Abstract

This paper is strongly based on two powerful general theorems proved by Ikebe, et.al in 1993[15] and 1996[13], which will be referred to as Theorem A and Theorem B in this paper. They were recently published and justify the approximate computations of simple eigenvalues of infinite matrices of certain types by truncation, giving an extremely accurate error estimates. So far, they have applied to some important problems in engineering, such as computing the zeros of some special functions, and the eigenvalues of some differential equations. Such applications include (A)the computation of the zeros of $J_\nu(z)$ (where $J_\nu(z)$ denotes the Bessel function of the first kind of order ν, (B)the computation of the zeros of $zJ'_\nu(z) + HJ_\nu(z)$, and (C)the (ordinary) EVP (EigenValue Problem) of the Mathieu differential equation.

There are two main objectives in this paper. The first one is to expand the range of applications which either Theorem A or Theorem B may apply. It is shown in this article that for three more such problems, one is enabled to give methods for approximate solutions. They are enumerated as ⓐthe computation of the zeros of Coulomb wave function $F_L(\eta, \rho)$ and its first derivative, ⓑthe inverse EVP of the Mathieu differential equation, and ⓒthe ordinary and inverse EVP of the spheroidal wave equation.

With the consideration that the above stated 4 problems (A),(B),(C)(or ⓑ) and ⓒ (note that problem ⓐ is excluded) have something in common in their reformulated forms as infinite matrix eigenvalue problems, the generalization of such eigenvalue problems is attempted. Thus obtained generalized eigenvalue problems may be applied by Theorem B, and also are easily transformed into another type of eigenvalue problems, which is indeed subsumed into the class Theorem A may apply. In fact, this transformed problem can be regarded as the inverse problem to the first one.

Another objective is to contribute to the reinforcement or enhancement of Theorem A or Theorem B. One thing is to give a method for the computation of double eigenvalues of matrices which neither Theorem A nor Theorem B may apply. It will be proved that a lemma and a theorem exist concerning double eigenvalues of matrices only for the above generalized eigenvalue problems. They play important roles in the computation of double eigenvalues (or in the theoretical proof in case there exist no such double eigenvalues). The algorithm for the computation of such values is also presented. The other item likely to be counted as an enhancement for the theorems will be the simplification of the error estimate given in Theorem A. An explicit and closed error estimate is realized for ⓐ. This method might apply to the more generalized problems in the future.