Appendix B

Chernikova-Khalatnikov Theory

Fig. B.1 shows schematic drowing of the direction of propagation of waves in the case of a second sound wave incident onto a He II free surface as indicated by the *arrow* $E1$. There appear two reflected waves in the

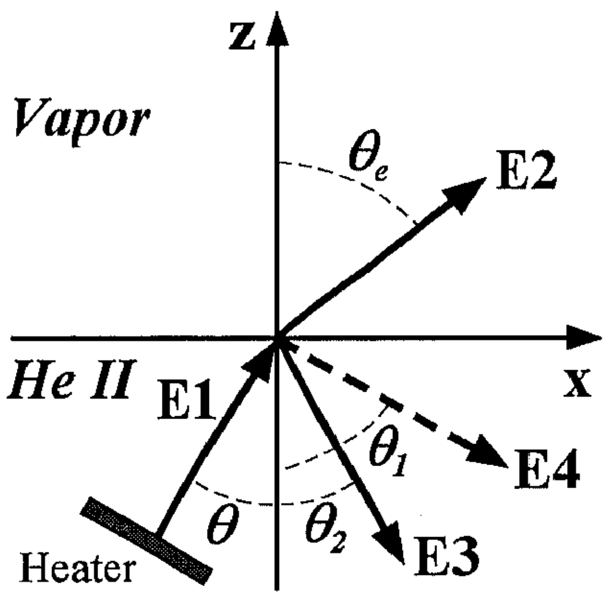

Fig. B.1: The schematic drowing of He II evaporation phenomena induced by a thermal pulse impingement onto a He II-vapor interfece with some angle θ. $E1$: the inpimging thermal pulse, $E2$: the evaporation wave, $E3$: the reflected thermal pulse, $E4$: the generated first sound wave. The xz is the plane of incidence of a thermal pulse and θ, θ_1, θ_2 and θ_e are the angles between the z-axis and the directions of propagation of the waves respectively.

liquid, a reflected second sound wave as indicated by the *arrow* $E3$ and a generated first sound wave by the *arrow* $E4$, and an evaporation wave in the vapor by the *arrow* $E2$. The plane x-y is taken as boundary plane. From the homogeneity of the problem in this plane all waves have the same wave-vectores K_x and K_y . From this there immediately follows a relation determining the direction of propagation of the waves which are excited. The xz is the plane of incidence of the thermal pulse and θ, θ_1, θ_2 and θ_e are the angles between the z-axis and the directions of propagation of the waves respectively. The impingimg second sound, the reflrcted second sound, the first sound and the evaporation wave are indicated by the subscripts i, r, $1st$ and e, respectively. All waves lie in the same plane.

$$\sin\theta_1 = \frac{a_{10}}{a_{20}}\sin\theta,\ \ \theta_2 = \theta,\ \ \sin\theta_e = \frac{a_0}{a_{20}} \tag{B.1}$$

where a_{10}, a_{20} and a_0 are the velocities of first and second sound waves and the velocity of sound wave in the vapor, respectively. The quantity $\zeta(x,t)$ characterizing the oscillations of the interface is given by

$$\zeta\left(x,t\right) = \Delta\zeta \exp\left[-i\omega t + ix\left(\frac{\omega}{a_{20}}\right)\sin\theta\right] \tag{B.2}$$

where ω is the frequency. The amplitude of the frequency is indicated by $\Delta\zeta$. The conditions which must be satisfied on a He II free surface are presented as follows;

(a) The force equation on the boundary

$$\Delta P_{1st} - \Delta P_e = \gamma\Delta\zeta\frac{\omega^2}{a_{20}^2}\sin^2\theta, \tag{B.3}$$

where γ and ΔP indicate the coefficient of surface tension and the pressure amplitude.

(b) The equality of the matter current densities on both sides of the boundary

$$-\frac{\cos\theta_1}{u_{10}}\Delta P_{1st} + i\omega\rho_0\Delta\zeta = \frac{\cos\theta_e}{a_0}\Delta P_e + i\omega\rho_{0,v}\Delta\zeta. \tag{B.4}$$

where the subscript v and the no subscript indicate the physical value in the vapor phase and that in the liquid phase, respectively.

(c) The equality of the energy-flux densities

$$W\left(-\frac{\cos\theta_1}{a_{10}}\Delta P_{1st}+i\omega\rho_0\Delta\zeta\right)+\rho_0\frac{\rho s}{\rho n}\frac{s^2T_0}{a_{20}}\cos\theta\left(\Delta T_i-\Delta T_r\right)$$
$$=W_e\left(\frac{\cos\theta_e}{a_0}\Delta P_e+i\omega\rho_{0,v}\Delta\zeta\right), \quad \text{(B.5)}$$

where W, s and ΔT are the heat function, the entropy and the temperature amplitude of waves, respectively.

(d) The equality of temperatures

$$\Delta T_i+\Delta T_r=\frac{T_0\alpha}{c_{P,v}}\Delta P_e+\Delta T_e, \quad \text{(B.6)}$$

where α and $c_{P,v}$ are the coefficient of thermal expansion of vapor and the specific heat of vapor in the constant pressure.

(e) The equality of chemical potentials

$$\frac{1}{\rho_0}\Delta P_{1st}-s\left(\Delta T_i+\Delta T_r\right)=\frac{1}{\rho_{0,v}}\Delta P_e-s_v\left(\frac{\alpha T_0}{c_{P,v}}\Delta P_e+\Delta T_e\right) \quad \text{(B.7)}$$

As the displacement of the interface is negligibly small as compared with the wave length, the surface tension term ($\gamma\omega/\rho_0 a_{10}a_{20}^2 << 1$) in Eq. (B.3) may be neglected. Furthermore if $s/s_v << 1$ and $\rho_{0,v}/\rho_0 << 1$, Eqs. (B.3) and (B.7) are simplified as follows;

$$\frac{\Delta P_e}{\Delta T_i}=\frac{\Delta P_{1st}}{\Delta T_i}=\frac{2\rho_0\frac{\rho_s}{\rho_n}\frac{s}{s_v}\frac{a_0}{a_{20}}\cos\theta}{\cos\theta_e+\frac{\rho_s}{\rho_n}\frac{\rho_0}{\rho_{0,v}}\left(\frac{s}{s_v}\right)^2\frac{a_0}{a_{20}}\cos\theta}, \quad \text{(B.8)}$$

$$\frac{\Delta T_r}{\Delta T_i}=\frac{-\cos\theta_e+\frac{\rho_s}{\rho_n}\frac{\rho_0}{\rho_{0,v}}\left(\frac{s}{s_v}\right)^2\frac{a_0}{a_{20}}\cos\theta}{\cos\theta_e+\frac{\rho_s}{\rho_n}\frac{\rho_0}{\rho_{0,v}}\left(\frac{s}{s_v}\right)^2\frac{a_0}{a_{20}}\cos\theta}, \quad \text{(B.9)}$$

where

$$\cos\theta_e=\sqrt{1-\frac{a_0^2}{a_{20}^2}\sin^2\theta}. \quad \text{(B.10)}$$

Eq. (B.9) indicates the reflection coefficient R_{22} of a second sound wave from a He II free surface.