SE.	$(5x+n+1)N/2$ SE.
Delay	nD_b	$(n+m)D_b$	$nKD_b + \alpha$	$(n+x)D_b$
Flexibility	none	good	difficult	good

Table 3.1: Comparisons of hybrid dilated banyan network with delta, rerouting banyan and tandem network