# Chapter 2

# Background

In ATM networks, self-routing networks are used for constructing an interconnecting fabric of the switching system. This chapter suggests banyan networks and dilated banyan networks which are popular methods for realizing self-routing networks, and shows their output rates when their input rates are fixed.

## 2.1 Banyan Networks

A banyan network is defined as a network with an unique part between each input/output pair, where each stage performs a fixed permutation on the incoming lines, and then routes them through a column of $2 \times 2$ banyan switching elements to the output as shown in Fig. 2.1.

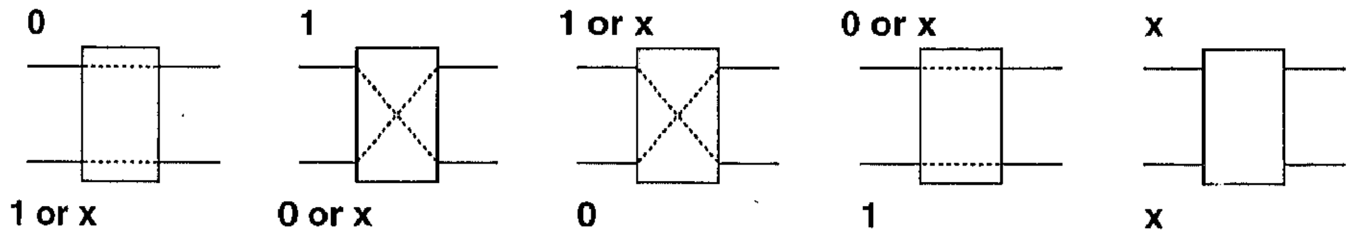


Figure 2.1: $2 \times 2$ banyan switching elements.

Each banyan switching elements are connected each other to construct a banyan network. Fig. 2.2 shows an 8-input and 8-output $(8 \times 8)$ banyan network. It has three columns of $2 \times 2$ banyan switching elements called stages. An $N \times N$ banyan network has $n = \log_2 N$ stages where $N$ is a power of 2. Its input and output ports are labeled $d_0$,$d_1$,..., and $d_{n-1}$.

The routing algorithm of the banyan network is as follows. Each cell contains an $n$ bits destination binary address of its requested output port. In the $k$-th $(0 \leq k \leq n-1)$ stage, a banyan switching element which receives a cell examines the $n$-th most significant bit in the destination address and transmit the cell to its output port based on the value of the bit. If the bit is 0, the upper output port is selected, and if the bit is 1, the lower output port is selected. If only one cell selects a particular output port, it is forwarded on the single output link. When two cells arrive at a banyan switching element at the same time and select the same output port, link debate has occurred. One cell is randomly selected and forwarded, but the other is blocked.

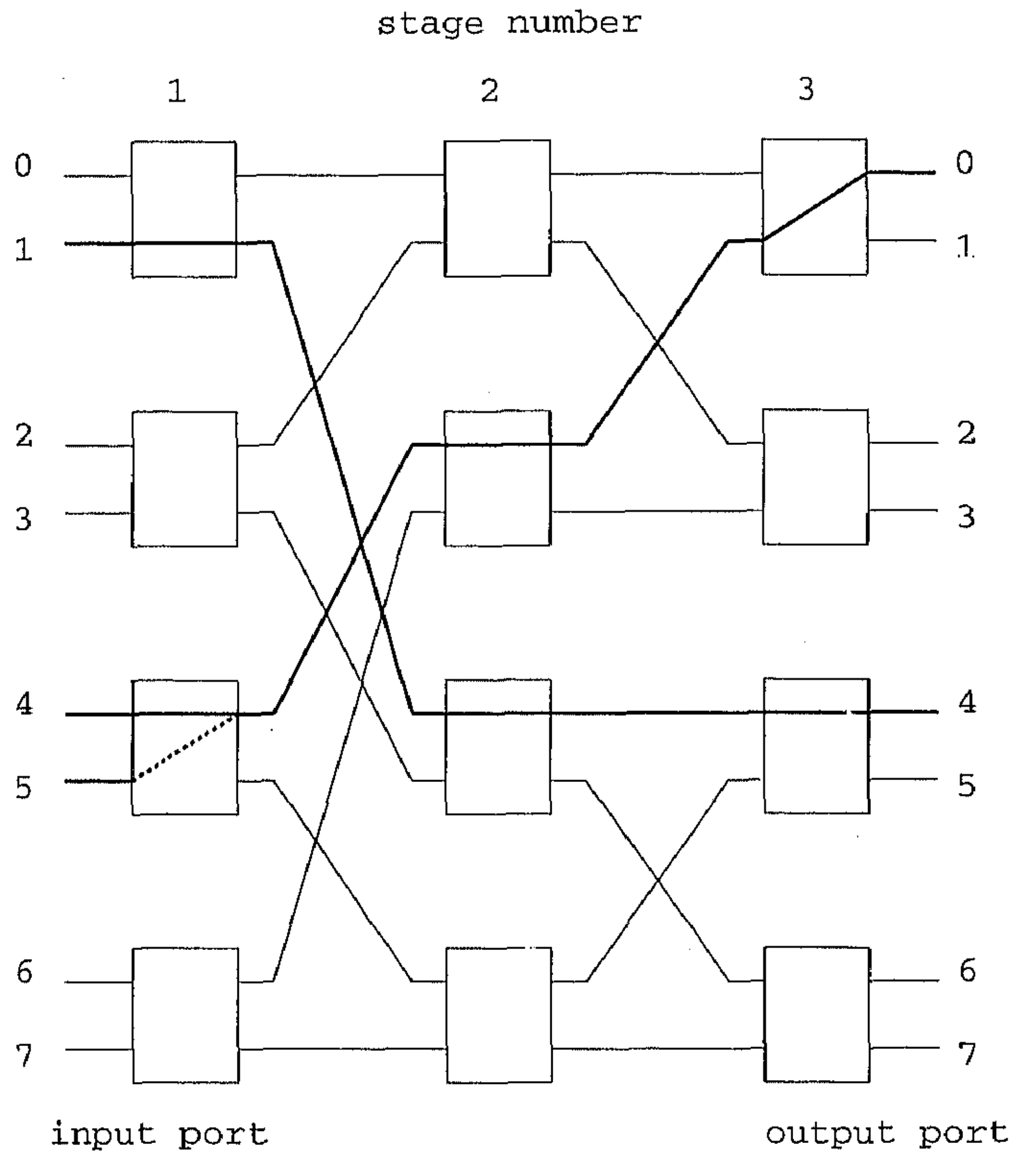


Figure 2.2: 8 × 8 banyan network with three stages.

Fig. 2.2 illustrates the attempted routing of three cells from 1 to 4, from 4 to 0 and from 5 to 3, respectively. For the first cell from the first input port to the fourth output port, no blocking occurs in the routing. However, the other two cells are involved in same link debate in the first stage and only one cell from the fourth input port is forwarded to the second stage. Since the blocked cells in the banyan switching element are lost, it causes that the performance of the banyan network is decreased. In the next section, the improved network of the banyan network is introduced.

For performance analysis of a banyan network, assume that the probability of a cell arrival rate at an input port is fixed to $p$ [26]-[28]. The probability $p$ is named as an input rate. From the construction of a banyan network, all probabilities of the cell arrival rate at any switching elements in the same stage are equal. Let $p_m$ be the probability at the $m$-th stage. Then it satisfies the recurrence relation[29]:

$$p_{m+1} = 1 - (1 - \frac{p_m}{2})^2 \tag{2.1}$$

with boundary condition $p_0 = p$. For more detail discussion of the above relation, see Appendix A.1.1.

## 2.2 Dilated Banyan Networks

A $d$-dilated banyan network[16] is obtained from an original banyan network in the previous section by replacing each link by independent $d$ links (in this thesis, 2-dilated is used). A $16 \times 16$ 2-dilated banyan network is shown in Fig. 2.3. This extension can be constructed by multiplexing space of each link in the original banyan network so that it can support up to $d$ connections.

The routing algorithm for 2-dilated banyan network is a simple generalization of that used in the previous section. Each cell carries an $n$ bit destination address, and a 2-dilated banyan switching element in the $k$-stage selects its output port based on the $k$-th most significant bit of the destination as in the banyan network. When one or two cells arrive at a 2-dilated banyan switching element and select the same output port, each cell can be forwarded. If three or more cells arrive at a 2-dilated banyan switching element and request the same output port, then two cells are randomly selected and forwarded to the next stage but the others are blocked. For example, five cells are entering in Fig. 2.3. Two cells from the fourth and fifth input ports are selects upper output port at the second stage, and then both of them are forwarded to the third stage. The other three cell are also selects upper output port at the second stage, however, the cell from the eleventh input port is blocked.

Note that the switching elements at the first and final stages works different from ones of the other stages. The former switching element has two input and four output ports, and blocks no cells. The latter one has four input port but two output ports. Therefore, its cell loss rate is much greater than the ones at the other stages. In this thesis, the switching element at

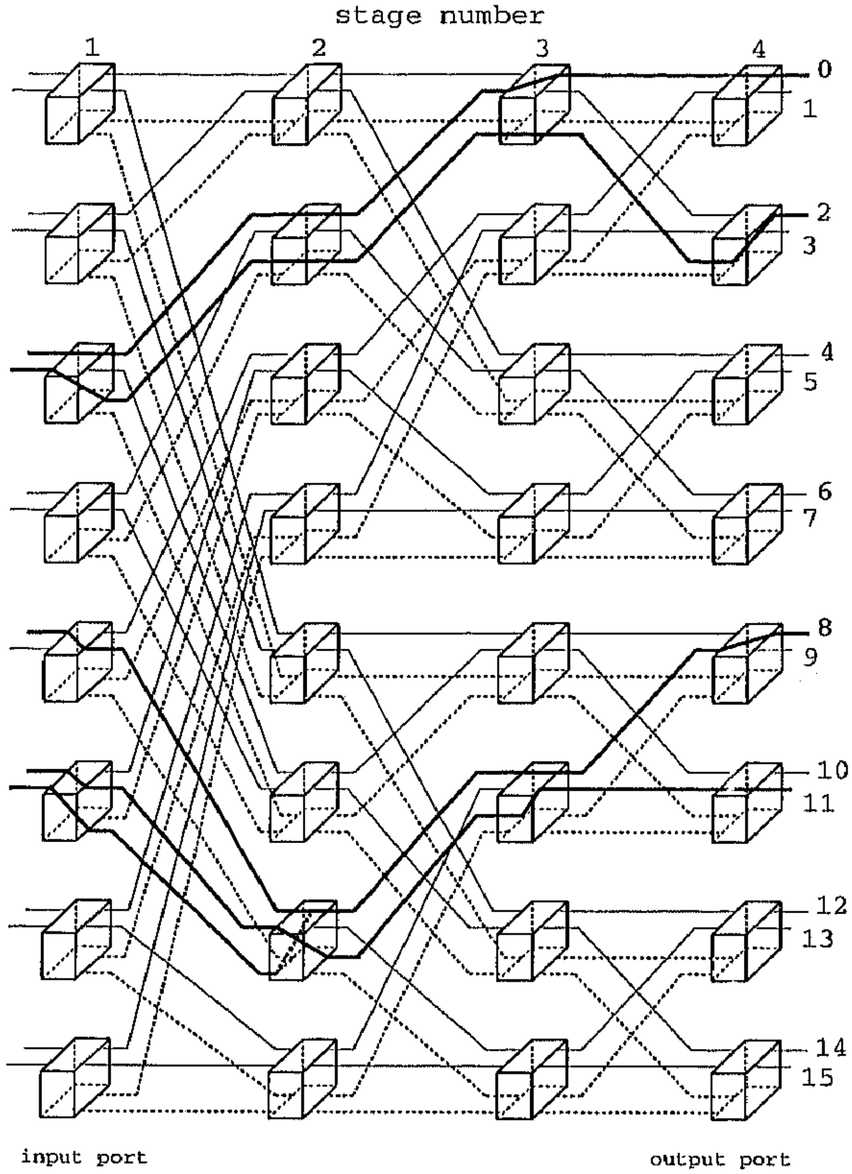


Figure 2.3: 16 × 16 2-dilated banyan network with four stages.

both stages are called 2 × 4 re-arrangeable input switching element and 4 × 2 re-arrangeable output switching element[30], respectively.

As in a banyan network, the output rate of each switching element can be obtained, when the input rate of the 2-dilated banyan network is fixed to $p$. Let $p_m$ be the output rate of switching elements at the $m$-th stage, and $p_0 = p$. At the first stage, the output rate is calculated as follows:

$$p_1 = \frac{1}{2}p. \tag{2.2}$$

When $1 \leq m \leq n-2$ and $n$ is the number of the stages of the 2-dilated banyan network,

$p_m$ satisfies the recurrence relation in below:

$$p_{m+1} = p_m - \frac{1}{4}{p_m}^3 + \frac{1}{16}{p_m}^4. \tag{2.3}$$

At the final stage, the output rate $p_n$ results in the following relation:

$$p_n = 2p_{n-1} - \frac{3}{2}p^2{}_{n-1} + \frac{1}{2}p^3{}_{n-1} - \frac{1}{16}p^4{}_{n-1}. \tag{2.4}$$

For more detailed inductions, see Appendix A.1.2.

## 2.3 Output Rates of the Original and the 2-Dilated Banyan Networks

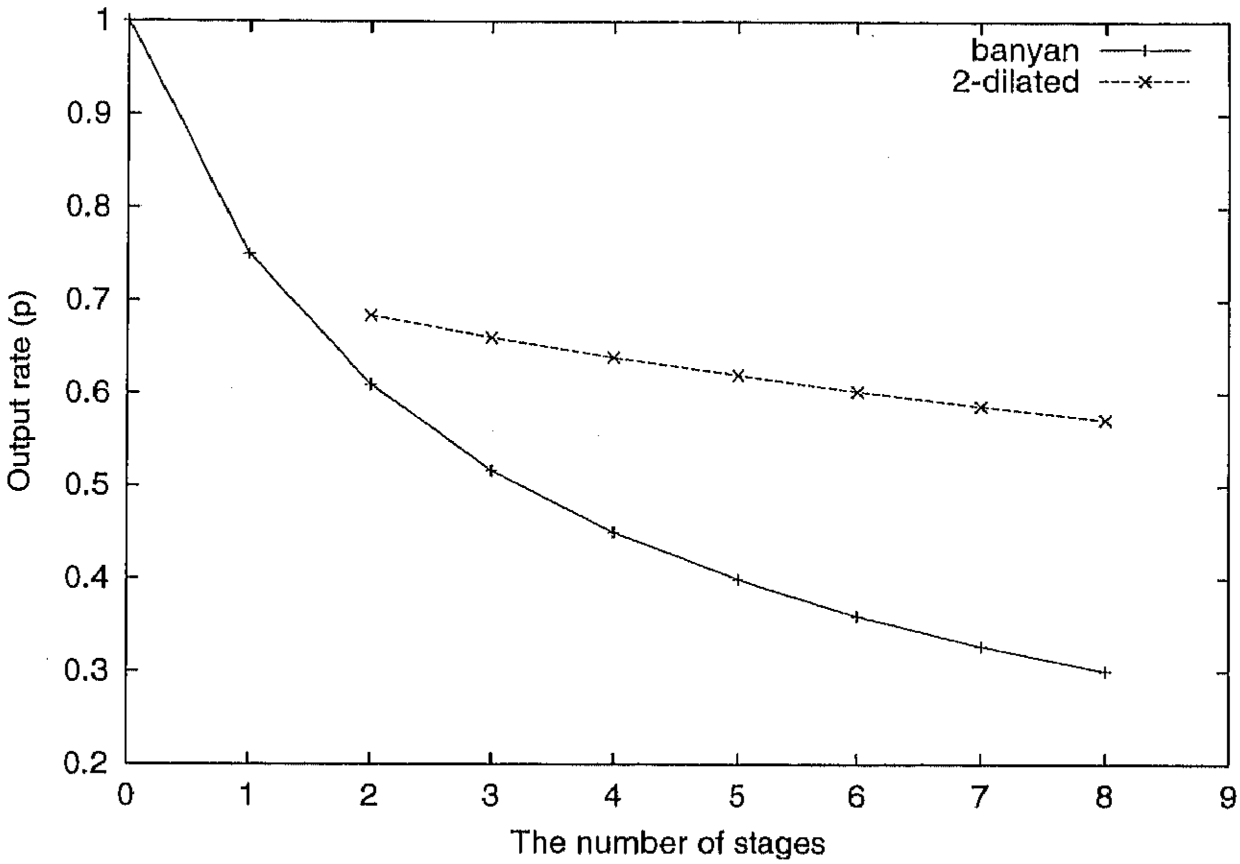


Figure 2.4: The output rate of original banyan and 2-dilated banyan network at input rate = 1.0.

Fig. 2.4 depicts the output rates of the original and the 2-dilated banyan networks when all input rates of each network are fixed at 1.0. This results at each stages are obtained under the following conditions[26]-[29]:

1. Each network does not have any sort of cell buffers.

2. Every processor transmitting to each network generates random and independent cells.

3. Cells arriving at an input of each network are uniformly distributed all over the outputs of the network.

4. The probability of output rate in a switching element becomes the probability of input rate into the next switching element connected by a link.

5. A cell may have link debation or will be successfully forwarded. The cell is independent of the previous cell.

6. The blocked cells are neglected. Then, the cells issued at the next time cycle are independent to the previous ones.

7. A link (and a bypass) between two switch elements can carry only one cell in each time cycle.

The above conditions are assumed for the output rate calculations through this thesis. Note that a bypass in condition 7 will be introduced in chapter 4.

From Fig. 2.4, the output rates of the 2-dilated banyan network are up to 90.3 percent greater than these of the banyan network. However the size of the switching element of the former network is more than two times of the latter's from Appendix A.2. Thus, the cost of the former one should be more greater than that of the latter one. In the next chapter, hybrid dilated banyan networks are proposed to improve this problem.