APPENDIX 2

MATHEMATICS DEMONSTRATION FOR THE EQUAL WAVELENGTHS AT THE INTERFACES I AND II

For a vapor blanket shown in fig. A-1.

At two gas-liquid interfaces, waves 1 and 2 exist. If c is wave velocity, λ is wavelength, at any time t, waves 1 and 2 can be expressed as:

$$\eta_1 = \eta_{01} \sin k_1 (x - c_1 t); \quad \eta_2 = D_B + \eta_{02} \sin k_2 (x - c_2 t)$$

where k_1, k_2 are wave numbers and are written as:

$k_1 = 2\pi / \lambda_1, \quad k_2 = 2\pi / \lambda_2.$

For η (wave height) is much smaller than wavelength λ, it can be assumed that:

$$k_{(1,2)}\eta_{(1,2)} = 2\pi\eta_{(1,2)} / \lambda_{(1,2)} \approx 0$$

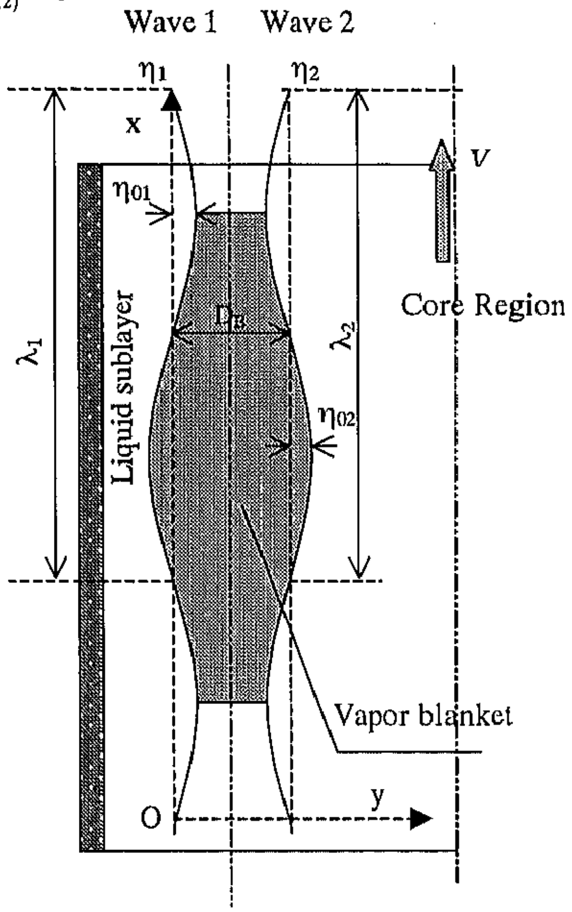

Fig.A-1 Schematic Representation of a Vapor Blanket

To search the velocity potential ϕ in the vapor blanket, suppose the velocity on x, y directions are u and v respectively

$$u = -\frac{\partial \phi}{\partial x};\quad v = -\frac{\partial \phi}{\partial y}$$

With the conservation equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) = 0$,

The velocity potential ϕ is written as:

$$\phi = -u_B x + (A\cosh ky + B\sinh ky)\cos k(x - ct)$$

(1) Search velocity potential in the vapor blanket from the wave 1

$$\phi_{g1} = -u_B x + (A_1 \cosh k_1 y + B_1 \sinh k_1 y)\cos k_1(x - c_1 t) \qquad \text{(A-1)}$$

With boundary conditions:

$$\begin{cases} y = D_B + \eta_2, v_g = -\left(\frac{\partial \phi_{g1}}{\partial y}\right)_{y=D_B+\eta_2} = \frac{d\eta_2}{dt} = \frac{\partial \eta_2}{\partial t} + u_B \frac{\partial \eta_2}{\partial x}; \\ y = \eta_1, v_g = -\left(\frac{\partial \phi_{g1}}{\partial y}\right)_{y=\eta_1} = \frac{d\eta_1}{dt} = \frac{\partial \eta_1}{\partial t} + u_B \frac{\partial \eta_1}{\partial x} \end{cases}$$

A_1 and B_1 are obtained:

$$\begin{cases} A_1 = \dfrac{\dfrac{\cos k_2(x - c_2 t)}{\cos k_1(x - c_1 t)}(c_2 - u_B)k_2\eta_{02} - (c_1 - u_B)\eta_{01}k_1 \cosh k_1 D_B}{k_1 \sinh(k_1 D_B)}; \\ B_1 = (c_1 - u_B)\eta_{01} \end{cases}$$

(2) Search the velocity potential in the vapor blanket from the wave 2

$$\phi_{g2} = -u_B x + (A_2 \cosh k_2 y + B_2 \sinh k_2 y)\cos k_2(x - c_2 t) \qquad \text{(A-2)}$$

With boundary conditions:

$$\begin{cases} y = D_B + \eta_2, v_g = -\left(\frac{\partial \phi_{g2}}{\partial y}\right)_{y=D_B+\eta_2} = \frac{d\eta_2}{dt} = \frac{\partial \eta_2}{\partial t} + u_B \frac{\partial \eta_2}{\partial x} \\ y = \eta_1, v_g = -\left(\frac{\partial \phi_{g2}}{\partial y}\right)_{y=\eta_1} = \frac{d\eta_1}{dt} = \frac{\partial \eta_1}{\partial t} + u_B \frac{\partial \eta_1}{\partial x} \end{cases}$$

A_2, B_2 are got as:

$$\begin{cases} A_2 = \dfrac{(c_2 - u_B)\eta_{02} - \dfrac{(c_1 - u_B)\eta_{01}k_1 \cos k_1(x - c_1 t)}{k_2 \cos k_2(x - c_2 t)}\cosh k_2 D_B}{\sinh k_2 D_B} \\ B_2 = \dfrac{(c_1 - u_B)\eta_{01}k_1 \cos k_1(x - c_1 t)}{k_2 \cos k_2(x - c_2 t)} \end{cases}$$

Rearranging the eqs.A-1 and A-2 and by assuming

$$\begin{cases} a = k_2\eta_{02}(c_2 - u_B)\cos(k_2(x - c_2 t)) \\ b = k_1\eta_{01}(c_1 - u_B)\cos(k_1(x - c_1 t)) \end{cases}$$

ϕ_{g1} and ϕ_{g2} are got as:

$$\begin{cases} \phi_{g1} = a\dfrac{\cosh k_1 y}{k_1 \sinh k_1 D_B} + b\left[\dfrac{\sinh k_1 y}{k_1} - \dfrac{\cosh k_1 y}{k_1 tghk_1 D_B}\right] \\ \phi_{g2} = a\dfrac{\cosh k_2 y}{k_2 \sinh k_2 D_B} + b\left[\dfrac{\sinh k_2 y}{k_2} - \dfrac{\cosh k_2 y}{k_2 tghk_2 D_B}\right] \end{cases} \qquad \text{(A-3)}$$

Because ϕ_{g1} and ϕ_{g2} are both the velocity potential in the vapor blanket, $\phi_{g1} = \phi_{g2}$

(A-4)

The solution of eq. A-4 is:
$k_1 = k_2$ or $k_1 = -k_2$
For $k_1 = 2\pi/\lambda_1$, $k_2 = 2\pi/\lambda_2$,
$\lambda_1 = \lambda_2$ is demonstrated.

For the two waves at the interfaces I and II, wave phases are determined by $k_1 c_1 t$ and $k_2 c_2 t$ respectively.
$k_1 = k_2$ means the two waves are of same phase at initial (t=0) condition.
$k_1 = -k_2$ means the two waves are of opposite phase at initial (t=0) condition.
In the vapor developing process, there exist a series value of time t, at which the two waves come to opposite phases.

If $k_1 = k_2$, the time t is calculated as: $t = \dfrac{(2n+1)\pi}{k|c_2 - c_1|}$ (n=0, 1, 2, 3 ...)

If $k_1 = -k_2$, the time t is calculated as: $t = \dfrac{2n\pi}{k|c_2 - c_1|}$ (n=0, 1, 2, 3 ...)

A stable vapor blanket is therefore assumed containing only one complete wavelength, that is to say, $L_B = \lambda$ (the length of vapor blanket equals to the wavelength). Otherwise, if a vapor blanket contains more than one wavelength, the vapor blanket is considered to be unstable. It easily breaks down to form stable vapor blanket at the blanket thinnest points, which is reached when the two waves just come to the opposite phases.