A Study on Validated Computation of Zeros of Analytic Functions

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Abstract

In this thesis, we consider the problem of computing the zeros of analytic functions that lie inside a circular region of the complex plane. A validated method for bounding cluster of zeros of analytic functions and a validated method for locating the zeros in the cluster are proposed. First we consider the problem of computing the zeros of polynomials which have multiple zeros or clusters of zeros. An eigenvalue method is presented which constructs a new companion matrix whose eigenvalues are simple and the zeros of the given polynomial. This method can calculate multiple zeros and their multiplicities accurately, and it is also efficient in calculating the center of the cluster of zeros and the number of zeros in the cluster.

One problem in computing zeros of polynomials using eigenvalue method is that a small perturbation in the elements of a companion matrix may produce significant changes in the eigenvalues of the companion matrix calculated by the QR method. In order to improve the accuracy of the elements of the new companion matrix, we propose a new method to compute the companion matrix. The method uses a three-term recurrence algorithm based on Euclid's algorithm to calculate the values of the polynomial with arithmetic operations of $O(n^2)$, where n is the degree of the polynomial, then constructs the companion matrix and calculates the eigenvalues of it by the QR method. The eigenvalue computation is carried out using floating point arithmetic and other computations are carried out in multiple precision arithmetic. In this way, increase of overall computing time can be contained. This method is efficient in computing multiple zeros or the center of the cluster of zeros of high degree polynomials.

Next we consider the problem of computing the zeros of analytic functions that lie inside a circular region. For the determination of multiple or cluster of zeros of an analytic function $f(z)$, factoring methods can find such zeros as a polynomial. The computation of coefficients of a polynomial of which zeros are close is more stable than the determination of locations of close zeros. For this reason, we present a validated method to compute an upper bound of $|f(z)|$ on the boundary of a disk in which $f(z)$ is analytic, and then perform a validated computation of the Taylor coefficients of $f(z)$. Based on these results, a factorization method is used to calculate an interval coefficient polynomial which includes a polynomial factor of $f(z)$. Circular complex interval arithmetic is used in the computation and the results are numerically validated.

Given an analytic function $f(z)$ with a cluster of zeros around the origin, we present a method to compute the number m of zeros in the cluster and compute a disk centered at the origin which contains exactly m zeros of $f(z)$. This method is based on the validated computation of the n-degree Taylor polynomial $p(z)$ of $f(z)$, and uses some well known formulae for bounding zeros of polynomials to calculate a disk containing the cluster of zeros of $p(z)$. Rouché's theorem is used to verify that the disk contains the cluster of zeros of $f(z)$. The computations are performed using circular complex arithmetic and the results are mathematically correct.

A number of numerical examples are presented to illustrate the efficiency of the methods presented in this thesis.

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