Chapter 6

Application to superdeformed state for ^{40}Ca

The 0_3^+ state for ^{40}Ca is a bandhead of a superdeformed band and the configuration of the state is considered as 8-particle 8-hole (8p-8h) excitation [34]. For example, Zheng *et al.* predict SD band with a deformed SHF calculation with axial symmetry imposed on the density [38]. However, a shape of the 0_3^+ state becomes slightly triaxial, by the calculation without the spatial symmetry restriction in SHF [39], where γ-deformation is about 10°. In this chapter, we apply our formulation to the SD band of ^{40}Ca.

6.1 Description of SD band of ^{40}Ca with SHF plus RPA

In this section, we show the numerical results for the band structure of the superdeformed states of ^{40}Ca and our formulation can successfully reproduce the experimental values of SD band for ^{40}Ca.

Table 6.1 shows numerical results of the SHF plus RPA calculation of ground state and 8p-8h state for ^{40}Ca. We partition properties into four category in Table 6.1. First is the numerical results for the ground state obtained by means of the SHF calculation. $E_{\rm HF}^{\rm gr}$ is a binding energy of an intrinsic ground state. Second is the numerical results for the 8p-8h state obtained by means of the SHF calculation. $E_{\rm HF}^{\rm 8p8h}$ is a HF energy of an intrinsic 8p-8h state. The deformation parameter β and γ of the 8p-8h state are defined by Eq. (4.49). $\langle J_\nu^2 \rangle$ is an HF expectation value of a squared angular momentum operator with respect to the 8p-8h state. Third is the numerical results for the 8p-8h state in terms of the RPA calculation. $E_{\rm cor}^{{\rm rot},\nu}$ is a RPA correlation energy of a spurious rotation around the ν-axis and given in terms of Eq. (4.71) as

$$E_{\rm cor}^{{\rm rot},\nu} = \frac{1}{4}|\hbar\omega_\lambda| \sum_{ix} \left|\phi_i^{(+)\lambda}(x)\right|^2. \tag{6.1}$$

Fourth is the numerical results in terms of above quantities. $E_{\rm RPA}^{\rm 8p8h}$ is an energy of the 8p-8h state including the RPA correlation energy for rotation, which is defined as

$$E_{\rm RPA}^{\rm 8p8h} = E_{\rm HF}^{\rm 8p8h} - \sum_{\nu=x,y,z} E_{\rm cor}^{{\rm rot},\nu}. \tag{6.2}$$

$E_x(0_3^+)$ is excitation energy of 0_3^+ state defined as

$$E_x(0_3^+) = E_{\rm RPA}^{\rm 8p8h} - E_{\rm HF}^{\rm gr}. \tag{6.3}$$

$\mathcal{J}_\nu$ is the moment of inertia corresponding to a rotation around the i-axis, which correspond to that of Thouless-Valatin's. The moment of inertia is estimated in terms of Eq. (4.75) as

$$\mathcal{J}_\nu = \frac{1}{2}\frac{\langle {\rm HF}_{\rm 8p8h}|\hat{J}_\nu^2|{\rm HF}_{\rm 8p8h}\rangle}{E_{\rm cor}^{{\rm rot},\nu}}. \tag{6.4}$$

fThe shape of the 8p-8h state for each of the several Skyrme parameters is not axial but weakly triaxial superdeformation in our calculation. Our formulation enables us to evaluate the excitation energy of the 0_3^+ state taking into account the zero-point rotational correlation energy and the moment of inertia corresponding to the Thouless-Valatin formula. It is to be emphasized that the zero-point rotational correlation energy around z-axis, $E_{\rm cor}^{{\rm rot},z}$, is not small in comparison with those around x and y-axis. Thus, excitation energy of 0_3^+ with $\gamma \sim -10°$ is lower than the ones with $\gamma = 0°$ by the magnitude of $E_{\rm cor}^{{\rm rot},z}$.

In the case of the triaxial deformation, there exist three moments of inertia around each of the axes, $\mathcal{J}_\nu$. And in terms of $A_\nu \equiv \hbar^2/2\mathcal{J}_\nu$, a rotational spectrum with angular momentum I is given as [79]

$$E_{\rm rot}(I) = \frac{1}{2}(A_1 + A_3)I(I+1) + \frac{1}{2}(A_1 - A_3)E_{\tau I}(\kappa), \tag{6.5}$$

where $E_{\tau I}(\kappa)$ is an eigenvalue of a matrix defined as

$$H(\kappa) = I_1^2 + \kappa I_2^2 - I_3^2, \qquad \kappa = \frac{2A_2 - A_1 - A_3}{A_1 - A_3}. \tag{6.6}$$

In Fig. 6.1, we show SD band for ^{40}Ca with several Skyrme interactions. This figure is constructed using the excitation energy of 0_3^+ state and three moment of inertia $\mathcal{J}_\nu$ in Table. 6.1. One can confirm that the experimental value is well reproduced with the SkI4 interaction. These results are amazing because the levels are obtained by fully microscopic calculation.

Table 6.1: The numerical results of the ground states and the 8p-8h states in ^{40}Ca. See the text about quantities in the first column.

	SIII	Z_σ	SkX	SkI4	SkO
$E_{\rm HF}^{\rm gr}$	-341.32	-342.34	-340.93	-343.95	-344.29
$E_{\rm HF}^{\rm 8p8h}$	-330.51	-332.26	-328.77	-331.86	-331.84
β	0.599	0.603	0.599	0.607	0.600
γ	-7.16	-9.09	-8.69	-9.68	-8.98
$\langle J_x^2 \rangle$	36.27	37.30	37.48	37.87	37.20
$\langle J_y^2 \rangle$	27.22	26.00	26.61	25.78	26.21
$\langle J_z^2 \rangle$	2.44	4.01	3.63	4.65	3.89
$E_{\rm cor}^{{\rm rot},x}$	1.79	1.57	1.57	2.21	1.97
$E_{\rm cor}^{{\rm rot},y}$	1.40	1.17	1.17	1.56	1.40
$E_{\rm cor}^{{\rm rot},z}$	1.27	0.85	1.01	1.96	1.51
$E_{\rm RPA}^{\rm 8p8h}$	-334.98	-335.86	-332.53	-337.60	-336.71
$E_x(0_3^+)$	6.34	6.49	8.41	6.36	7.58
$\mathcal{J}_x$	10.13	11.85	11.87	8.55	9.46
$\mathcal{J}_y$	9.69	11.08	11.39	8.24	9.39
$\mathcal{J}_z$	0.96	2.36	1.81	1.19	1.29

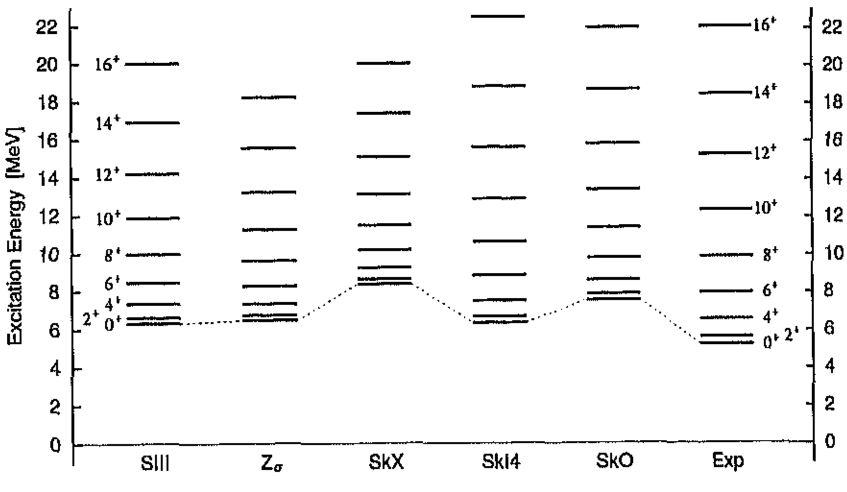

Figure 6.1: SD band for ^{40}Ca. See Fig. 1.1 about the experimental values (EXP).

6.2 Octupole softness

In this section, we study octupole softness for SD band of ^{40}Ca.

We calculate two cases: One is the case of the axial superdeformed nucleus ($\beta = 0.6$, $\gamma = 0°$). The other is the case of the triaxial superdeformed nucleus ($\beta = 0.6$, $\gamma \sim -10°$). The single-particle wave functions of the triaxial superdeformation are obtained by using the randomly chosen twenty odd parity single-particle wave functions and twenty even parity single-particle wave functions as initial values of imaginary time step method for solving the SHF equation. The single-particle wave functions of axial superdeformation are obtained by using the single-particle wave functions with axial density as initial values of imaginary time step method[1]. The HF energy of 8p-8h state with triaxial superdeformation is slightly lower than the one with axial superdeformation (e.g. 0.144 MeV for SIII force). We solve the RPA equations on top of the 8p-8h states with axial superdeformation and triaxial superdeformation respectively[2].

In Fig. 6.2, we show the excitation energies of low-lying isoscalar odd-parity states associated with the bandhead of the SD band of ^{40}Ca. In the case of the axial superdeformation in Fig. 6.2, the intrinsic electric octupole transition probabilities are large for the $K^\pi = 1^-$ state and $K^\pi = 0^-$ state for all of the displayed interactions. These results directly correspond to the octupole softness pointed out in Ref. [39], where the 8p-8h state with axial superdeformation is extremely soft with respect to the β_{30} and β_{31} octupole deformation. In Fig 6.2, the levels of $K^\pi = 1^-$ and 3^- states on axially superdeformed state split into two levels of $(s_x^\lambda, s_y^\lambda, s_z^\lambda) = (-,+,+)$ and $(+,-,+)$ states on triaxially superdeformed state respectively. The levels of $K^\pi = 2^-$ state on axially superdeformed state split into two levels of $(+,+,-)$ and $(-,-,-)$ states on triaxially superdeformed state. The level of $K^\pi = 0^-$ state on axially superdeformed state corresponds to the level of $(-,-,-)$ state on triaxially superdeformed state. The levels on the triaxially superdeformed state are labelled $(s_x^\lambda, s_y^\lambda, s_z^\lambda)_K$, where the state with $(s_x^\lambda, s_y^\lambda, s_z^\lambda)_K$ is the $(s_x^\lambda, s_y^\lambda, s_z^\lambda)$ state split from the K^π state on the axially superdeformed state. In Fig. 6.2, all of the levels on the triaxially superdeformed state other than the levels of $(+,-,+)_1$ state with SIII, SkX, SkO

[1]The imaginary time step method holds symmetry of the density made of initial single-particle wave functions.

[2]In the case of 8p-8h state with axial superdeformation, the excited state with quantum number $(s_x^\lambda, s_y^\lambda, s_z^\lambda) = (-,-,+)$ is unstable because the 8p-8h state with axial superdeformation is γ-unstable. We can calculate the RPA equations on the 8p-8h states with axial superdeformation for the other excited state provided those are stable with respect to the octupole deformation.

interactions are lower than corresponding levels on the axially superdeformed state. We can see that the levels of $(+,-,+)_1$ state with SIII, Z_σ and SkI4 has very low-lying excitation energy. Furthermore, the levels of $(+,-,+)_1$ state with SkX and SkO are lower than 0 MeV in the figure, where we display the pure imaginary eigenvalues of the RPA equations as minus energies. This means that the 8p-8h states with triaxial superdeformation for SkX and SkO interactions are unstable with respect to the octupole deformation. In the case of triaxial superdeformation in Fig. 6.2, the intrinsic electric octupole transition probabilities are large for the $(s_x^\lambda, s_y^\lambda, s_z^\lambda)_K = (+,-,+)_1$, $(-,+,+)_1$, $(+,+,-)_2$, $(+,+,-)_0$, and $(-,+,+)_3$ states for all of the interactions other than the $(+,-,+)_1$ states of SkX and SkO interactions and the $(+,-,+)_3$ state of Z_σ interaction. The main component of the reduced transition probability of $(+,+,-)_2$ state is not $\widetilde{Y}_{32}$ but $\widetilde{Y}_{30}$, e.g, 0.5 and 6.7 respectively for SIII interaction. Just as in the axial case, the 8p-8h state with triaxial superdeformation is also soft with respect to the β_{30} and $\beta_{3\pm1}$ octupole deformations for SIII, Z_σ and SkI4 interactions. The 8p-8h state with triaxial superdeformation is unstable with respect to the β_{31} octupole deformation for SkX and SkO interactions.

The obtained excited levels in Fig. 6.2 predict the existence of the odd parity excited bands associated with the SD band.

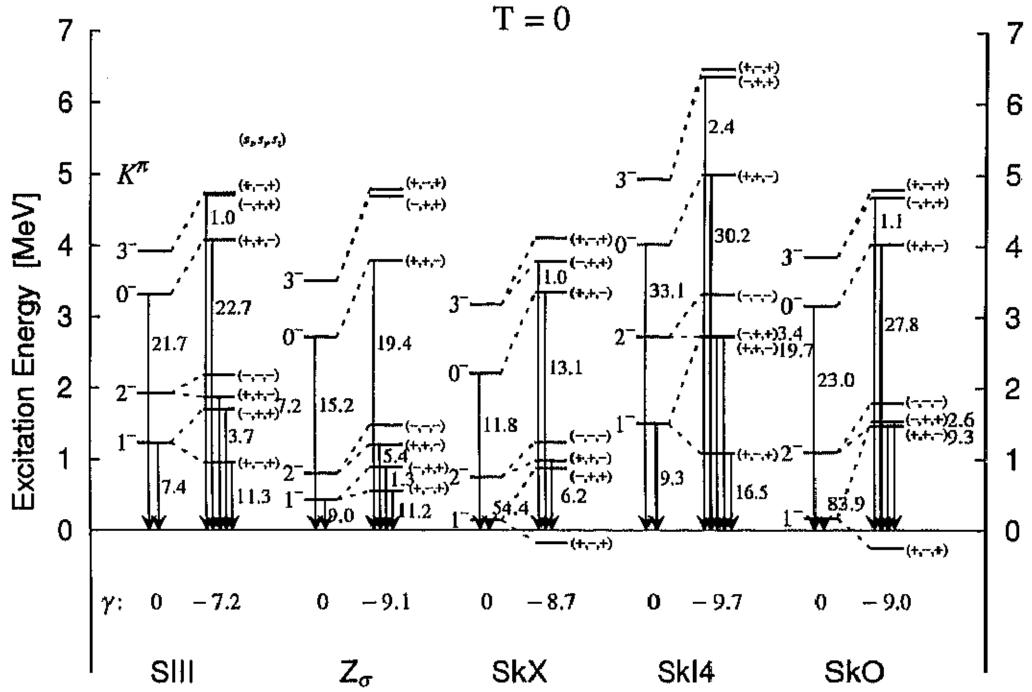

Figure 6.2: Excitation energies of low-lying isoscalar odd-parity states associated with the bandhead of the SD band of ^{40}Ca. Arrow is intrinsic reduced transition probability B(E3) which is more than 1.0 W.u. The excitation energy is calculated in two cases, $\beta = 0.6$, $\gamma = 0°$ and $\beta = 0.6$, $\gamma \sim -10°$ for each of the Skyrme interaction. In prolate case ($\gamma = 0$), levels are labeled by K^π. In triaxial case ($\gamma \neq 0$), levels are labeled by quantum numbers $(s_x^\lambda, s_y^\lambda, s_z^\lambda)$ in Eq. (4.68).