Chapter 7

Summary and Future problems

7.1 Summary

In this thesis, we propose the formulation of self-consistent Skyrme-Hartree-Fock (SHF) plus random phase approximation (RPA) calculation of low-lying states in the three dimensional Cartesian mesh representation for the even-even nuclei with reflection symmetry with respect to $x = 0$, $y = 0$ and $z = 0$ planes. Our formulation enables us to calculate the accurate excitation energies and the accurate intrinsic reduced transition probabilities of spherical, axial and triaxial nuclei. The formulation also enables us to calculate the RPA correlation energies of spurious states of the translation and the rotation, the collective mass of the translation and the moment of inertia of the rotation. We do not consider the pairing correlations in our formulation.

The SHF equation in coordinate representation and the RPA equation in mixed configuration space of the coordinate and hole orbitals are given. We make use of the properties of the equations under time-reversal.

We propose four numerical methods. First is the efficient solution method of the SHF equation in the 3D Cartesian mesh representation. Second is the solution method of the RPA equation in the 3D Cartesian mesh representation, by which we can numerically treat the spurious solutions of the RPA equation with pure imaginary eigenvalues. Third is a method for accurately calculating in the 3D Cartesian mesh representation. We take an average of numerical results of two mesh calculations, where one of the two meshes consists of even number of mesh points for one direction of cubic box and the other is odd number of mesh points. We show that the average values in terms of the Lagrange mesh method are very close to the converged values even if the coarse mesh size is used in the calculation. Fourth is the method of calculating the RPA correlation energy of spurious state and obtaining the collective mass for rotation as well as translation of the nuclei.

We compare the numerical results of our formulation with the other's in detail. We examine unknown instability induced by the $\boldsymbol{s} \cdot \Delta \boldsymbol{s}$ terms of Skyrme energy functional. We remove the $\boldsymbol{s} \cdot \Delta \boldsymbol{s}$ terms for several Skyrme energy functional in order to avoid the instability.

We apply the formulation to the superdeformed state of ^{40}Ca. The formulation with SkI4 interaction successfully reproduces the SD band of ^{40}Ca. According to the calculation, it is suggested that the 8p-8h state with triaxial superdeformation of ^{40}Ca tends to be soft with respect to β_{30} and $\beta_{3\pm1}$ octupole deformations for SIII, Z_σ and SkI4 interactions and unstable with respect to the β_{31} octupole deformation for SkX and SkO interactions. The isoscalar odd-parity low-lying levels obtained on top of the 8p-8h state predict the existence of the odd parity excited bands associated with the SD band.

7.2 Future problems

Skyrme interaction

At present, it is uncertain whether the Skyrme interactions are able to reproduce the low-lying excited states of deformed nuclei. In spherical nuclei, the low-lying states of the spherical nuclei in terms of the RPA with several Skyrme interactions were studied [19, 20, 22]. We should study low-lying states of deformed nuclei as well as spherical nuclei within the framework of SHF plus RPA.

Pairing correlation

The pairing correlation is not considered in our formulation. In order to describe the low-lying states of open shell nuclei, it is necessary to expand our formulation into the self-consistent Skyrme-Hartree-Fock-BCS plus quasiparticle RPA (QRPA) or the self-consistent Skyrme-Hartree-Fock-Bogoliubov plus QRPA. Such expanded formulation may also enable us to evaluate the RPA correlation energy of rotation. This is useful for evaluating total binding energy taking into account the zero-point rotational energy correction for deformed nuclei.

Method of obtaining the accurate results in the mesh representation

In subsection 4.3, we show that the method of obtaining the accurate results in the mesh representation. However, we do not explain why we obtain the accurate results. It should be examined in future.

$\boldsymbol{s} \cdot \triangle \boldsymbol{s}$ term

In subsection 4.6, we study an unknown instability which occurs in the RPA calculation, induced by $\boldsymbol{s} \cdot \triangle \boldsymbol{s}$ terms in Skyrme energy functional. One supposes that this effect is undesirable in the RPA calculation. However, this may enable us to fit the excited levels in terms of the parameter $C_t^{\triangle s}$, because the $\boldsymbol{s} \cdot \triangle \boldsymbol{s}$ terms do not contribute to the ground state properties of even-even nuclei (cf. [23]).

Exchange contribution of Coulomb interaction

In this thesis, we approximate the exchange contribution of Coulomb interaction as Slater approximation. This influences the proton single-particle energies [80]. Possibly, the influence of the approximation on the low-lying states cannot be ignored. This influence on the low-lying state should be examined in future.