Appendix A

Azimuthal Angular Correlations

In addition to possible partial-rate asymmetries, CP violation may be accessed in charmless $B \to VV$ (where both vectors decay into two pseudo scalars) decays via azimuthal angular correlations between the two vector particles.

As shown in Fig. A.1, we define θ_1 is the polar angle of the π^+ (ρ^0 daughter) in the rest system of the ρ^0 with respect to the helicity axis; similarly θ_2 and ϕ are the polar and azimuthal angle of the π^+ (ρ^+ daughter) in the ρ^+ rest system with respect to the helicity axis of the ρ^+; i.e. ϕ is the angle between the planes of the two decays $\rho^0 \to \pi^+\pi^-$ and $\rho^+ \to \pi^+\pi^0$.

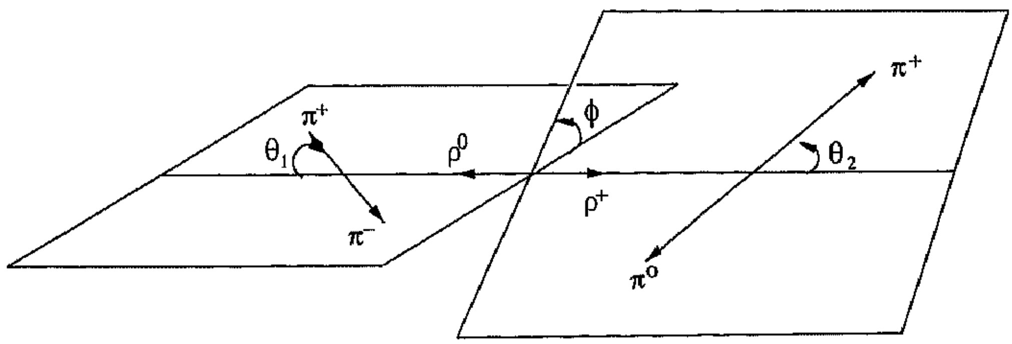

Figure A.1: The helicity frame for $B^+ \to \rho^+\rho^0$.

With this definition, the angular distribution for the decay has the following form,

$$\begin{aligned}\frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} \sim & \frac{1}{4}\frac{\Gamma_T}{\Gamma}\sin^2\theta_1\sin^2\theta_2 + \frac{\Gamma_L}{\Gamma}\cos^2\theta_1\cos^2\theta_2 \\ & + \frac{1}{4}\sin 2\theta_1 \sin 2\theta_2(\alpha_1\cos\phi - \beta_1\sin\phi) \\ & + \frac{1}{2}\sin^2\theta_1\sin^2\theta_2(\alpha_2\cos 2\phi - \beta_2\sin 2\phi),\end{aligned}$$

where the coefficients are given by,

$$\frac{\Gamma_T}{\Gamma} = \frac{|H_{+1}|^2 + |H_{-1}|^2}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2} \qquad \frac{\Gamma_L}{\Gamma} = \frac{|H_0|^2}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}$$
$$\alpha_1 = \frac{Re(H_{+1}H_0^* + H_{-1}H_0^*)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2} \qquad \beta_1 = \frac{Im(H_{+1}H_0^* - H_{-1}H_0^*)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}$$
$$\alpha_2 = \frac{Re(H_{+1}H_{-1}^*)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2} \qquad \beta_2 = \frac{Im(H_{+1}H_{-1}^*)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}.$$

As described, direct CP-violating observables require the interference of two amplitudes with difference weak phases, ϕ. We decompose the amplitudes into contributions proportional to $V_{ub}V_{us}^*$ and $V_{cb}V_{cs}^*$. By letting $v_u \equiv V_{ub}V_{us}^*$, and $v_c \equiv V_{cb}V_{cs}^*$, we have

$$A = v_u A^{(u)} + v_c A^{(c)},$$

where A stands for a generic decay or helicity amplitude.

The decay rate, as well as parameters α_i $(i = 1, 2)$, can signal CP violation only one compares them with the corresponding quantities of the charge conjugate decay channel, and when both, non-vanishing weak phase differences $(\phi_u - \phi_c)$ and strong phase shifts $(\delta_u - \delta_c)$ are present. For instance, the decay rate asymmetry

$$\Gamma - \bar{\Gamma} \sim Im[v_u v_c^*] \cdot Im[A_u A_c^*] \sim \sin(\phi_u - \phi_c) \cdot \sin(\delta_u - \delta_c). \tag{A.1}$$

The decay parameters β_i $(i = 1, 2)$ can have non-zero values in the presence of either weak or strong phases alone. By comparison with the parameters $\bar{\beta}_i$ (β_i^{CP}) of the C (CP) conjugate decay, we can, in principle, establish a weak phase difference even for vanishing strong phases

$$\beta_i + \bar{\beta}_i = \beta_i - \beta_i^{CP} \sim Im[v_u v_c^*] \cdot Re[A_u A_c^*] \sim \sin(\phi_u - \phi_c) \cdot \cos(\delta_u - \delta_c). \tag{A.2}$$

Or measure the strong phase shifts even for negligible weak phases

$$\beta_i - \bar{\beta}_i = \beta_i + \beta_i^{CP} \sim Re[v_j v_k^*] \cdot Im[A_j A_k^*] \sim \cos(\phi_j - \phi_k) \cdot \cos(\delta_j - \delta_k), \tag{A.3}$$

where the relative sign between $\bar{\beta}_i$ and β_i^{CP} is due to the parity reflection.

From Equations A.1 through A.3, we see that, if there are no CP-violating weak phases then $\beta_i = -\bar{\beta}_i$, and $\alpha_i = \bar{\alpha}_i$, and $\Gamma = \bar{\Gamma}$, while the absence of strong phases implies $\beta_i = \bar{\beta}_i$ (of course, $\alpha_i = \bar{\alpha}_i$, and $\Gamma = \bar{\Gamma}$)

The presence of strong phases is unambiguously demonstrated by a partial rate asymmetry as well as angular correlations

$$\frac{2\pi}{\Gamma}\frac{d\Gamma}{d\phi} - \frac{2\pi}{\bar{\Gamma}}\frac{d\bar{\Gamma}}{d\phi} = -(\alpha_2 - \bar{\alpha}_2)\cos 2\phi - (\beta_2 - \bar{\beta}_2)\sin 2\phi.$$

Other terms can be isolated by examining the ϕ dependence of the differential rate difference between same hemisphere (SH) events (e.g. $\theta_1 > 0$, $\theta_2 < \pi/2$) and opposite hemisphere (OH) events (e.g. $0 < \theta_1 < \pi/2$, $\pi/2 < \theta_2 < \pi$),

$$\frac{2\pi}{\Gamma}(\frac{d\Gamma^{OH}}{d\phi} - \frac{d\Gamma^{SH}}{d\phi}) - \frac{2\pi}{\bar{\Gamma}}(\frac{d\bar{\Gamma}^{OH}}{d\phi} - \frac{d\bar{\Gamma}^{SH}}{d\phi}) = -\frac{1}{2}\{(\alpha_1 - \bar{\alpha}_1)\cos\phi - (\beta_1 - \bar{\beta}_1)\sin\phi\}.$$

In general the dominant terms in the angular coefficients are Γ_T/Γ, Γ_L/Γ, α_1 and α_2, the terms β_1 and β_2 are small. It is found that the small angular coefficients are not significantly degraded by the strong phases.