Appendix A

Numerical Simulation of Vapor Flow Region

The vapor flow region is numerically calculated by solving the one-dimensional Navier-Stokes equation supplemented with a set of boundary conditions at the phase boundary derived from the kinetic theory of gases on the basis of the BGK model equation.

A.1 One-Dimensional Equations

The one-dimensional equations that are the equality of the mass current density, the equality of the momentum flux density and the equality of the energy flux density are written in a vector form as;

$$\frac{\partial U}{\partial t}+\frac{\partial E}{\partial x}=\frac{\partial R}{\partial x}, \tag{A.1}$$

where one has

$$U=\begin{bmatrix}\rho\\ m\\ e\end{bmatrix},\ E=\begin{bmatrix}m\\ \frac{m^2}{\rho}+P\\ \frac{m(e+P)}{\rho}\end{bmatrix},$$

$$R=\begin{bmatrix}0\\ \frac{4}{3}\frac{1}{Re}\frac{du}{dx}\\ \frac{1}{\gamma-1}\frac{1}{Pr\cdot Re}\frac{dT}{dx}+\frac{4}{3}\frac{1}{Re}u\frac{du}{dx}\end{bmatrix}, \tag{A.2}$$

where $\rho, m, e, u, P, T, \gamma$, Re and Pr are the density, the mass flux, the total energy, the velocity, the pressure, the temperature, the specific heat ratio, the Reynolds number and the Prandtl number of the vapor, respectively.

A.2 Boundary Condition

The slip boundary condition solved by Sone and Onishi[5] is applied to the boundary condition of this numerical simulation because the inside of the Knudsen layer can not be treated by the Naver-Stokes equation as described in Sec. 1.1.

A.3 Finite Difference Computation Scheme

In this study, the MacCormack finite difference scheme with the flux-corrected transport algorithm is adopted to solve the Navier-Stokes equation.

A.3.1 MacCormack Finite Difference Scheme

The scheme shown below has a second order accuracy in both time and space.

$$U_i^* = U_i^n - \frac{\Delta t}{\Delta x}\left\{(E_i^n - E_{i-1}^n) - (R_i^n - R_{i-1}^n)\right\} \tag{A.3}$$

$$U_i^{n+1} = \frac{1}{2}\left(U_i^n + U_i^*\right) - \frac{\Delta t}{2\Delta x}\left\{(E_{i+1}^* - E_i^*) - (R_{i+1}^* - R_i^*)\right\} \tag{A.4}$$

where subscript i, n and $*$ indicate i-th spatial step, n-th time step and predictor step, respectively. Eqs. (A.3) and (A.4) are the computations at the predictor and corrector steps, respectively. The MacCormack scheme may be considered to belong to a kind of two step Lax-Wendroff scheme family. In difference of two scheme, the advantage of the MacCormack scheme is that the calculation of the middle point values such as those at glid points $i + 1/2$ or $i - 1/2$ are not required.

A.3.2 Flux-corrected Transport Algorithm

The numerical computation of parabolic equations by a second order accuracy scheme in space becomes absolutely unstable and is necessarily accompanied by numerical oscillation. To avoid the problem, the method of artificial viscosity is generally used. But it often reduces the magnitude of physical discontinuity. In this study, the FCT (flux-corrected transport) algorithm is applied to the MacCormack scheme. It is of indeterminate order but yield realistic, accurate results.

The solution obtained at the corrector step by Eq. (A.4) is interpreted as U^{**} in what follows. The phoenical FCT algorithm consists of the following six steps:

i) Generate diffusive fluxes

$$F^d_{i+\frac{1}{2}} = \nu_{i+\frac{1}{2}} \left(U^n_{i+1} - U^n_i\right) \tag{A.5}$$

$$\nu_{i+\frac{1}{2}} = \eta_0 + \eta_1 \left(u_{i+\frac{1}{2}} \frac{\Delta t}{\Delta x}\right)^2 \tag{A.6}$$

ii) Generate antidiffusive fluxes

$$F^{ad}_{i+\frac{1}{2}} = \mu_{i+\frac{1}{2}} \left(U^{**}_{i+1} - U^{**}_i\right) \tag{A.7}$$

$$\mu_{i+\frac{1}{2}} = \eta_0 + \eta_1 \left(u_{i+\frac{1}{2}} \frac{\Delta t}{\Delta x}\right)^2 \tag{A.8}$$

iii) Diffuse the solution

$$U^{***}_i = U^{**}_i + F^d_{i+\frac{1}{2}} - F^d_{i-\frac{1}{2}} \tag{A.9}$$

iv) Calculate first difference of U_i^{***}

$$\Delta U^{***}_{i+\frac{1}{2}} = U^{**}_{i+1} - U^{**}_i \tag{A.10}$$

v) Limit the antidiffusive fluxes

$$S = sgnE^{ad}_{i+\frac{1}{2}} \tag{A.11}$$

$$F^{cad}_{i+\frac{1}{2}} = S \cdot \max\left[0, \min\left\{S \cdot \Delta U^{***}_{i-\frac{1}{2}}, \left|F^{ad}_{i+\frac{1}{2}}\right|, S \cdot \Delta U^{***}_{i+\frac{3}{2}}\right\}\right] \tag{A.12}$$

vi) Antidiffuse the solution

$$U_i^{n+1} = U_i^{***} - \left(F_{i+\frac{1}{2}}^{cad} - F_{i-\frac{1}{2}}^{cad} \right) \tag{A.13}$$

In the present computation one has $u_{i+\frac{1}{2}} = 0.5(u_i + u_{i-1})$ and, $\eta_0 = 1/6$, $\eta_1 = 1/3$, $\eta_2 = -1/6$.