Appendix A

Appendix

A.1 Hypercoordinate in 2-hyperaxis

Given an object p in a 2-hyperaxis using 3 pivots, $p_1p_2p_3$, as shown in Figure A.2(A). With Equation (5.6), the object p's first relative coordinate $D(p|_{p_1p_2p_3}, p_1p_2)$ can be computed. The second relative coordinate $D(p|_{p_1p_2p_3}, p_1p_2p_3)$ is illustrated in Figure A.2(B).

A.2 The number of pivot object from k to $k+1$

We compute the relative coordinate $D(p, p_1p_2...p_k)$ using an inductive method. Figure A.4 shows a hyperaxis formed by $k+1$ pivot objects. The formulation is given in Figure A.1 and Figure A.3.

In the $\triangle p_1p_3p_3|_{p_1p_2}$, the following equation holds:

$$(dist(p_3, p_3|_{p_1p_2}))^2 = (dist(p_1, p_3))^2 - (D(p_3, p_1p_2))^2$$

In the $\triangle p'p|_{p_1p_2}p_3|_{p_1p_2}$,(note:$p' = p|_{p_1p_2p_3}$)

$$(dist(p', p_3|_{p_1p_2}))^2 = D^2(p, p_1p_2p_3) + (D(p_3, p_1p_2) - D(p, p_1p_2))^2$$

In the $\triangle p_1pp_3$,

$$\because \quad (dist(p_1, p|_{p_1p_3}))^2 - (dist(p_3, p|p_1p_3))^2 = (dist(p, p_1))^2 - (dist(p, p_3))^2$$

In the $\triangle p_1p'p|_{p_1p_3}$ $\triangle p'p|_{p_1p_2}p_3$

$$\because pp' \perp p_1p_3 \ \cap \ pp|_{p_1p_3} \perp p_1p_3 \Rightarrow p'p|_{p_1p_3} \perp p_1p_3$$

$$\begin{aligned} \therefore \quad (dist(p', p_3))^2 &= (dist(p', p_1))^2 - (dist(p_1, p|_{p_1p_3}))^2 + (dist(p_3, p|_{p_1p_3}))^2 \\ &= (D^2(p, p_1p_2p_3) + D^2(p, p_1p_2)) - (dist(p, p_1))^2 + (dist(p, p_3))^2 \end{aligned}$$

From the Figure A.2 (B), apply the cosine law in $\triangle p'p_3p_3|_{p_1p_2}$. $p_3p_3|_{p_1p_2}$, $p'p_3|_{p_1p_2}$

$$\begin{aligned} D(p, p_1p_2p_3) &= \frac{(dist(p', p_3|_{p_1p_2}))^2 - (dist(p', p_3))^2 + (dist(p_3, p|_{p_1p_2}))^2}{2dist(p_3, p_3|_{p_1p_2})} \\ &= \frac{(dist(p,p_1))^2 - (dist(p,p_3))^2 + (dist(p_1,p_3))^2 - 2D(p_3,p_1p_2) \cdot D(p,p_1p_2)}{2\sqrt{(dist(p_1,p_3))^2 - (D(p_3,p_1p_2))^2}} \end{aligned}$$

Figure A.1: HyperMap: Expressions for Computing Relative Coordinate

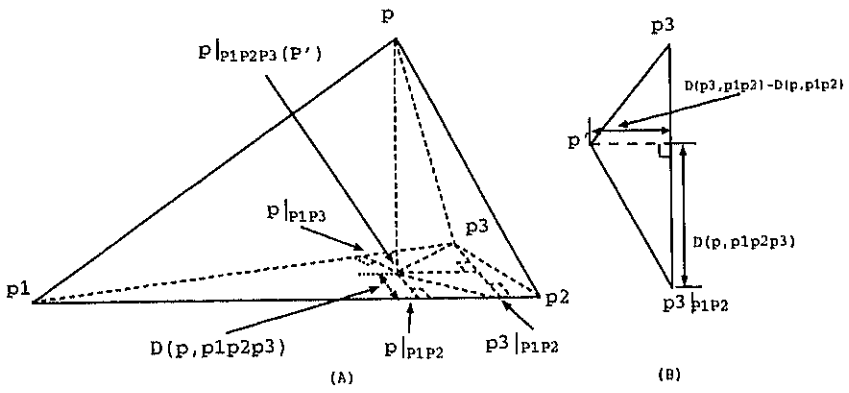

Figure A.2: Relative Coordinate Computing in 2-hyperaxis (Note p' denotes $p|_{p_1p_2p_3}$)

In the $\triangle p_1 p_{k+1} p_{k+1}|_{p_1 p_2 \ldots p_k}$, the following equation holds(Lemma 1):

$$(dist(p_{k+1}, p_{k+1}|_{p_1 p_2 \ldots p_k}))^2 = (dist(p_1, p_{k+1}))^2 - \sum_{j=2}^{k}(D(p_{k+1}, p_1 p_2 \ldots p_j))^2$$

In the $\triangle p' p|_{p_1 p_2 \ldots p_k} p_{k+1}|_{p_1 p_2 \ldots p_k}$,

$$(dist(p', p_{k+1}|_{p_1 p_2 \ldots p_k}))^2 = D^2(p, p_1 p_2 \ldots p_k p_{k+1}) + \sum_{j=2}^{k}(D(p_{k+1}, p_1 p_2 \ldots p_k) - D(p, p_1 p_2 \ldots p_k))^2$$

In the $\triangle p_1 p p_{k+1}$,

$$\because \quad (dist(p_1, p|_{p_1 p_{k+1}}))^2 - (dist(p_{k+1}, p|p_1 p_{k+1}))^2 = (dist(p, p_1))^2 - (dist(p, p_{k+1}))^2$$

In $\triangle p p' p|_{p_1 p_{k+1}}, \triangle p' p|_{p_1 p_{k+1}} p_{k+1}$

$$pp' \perp p_1 p_{k+1} \cap pp|_{p_1 p_{k+1}} \perp p_1 p_{k+1} \Rightarrow p' p|_{p_1 p_{k+1}} \perp p_1 p_{k+1}$$

$$\therefore \quad (dist(p', p_{k+1}))^2 = (dist(p', p_1))^2 - (dist(p_1, p|_{p_1 p_{k+1}}))^2 + (dist(p_{k+1}, p|_{p_1 p_{k+1}}))^2$$
$$= (\sum_{j=2}^{k+1} D^2(p, p_1 p_2 \ldots p_j) - (dist(p, p_1))^2 + (dist(p, p_{k+1}))^2$$

From the Figure A.4 (B),
apply the cosine law in $\triangle p' p_{k+1} p_{k+1}|_{p_1 p_2 \ldots p_k}$.

$$D(p, p_1 p_2 \ldots p_{k+1}) = \frac{(dist(p', p_{k+1}|_{p_1 p_2 \ldots p_k}))^2 - (dist(p', p_{k+1}))^2 + (dist(p_{k+1}, p|_{p_1 p_2 \ldots p_k}))^2}{2dist(p_{k+1}, p_{k+1}|_{p_1 p_2 \ldots p_k})}$$
$$= \frac{(dist(p, p_1))^2 - (dist(p, p_{k+1}))^2 + (dist(p_1, p_{k+1}))^2 - 2\sum_{i=2}^{k} D(p_{k+1}, p_1 p_2 \ldots p_k) \cdot D(p, p_1 p_2 \ldots p_k)}{2\sqrt{(dist(p_1, p_{k+1}))^2 - \sum_{j=2}^{k} D(p_{k+1}, p_1 p_2 \ldots p_j)^2}}$$

Figure A.3: HyperMap: Expressions for Computing Relative Coordinate

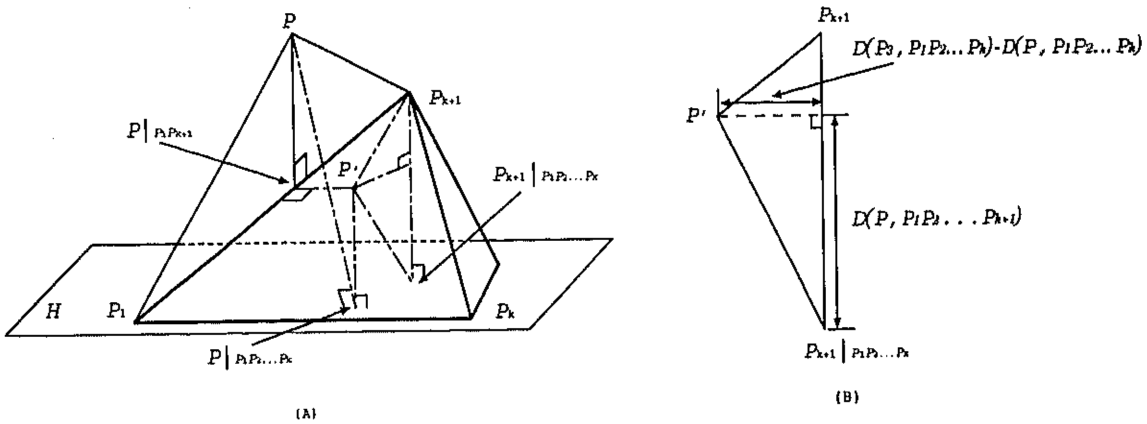

Figure A.4: Relative Coordinate Computing in a k-hyperaxis.