# Appendix B

# DCP Violation via $\rho\omega$ Interference

In $B \to \rho^0(\omega)\rho^+$, a large direct $CP$ asymmetries via $\rho\omega$ interference is predicted [16]. By measuring the $\rho\omega$ interference, we can, in principle, derive the weak phase $\phi_2$.

Figure B.1 shows the quark level diagrams for the decays of $B^+ \to \rho^+\rho^0$ and $B^+ \to \rho^+\omega$. The tree process gives the final state of $\rho^0\rho^+$ and $\omega\rho^+$, while the penguin process contributes solely to the final state of $\omega\rho^+$.

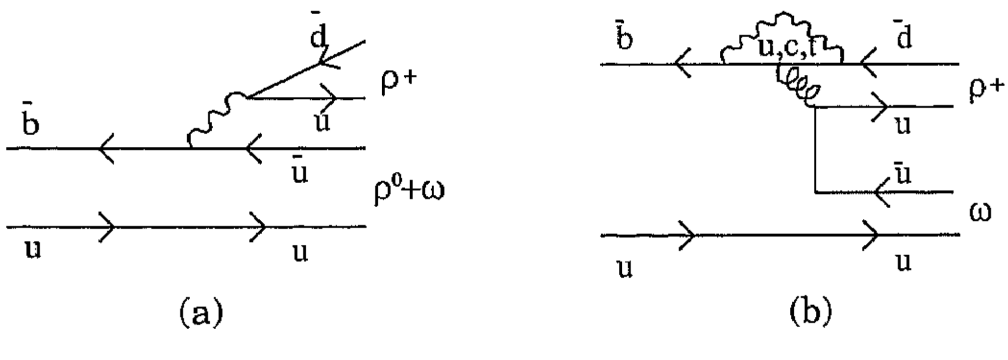


Figure B.1: (a) The tree diagram for the decays of $B^+ \to \rho^+\rho^0$ and $B^+ \to \rho^+\omega$. (b) The penguin diagram for the decay $B^+ \to \rho^+\omega$.

In the $B^\mp \to \pi^+\pi^-\rho^\mp$ system, we define the rate $CP$ asymmetry as

$$\mathcal{A}_{CP} = \frac{|\bar{A}_{\bar{f}}|^2 - |A_f|^2}{|\bar{A}_{\bar{f}}|^2 + |A_f|^2} = \frac{2r\sin(\Delta\delta)\sin(\Delta\phi)}{1 + r^2 + 2r\cos(\Delta\delta)\cos(\Delta\phi)},$$

where $\Delta\delta$ ($\Delta\phi$) is the strong phase (the weak phase) difference. The amplitudes ( $A_f$ and $\bar{A}_{\bar{f}}$ )

$$A_f \equiv \langle\pi^+\pi^-\rho^+|\mathcal{H}^T|B^+\rangle + \langle\pi^+\pi^-\rho^+|\mathcal{H}^P|B^+\rangle = \langle\pi^+\pi^-\rho^+|\mathcal{H}^T|B^+\rangle[1 + re^{i\Delta\delta}e^{i\Delta\phi}],$$

$$\bar{A}_{\bar{f}} \equiv \langle \pi^+\pi^-\rho^-|\mathcal{H}^T|B^-\rangle + \langle \pi^+\pi^-\rho^-|\mathcal{H}^P|B^-\rangle = \langle \pi^+\pi^-\rho^-|\mathcal{H}^T|B^-\rangle[1 + re^{i\Delta\delta}e^{-i\Delta\phi}],$$

where $\mathcal{H}^T$ is the tree Hamiltonian, and $\mathcal{H}^P$ is the penguin Hamiltonian, respectively. $r$ stands for the absolute ratio of penguin to tree,

$$r \equiv \left| \frac{\langle \pi^+\pi^-\rho^+|\mathcal{H}^P|B^+\rangle}{\langle \pi^+\pi^-\rho^+|\mathcal{H}^T|B^+\rangle} \right|.$$

Furthermore, we define the

$$\alpha e^{i\Delta\delta_\alpha} \equiv \frac{\langle \omega\rho^+|\mathcal{H}^T|B^+\rangle}{\langle \rho^0\rho^+|\mathcal{H}^T|B^+\rangle},$$

$$\beta e^{i\Delta\delta_\beta} \equiv \frac{\langle \rho^0\rho^+|\mathcal{H}^P|B^+\rangle}{\langle \omega\rho^+|\mathcal{H}^P|B^+\rangle},$$

$$r' e^{i(\Delta\delta_q + \Delta\phi)} \equiv \frac{\langle \omega\rho^+|\mathcal{H}^P|B^+\rangle}{\langle \rho^0\rho^+|\mathcal{H}^T|B^+\rangle},$$

where $\Delta\delta_\alpha$, $\Delta\delta_\beta$ and $\Delta\delta_q$ are the strong phase differences at short distance, and $\Delta\phi$ is the weak phase difference.

The existence of the short distance absorptive triggers a partial rate $CP$ asymmetry. From the above definition, for $B^+ \to \rho^+\rho^0$, we have

$$\frac{\langle \rho^0\rho^+|\mathcal{H}^P|B^+\rangle}{\langle \rho^0\rho^+|\mathcal{H}^T|B^+\rangle} = \frac{\langle \rho^0\rho^+|\mathcal{H}^P|B^+\rangle}{\langle \omega\rho^+|\mathcal{H}^P|B^+\rangle} \frac{\langle \omega\rho^+|\mathcal{H}^P|B^+\rangle}{\langle \rho^0\rho^+|\mathcal{H}^T|B^+\rangle} = \beta r' e^{i(\Delta\delta_q + \Delta\phi)}.$$

The $\mathcal{A}_{CP}(B^\mp \to \rho^\mp\rho^0)$ is given by,

$$\frac{\Gamma(B^- \to \rho^0\rho^-) - \Gamma(B^+ \to \rho^0\rho^+)}{\Gamma(B^- \to \rho^0\rho^-) + \Gamma(B^+ \to \rho^0\rho^+)} = \frac{2\beta r' \sin(\Delta\delta_q + \Delta\delta_\beta)\sin(\Delta\phi)}{1 + \beta^2 r'^2 + 2\beta r' \sin(\Delta\delta_q + \Delta\delta_\beta)\sin(\Delta\phi)}. \quad \text{(B.1)}$$

Similarly, for $B^\mp \to \omega\rho^\mp$, we get

$$\frac{\Gamma(B^- \to \omega\rho^-) - \Gamma(B^+ \to \omega\rho^+)}{\Gamma(B^- \to \omega\rho^-) + \Gamma(B^+ \to \omega\rho^+)} = \frac{2\alpha^{-1} r' \sin(\Delta\delta_q - \Delta\delta_\alpha)\sin(\Delta\phi)}{1 + \alpha^{-2} r'^2 + 2\alpha^{-1} r' \sin(\Delta\delta_q - \Delta\delta_\alpha)\sin(\Delta\phi)}. \quad \text{(B.2)}$$

Also, we obtain the ratio of partial decay width

$$\frac{\Gamma(B^- \to \omega\rho^-) + \Gamma(B^+ \to \omega\rho^+)}{\Gamma(B^- \to \rho^0\rho^-) + \Gamma(B^+ \to \rho^0\rho^+)} = \frac{1 + \alpha^{-2} r'^2 + 2\alpha^{-1} r' \sin(\Delta\delta_q - \Delta\delta_\alpha)\sin(\Delta\phi)}{1 + \beta^2 r'^2 + 2\beta r' \sin(\Delta\delta_q + \Delta\delta_\beta)\sin(\Delta\phi)}. \quad \text{(B.3)}$$

Thus by measuring the $M(\pi^+\pi^-)$ spectra for $B^+$ and $B^-$ around $M(\omega) \pm \Gamma(\omega)$, which provide the information of pole position and magnitude, we can, in principle, derive the weak phase $\phi$ based on the Equations B.1 through B.3. These measurements have the advantage without assuming any theoretical models of the hadron matrix elements, and also the existence of the electroweak penguin operator does not affect this determination.

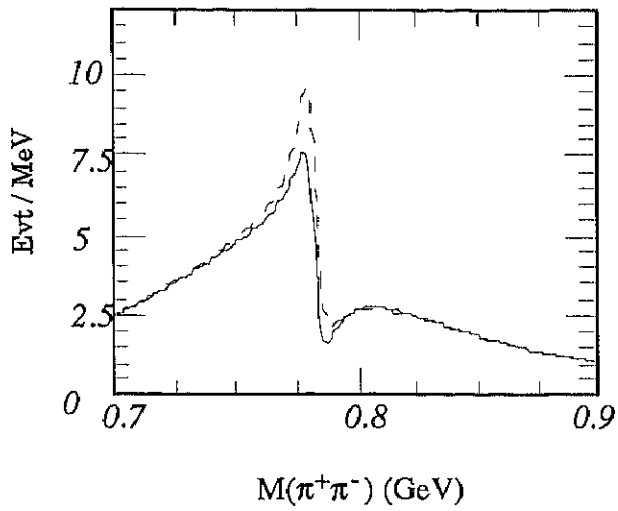


Figure B.2: The expected invariant mass spectra of $M(\pi^+\pi^-)$ ($N_c = 2$ and $k^2 = 0.5m_b^2$). Solid plot is for $B^+$, dashed plot is for the $B^-$. The asymmetry pattern is due to the $\rho\omega$ interference.

Figure B.2 shows the predicted asymmetry patterns of $M(\pi^+\pi^-)$ spectra by assuming the color suppression factor $N_c = 2$ , and the gluon momentum $k^2 = 0.5m_b^2$. Solid plot is for $B^+$, and dashed plot is for $B^-$.

Although the branching fraction of the isospin-violating decay $\omega \to \pi^+\pi^-$ is small (1.7%), the interference at the kinematical region $M(\pi^+\pi^-) \sim M(\omega) \pm \Gamma_\omega$ is enhanced by the $\omega$ pole. The $CP$ asymmetry appears in the deformation of the Breit-wigner shape of the $\rho^0 \to \pi^+\pi^-$ invariant mass spectrum. To carry out this measurement, a $M(\pi\pi)$ mass resolution is required to be better than the width of the $\omega$ meson (8.4 GeV) at the $\omega$ mass region.