## Notation

H	specimen's height, mm
$M_{\scriptscriptstyle T}$	applied torque, N·m
Nc	number of cyclic loading
Nl	number of loading set
f	frequency, Hz
p	mean normal stress, kPa
q	deviator stress, kPa
$\mathbf{r}_{i}$	specimen's inner radius, mm
$r_o$	specimen's outer radius, mm
u <sub>a</sub>	pore air pressure, kPa
$u_w$	pore water pressure, kPa
δ	cyclic torsional angle, rad
ф	angle of internal friction, deg
$\gamma_a$	double amplitude of torsional shear strain, —
$\gamma_{z\theta}$	torsional shear strain, —
η	effective stress ratio, —
ν	specific volume, m <sup>3</sup> /Mg
ρ	bulk density, Mg/m³
$\sigma_{l}$	major principal stress, kPa
$\sigma'_{i}$	effective major principal stress, kPa
$\sigma_2$	intermediate principal stress, kPa
$\sigma'_2$	effective intermediate principal stress, kPa

- σ<sub>3</sub> minor principal stress, kPa
- σ'<sub>3</sub> effective minor principal stress, kPa
- σ<sub>c</sub> confining stress, kPa
- σ'<sub>c</sub> effective confining stress, kPa
- σ<sub>h</sub> horizontal stress, kPa
- σ<sub>m</sub> mean normal stress, kPa
- σ'<sub>m</sub> effective mean normal stress, kPa
- σ'<sub>n</sub> effective normal stress, kPa
- σ<sub>oct</sub> octahedral normal stress, kPa
- σ<sub>v</sub> vertical stress, kPa
- τ shear stress, kPa
- τ<sub>A</sub> cyclic shear stress, kPa
- $\tau_{\rm f}$  shear stress at failure, kPa
- $\tau_{oot}$  octahedral shear stress, kPa
- τ, residual shear stress, kPa
- τ<sub>vh</sub> horizontal shear stress, kPa
- $\tau_{z\theta}$  cyclic torsional shear stress, kPa
- χ wet area per unit gross area,—

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