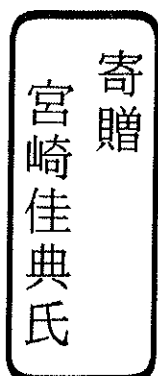


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Computations of Zeros of Special Functions  
and Eigenvalues of Differential Equations  
by Matrix Method

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