Chapter 3

Skyrme Hartree-Fock plus random phase approximation approach

Skyrme interaction was introduced by Skyrme [3, 4]. This interaction was applied to Hartree-Fock calculation by Vautherin and Brink [5] and Beiner *et al.* showed that Skyrme-Hartree-Fock method reproduce the binding energy and density distribution of spherical nuclei [8]. The Skyrme interaction, which has been widely used in Hartree-Fock calculation at present time, consist of the zero-range momentum-dependent two-body force plus the density dependent two-body force:

$$\begin{aligned} v_{\mathrm{Sk}}(\boldsymbol{r}) = {} & t_0(1+x_0P_\sigma)\delta(\boldsymbol{r}) + \tfrac{1}{2}t_1(1+x_1P_\sigma)[\delta(\boldsymbol{r})k^2 + k'^2\delta(\boldsymbol{r})] \\ & + t_2(1+x_2P_\sigma)\boldsymbol{k}'\cdot\delta(\boldsymbol{r})\boldsymbol{k} + iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}'\times\delta(\boldsymbol{r})\boldsymbol{k}, \\ & + \tfrac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha(\boldsymbol{R})\delta(\boldsymbol{r}), \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} \boldsymbol{r} &= \boldsymbol{r}_1 - \boldsymbol{r}_2 & \boldsymbol{R} &= \frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2} \\ \boldsymbol{k} &= \frac{1}{2i}(\overrightarrow{\nabla}_1 - \overrightarrow{\nabla}_2) & \boldsymbol{k}' &= -\frac{1}{2i}(\overleftarrow{\nabla}_1 - \overleftarrow{\nabla}_2). \end{aligned}$$

In the Skyrme Hartree-Fock approach, the total binding energy E is given by [1]

$$E = E_{\mathrm{kin}} + \int \mathrm{d}\boldsymbol{r}\,\mathcal{E}_{\mathrm{Sk}} + E_{\mathrm{Coul}} + E_{\mathrm{pair}} - E_{\mathrm{corr}}, \tag{3.2}$$

where E_{kin} is the kinetic energy, $\mathcal{E}_{\mathrm{Sk}}$ the Skyrme energy functional, E_{Coul} the Coulomb energy, E_{pair} the pair energy and E_{corr} corrections for spurious motion. The kinetic energy is

$$E_{\text{kin}} = \int d\boldsymbol{r}\, \mathcal{E}_{\text{kin}} = \frac{\hbar^2}{2m} \int d\boldsymbol{r}\, \tau, \tag{3.3}$$

where $\mathcal{E}_{\text{kin}}$ is the kinetic energy functional and $\tau \equiv \tau_{00}$ is the total kinetic density in (3.17). The Skyrme energy functional is explained in later section.

The Coulomb energy E_{Coul} is divided into the direct part and exchange part:

$$E_{\text{Coul}} = E_{\text{Coul,dir}} + E_{\text{Coul,ex}} \tag{3.4}$$

The direct part of the Coulomb energy is

$$\begin{aligned} E_{\text{Coul,dir}} &= \int d\boldsymbol{r}\, \mathcal{E}_{\text{Coul,dir}}(\boldsymbol{r}) \\ &= \frac{e}{2} \int d\boldsymbol{r}\, \rho_{\text{p}}(\boldsymbol{r}) \Phi_{\text{C}}(\boldsymbol{r}) \end{aligned} \tag{3.5}$$

with the Coulomb potential

$$\Phi_{\text{C}}(\boldsymbol{r}) = e \int d\boldsymbol{r}' \frac{\rho_{\text{p}}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}, \tag{3.6}$$

where $\mathcal{E}_{\text{Coul,dir}}$ is the direct part of the Coulomb energy functional and ρ_{p} is the proton density. The exchange part of the Coulomb energy is usually treated in the Slater approximation:

$$\begin{aligned} E_{\text{Coul,ex}} &= \int d\boldsymbol{r}\, \mathcal{E}_{\text{Coul,ex}}(\boldsymbol{r}) \\ &= -\frac{3e^2}{4} \left(\frac{3}{\pi}\right)^{1/3} \int d\boldsymbol{r}\, \rho_{\text{p}}^{4/3}(\boldsymbol{r}), \end{aligned} \tag{3.7}$$

where $\mathcal{E}_{\text{Coul,ex}}$ is the exchange part of the Coulomb energy functional. In this thesis, the pairing correlation is ignored.

In the Skyrme Hartree-Fock plus random phase approximation approach, the correction for spurious motion is treated as the RPA correlation energy of spurious states in (2.22) and done after variation. Then, it is convenient to write down the total binding energy of Eq. (3.3) as

$$E = E_{\text{int}} - E_{\text{corr}} = \int d\boldsymbol{r}\, \mathcal{E} - E_{\text{corr}}. \tag{3.8}$$

with the energy functional $\mathcal{E}$ is defined by

$$\mathcal{E} = \mathcal{E}_{\text{kin}} + \mathcal{E}_{\text{Sk}} + \mathcal{E}_{\text{Coul}} \tag{3.9}$$

where $\mathcal{E}_{\mathrm{Coul}} = \mathcal{E}_{\mathrm{Coul,dir}} + \mathcal{E}_{\mathrm{Coul,ex}}$. The single-particle hamiltonian in the Hartree-Fock equation and the transition hamiltonian in the RPA equation are derived from the first and the second derivatives of the total energy functional $\mathcal{E}$ with respect to the local densities in Eq. (3.17). Because both the single-particle hamiltonian and the transition hamiltonian are local,the three-dimensional Cartesian mesh calculation of the HF and RPA equations is very suitable. The treatments of the corrections of spurious motion are explained in subsection 4.1.1.

In this chapter, we explain Hartree-Fock plus RPA approach with the Skyrme effective interaction, where pairing correlation is neglected. In section 3.1, the most general single-particle density matrix is explained. In section 3.2, we derive the Hartree-Fock equation and the RPA equation with Skyrme interaction in the most general representation. In section 3.3, we assume that proton and neutron are not mixed in the single-particle wave function for the ground state. Based on the assumption, the Hartree-Fock equation and the RPA equation with Skyrme interaction are given. The RPA equation is divided into the ones without and with the charge-exchange excitation. In section 3.4, we concentrate on a problem for even-even nuclei and no-charge-exchange excitation. In such case, we can make use of the time-reversal properties of the local densities. Then, both the Hartree-Fock equation and the RPA equation with Skyrme interaction take simple forms. The Hartree-Fock equation and the RPA equation in this section is used in the actual numerical calculation presented in chapters 4, 5 and 6.

3.1 Most general single-particle density matrix

In general, the single-particle density matrix $\rho(x, x', t)$ is divided into scalar parts $\rho_{tt_3}(\boldsymbol{r}, \boldsymbol{r}', t)$ and vector parts $\boldsymbol{s}_{tt_3}(\boldsymbol{r}, \boldsymbol{r}', t)$ (cf. [55, 23]):

$$\begin{aligned}\rho(\boldsymbol{r}\sigma\tau, \boldsymbol{r}'\sigma'\tau', t) = \tfrac{1}{4}\Big[&\rho_{00}(\boldsymbol{r}, \boldsymbol{r}', t)\delta_{\sigma\sigma'}\delta_{\tau\tau'} + \boldsymbol{s}_{00}(\boldsymbol{r}, \boldsymbol{r}', t)\cdot\boldsymbol{\sigma}_{\sigma\sigma'}\delta_{\tau\tau'} \\ &+ \delta_{\sigma\sigma'}\sum_{t_3=-1}^{+1}\rho_{1t_3}(\boldsymbol{r}, \boldsymbol{r}', t)\tau^{t_3}_{\tau\tau'} + \sum_{t_3=-1}^{+1}\boldsymbol{s}_{1t_3}(\boldsymbol{r}, \boldsymbol{r}', t)\cdot\boldsymbol{\sigma}_{\sigma\sigma'}\tau^{t_3}_{\tau\tau'}\Big],\end{aligned} \tag{3.10}$$

where $\boldsymbol{\sigma}_{\sigma\sigma'} = \langle\sigma|\boldsymbol{\sigma}|\sigma'\rangle$ and $\tau^{t_3}_{\tau\tau'} = \langle\tau|\tau^{t_3}|\tau'\rangle$ are Pauli matricis of spin and isospin space respectively:

$$\sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \quad \sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{3.11a}$$

$$\tau^{+1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \tau^{-1} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \tau^{0} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3.11b}$$

The scalar parts $\rho_{tt_3}(\boldsymbol{r},\boldsymbol{r}',t)$ and the vector parts $\boldsymbol{s}_{tt_3}(\boldsymbol{r},\boldsymbol{r}',t)$ are written as

$$\rho_{00}(\boldsymbol{r},\boldsymbol{r}',t) = \sum_{\sigma\sigma'\tau\tau'} \rho(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau',t)\delta_{\sigma'\sigma}\delta_{\tau'\tau}, \tag{3.12a}$$

$$\rho_{1t_3}(\boldsymbol{r},\boldsymbol{r}',t) = \sum_{\sigma\sigma'\tau\tau'} \rho(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau',t)\delta_{\sigma'\sigma}\tau^{t_3}_{\tau'\tau}, \tag{3.12b}$$

$$\boldsymbol{s}_{00}(\boldsymbol{r},\boldsymbol{r}',t) = \sum_{\sigma\sigma'\tau\tau'} \rho(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau',t)\boldsymbol{\sigma}_{\sigma'\sigma}\delta_{\tau'\tau}, \tag{3.12c}$$

$$\boldsymbol{s}_{1t_3}(\boldsymbol{r},\boldsymbol{r}',t) = \sum_{\sigma\sigma'\tau\tau'} \rho(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau',t)\boldsymbol{\sigma}_{\sigma'\sigma}\tau^{t_3}_{\tau'\tau}. \tag{3.12d}$$

Putting Eq. (2.60a) and Eqs. (2.61) into Eq. (3.12), the scalar parts and the vector parts of the single-particle density matrix $\rho^{(0)}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')$, which is defined as Eq. (2.60b), are written as

$$\rho^{(0)}_{00}(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma\sigma'\tau\tau'} \rho^{(0)}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')\delta_{\sigma'\sigma}\delta_{\tau'\tau}, \tag{3.13a}$$

$$\rho^{(0)}_{1t_3}(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma\sigma'\tau\tau'} \rho^{(0)}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')\delta_{\sigma'\sigma}\tau^{t_3}_{\tau'\tau}, \tag{3.13b}$$

$$\boldsymbol{s}^{(0)}_{00}(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma\sigma'\tau\tau'} \rho^{(0)}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')\boldsymbol{\sigma}_{\sigma'\sigma}\delta_{\tau'\tau}, \tag{3.13c}$$

$$\boldsymbol{s}^{(0)}_{1t_3}(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma\sigma'\tau\tau'} \rho^{(0)}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')\boldsymbol{\sigma}_{\sigma'\sigma}\tau^{t_3}_{\tau'\tau}, \tag{3.13d}$$

and those of general transition density matrix $\rho^{\lambda}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')$, which are defined as Eqs. (2.62a), are written as

$$\rho^{\lambda}_{00}(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma\sigma'\tau\tau'} \rho^{\lambda}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')\delta_{\sigma'\sigma}\delta_{\tau'\tau}, \tag{3.14a}$$

$$\rho^{\lambda}_{1t_3}(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma\sigma'\tau\tau'} \rho^{\lambda}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')\delta_{\sigma'\sigma}\tau^{t_3}_{\tau'\tau}, \tag{3.14b}$$

$$\boldsymbol{s}^{\lambda}_{00}(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma\sigma'\tau\tau'} \rho^{\lambda}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')\boldsymbol{\sigma}_{\sigma'\sigma}\delta_{\tau'\tau}, \tag{3.14c}$$

$$\boldsymbol{s}^{\lambda}_{1t_3}(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma\sigma'\tau\tau'} \rho^{\lambda}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')\boldsymbol{\sigma}_{\sigma'\sigma}\tau^{t_3}_{\tau'\tau}. \tag{3.14d}$$

Similarly,

$$\rho_{00}^{(\pm)\lambda}(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma\sigma'\tau\tau'} \rho^{(\pm)\lambda}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')\delta_{\sigma'\sigma}\delta_{\tau'\tau}, \tag{3.15a}$$

$$\rho_{1t_3}^{(\pm)\lambda}(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma\sigma'\tau\tau'} \rho^{(\pm)\lambda}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')\delta_{\sigma'\sigma}\tau_{\tau'\tau}^{t_3}, \tag{3.15b}$$

$$\boldsymbol{s}_{00}^{(\pm)\lambda}(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma\sigma'\tau\tau'} \rho^{(\pm)\lambda}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')\boldsymbol{\sigma}_{\sigma'\sigma}\delta_{\tau'\tau}, \tag{3.15c}$$

$$\boldsymbol{s}_{1t_3}^{(\pm)\lambda}(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma\sigma'\tau\tau'} \rho^{(\pm)\lambda}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau')\boldsymbol{\sigma}_{\sigma'\sigma}\tau_{\tau'\tau}^{t_3}. \tag{3.15d}$$

Then, the scalar and vector part of general transition densities are divided into hermitian and anti-hermitian properties:

$$\rho_{tt_3}^{\lambda}(\boldsymbol{r},\boldsymbol{r}') = \rho_{tt_3}^{(+)\lambda}(\boldsymbol{r},\boldsymbol{r}') + \rho_{tt_3}^{(-)\lambda}(\boldsymbol{r},\boldsymbol{r}'), \tag{3.16a}$$

$$\boldsymbol{s}_{tt_3}^{\lambda}(\boldsymbol{r},\boldsymbol{r}') = \boldsymbol{s}_{tt_3}^{(+)\lambda}(\boldsymbol{r},\boldsymbol{r}') + \boldsymbol{s}_{tt_3}^{(-)\lambda}(\boldsymbol{r},\boldsymbol{r}'). \tag{3.16b}$$

In terms of Eq. (3.13), local densities and currents used in the Skyrme Hartree-Fock calculation are defined as

$$\rho_{tt_3}(\boldsymbol{r}) = \rho_{tt_3}^{(0)}(\boldsymbol{r},\boldsymbol{r}), \tag{3.17a}$$

$$\boldsymbol{s}_{tt_3}(\boldsymbol{r}) = \boldsymbol{s}_{tt_3}^{(0)}(\boldsymbol{r},\boldsymbol{r}), \tag{3.17b}$$

$$\tau_{tt_3}(\boldsymbol{r}) = \boldsymbol{\nabla}\cdot\boldsymbol{\nabla}'\rho_{tt_3}^{(0)}(\boldsymbol{r},\boldsymbol{r}')|_{r=r'}, \tag{3.17c}$$

$$\boldsymbol{T}_{tt_3}(\boldsymbol{r}) = \boldsymbol{\nabla}\cdot\boldsymbol{\nabla}'\boldsymbol{s}_{tt_3}^{(0)}(\boldsymbol{r},\boldsymbol{r}')|_{r=r'}, \tag{3.17d}$$

$$\boldsymbol{j}_{tt_3}(\boldsymbol{r}) = -\frac{i}{2}(\boldsymbol{\nabla}-\boldsymbol{\nabla}')\rho_{tt_3}^{(0)}(\boldsymbol{r},\boldsymbol{r}')|_{r=r'}, \tag{3.17e}$$

$$J_{tt_3,\mu\nu}(\boldsymbol{r}) = -\frac{i}{2}(\boldsymbol{\nabla}-\boldsymbol{\nabla}')_{\mu}s_{tt_3,\nu}^{(0)}(\boldsymbol{r},\boldsymbol{r}')|_{r=r'}. \tag{3.17f}$$

These are the density, spin density, kinetic density, kinetic spin density, current and spin-current tensor, respectively. These densities and currents are all real. The spin orbit current $\boldsymbol{J}_{tt_3}$ is also defined as

$$\boldsymbol{J}_{tt_3} = \sum_{\mu\nu\omega} \epsilon_{\mu\nu\omega} J_{tt_3,\mu\nu}\boldsymbol{e}_{\omega}, \tag{3.17g}$$

where $\epsilon_{\mu\nu\omega}$ is the Levi-Civita symbol and $\boldsymbol{e}_{\omega}$ is unit vector for ω-direction.

In terms of Eq. (3.14), local transition densities and currents used in the

RPA calculation are defined as

$$\rho_{tt_3}^{\lambda}(\boldsymbol{r}) = \rho_{tt_3}^{\lambda}(\boldsymbol{r}, \boldsymbol{r}), \tag{3.18a}$$
$$\boldsymbol{s}_{tt_3}^{\lambda}(\boldsymbol{r}) = \boldsymbol{s}_{tt_3}^{\lambda}(\boldsymbol{r}, \boldsymbol{r}), \tag{3.18b}$$
$$\tau_{tt_3}^{\lambda}(\boldsymbol{r}) = \nabla \cdot \nabla' \rho_{tt_3}^{\lambda}(\boldsymbol{r}, \boldsymbol{r}')|_{r=r'}, \tag{3.18c}$$
$$\boldsymbol{T}_{tt_3}^{\lambda}(\boldsymbol{r}) = \nabla \cdot \nabla' \boldsymbol{s}_{tt_3}^{\lambda}(\boldsymbol{r}, \boldsymbol{r}')|_{r=r'}, \tag{3.18d}$$
$$\boldsymbol{j}_{tt_3}^{\lambda}(\boldsymbol{r}) = -\frac{i}{2}(\nabla - \nabla') \rho_{tt_3}^{\lambda}(\boldsymbol{r}, \boldsymbol{r}')|_{r=r'}, \tag{3.18e}$$
$$J_{tt_3,\mu\nu}^{\lambda}(\boldsymbol{r}) = -\frac{i}{2}(\nabla - \nabla')_{\mu} s_{tt_3,\nu}^{\lambda}(\boldsymbol{r}, \boldsymbol{r}')|_{r=r'}, \tag{3.18f}$$
$$\boldsymbol{J}_{tt_3}^{\lambda} = \sum_{\mu\nu\omega} \epsilon_{\mu\nu\omega} J_{tt_3,\mu\nu}^{\lambda} \boldsymbol{e}_{\omega}. \tag{3.18g}$$

These are the transition density, transition spin density, transition kinetic density, transition kinetic spin density, transition current and transition spin-current tensor, respectively. Similarly,

$$\rho_{tt_3}^{(\pm)\nu}(\boldsymbol{r}) = \rho_{tt_3}^{(\pm)\nu}(\boldsymbol{r}, \boldsymbol{r}), \tag{3.19a}$$
$$\boldsymbol{s}_{tt_3}^{(\pm)\nu}(\boldsymbol{r}) = \boldsymbol{s}_{tt_3}^{(\pm)\nu}(\boldsymbol{r}, \boldsymbol{r}), \tag{3.19b}$$
$$\tau_{tt_3}^{(\pm)\nu}(\boldsymbol{r}) = \nabla \cdot \nabla' \rho_{tt_3}^{(\pm)\nu}(\boldsymbol{r}, \boldsymbol{r}')|_{r=r'}, \tag{3.19c}$$
$$\boldsymbol{T}_{tt_3}^{(\pm)\nu}(\boldsymbol{r}) = \nabla \cdot \nabla' \boldsymbol{s}_{tt_3}^{(\pm)\nu}(\boldsymbol{r}, \boldsymbol{r}')|_{r=r'}, \tag{3.19d}$$
$$\boldsymbol{j}_{tt_3}^{(\pm)\nu}(\boldsymbol{r}) = -\frac{i}{2}(\nabla - \nabla') \rho_{tt_3}^{(\pm)\nu}(\boldsymbol{r}, \boldsymbol{r}')|_{r=r'}, \tag{3.19e}$$
$$J_{tt_3,\mu\nu}^{(\pm)\nu}(\boldsymbol{r}) = -\frac{i}{2}(\nabla - \nabla')_{\mu} s_{tt_3,\nu}^{(\pm)\nu}(\boldsymbol{r}, \boldsymbol{r}')|_{r=r'}. \tag{3.19f}$$
$$\boldsymbol{J}_{tt_3}^{(\pm)\lambda} = \sum_{\mu\nu\omega} \epsilon_{\mu\nu\omega} J_{tt_3,\mu\nu}^{(\pm)\lambda} \boldsymbol{e}_{\omega}. \tag{3.19g}$$

Then, the transition densities and currents are divided into real and pure imaginary quantities:

$$\rho_{tt_3}^{\lambda}(\boldsymbol{r}) = \rho_{tt_3}^{(+)\lambda}(\boldsymbol{r}) + \rho_{tt_3}^{(-)\lambda}(\boldsymbol{r}), \tag{3.20a}$$
$$\boldsymbol{s}_{tt_3}^{\lambda}(\boldsymbol{r}) = \boldsymbol{s}_{tt_3}^{(+)\lambda}(\boldsymbol{r}) + \boldsymbol{s}_{tt_3}^{(-)\lambda}(\boldsymbol{r}), \tag{3.20b}$$
$$\tau_{tt_3}^{\lambda}(\boldsymbol{r}) = \tau_{tt_3}^{(+)\lambda}(\boldsymbol{r}) + \tau_{tt_3}^{(-)\lambda}(\boldsymbol{r}), \tag{3.20c}$$
$$\boldsymbol{T}_{tt_3}^{\lambda}(\boldsymbol{r}) = \boldsymbol{T}_{tt_3}^{(+)\lambda}(\boldsymbol{r}) + \boldsymbol{T}_{tt_3}^{(-)\lambda}(\boldsymbol{r}), \tag{3.20d}$$
$$\boldsymbol{j}_{tt_3}^{\lambda}(\boldsymbol{r}) = \boldsymbol{j}_{tt_3}^{(+)\lambda}(\boldsymbol{r}) + \boldsymbol{j}_{tt_3}^{(-)\lambda}(\boldsymbol{r}), \tag{3.20e}$$
$$\overleftrightarrow{J}_{tt_3}^{\lambda}(\boldsymbol{r}) = \overleftrightarrow{J}_{tt_3}^{(+)\lambda}(\boldsymbol{r}) + \overleftrightarrow{J}_{tt_3}^{(-)\lambda}(\boldsymbol{r}), \tag{3.20f}$$

where $\overleftrightarrow{J}$ is the abbreviation of tensor $J_{\mu\nu}$.

For even-even nuclei, putting Eq. (2.95) into (3.13), the time reversal of the scalar part and the vector part of density matrix, $\rho_{tt_3}^{(0)T}$ and $\boldsymbol{s}_{tt_3}^{(0)T}$, are written as

$$\rho_{tt_3}^{(0)T}(\boldsymbol{r},\boldsymbol{r}') = \rho_{tt_3}^{(0)*}(\boldsymbol{r},\boldsymbol{r}'), \tag{3.21a}$$

$$\boldsymbol{s}_{tt_3}^{(0)T}(\boldsymbol{r},\boldsymbol{r}') = -\boldsymbol{s}_{tt_3}^{(0)*}(\boldsymbol{r},\boldsymbol{r}'). \tag{3.21b}$$

and putting Eq. (2.96b) into (3.15), the time reversal of the scalar part and the vector part of density matrix, $\rho_{tt_3}^{(\pm)\lambda T}$ and $\boldsymbol{s}_{tt_3}^{(\pm)\lambda T}$, are written as

$$\rho_{tt_3}^{(\pm)\lambda T}(\boldsymbol{r},\boldsymbol{r}') = \rho_{tt_3}^{(\pm)\lambda *}(\boldsymbol{r},\boldsymbol{r}'), \tag{3.22a}$$

$$\boldsymbol{s}_{tt_3}^{(\pm)\lambda T}(\boldsymbol{r},\boldsymbol{r}') = -\boldsymbol{s}_{tt_3}^{(\pm)\lambda *}(\boldsymbol{r},\boldsymbol{r}'). \tag{3.22b}$$

Because $\rho^{(0)}(\boldsymbol{r},\boldsymbol{r})$, $\boldsymbol{s}^{(0)}(\boldsymbol{r},\boldsymbol{r})$, $\rho^{(+)}(\boldsymbol{r},\boldsymbol{r})$ and $\boldsymbol{s}^{(+)}(\boldsymbol{r},\boldsymbol{r})$ are real and $\rho^{(-)}(\boldsymbol{r},\boldsymbol{r})$ and $\boldsymbol{s}^{(-)}(\boldsymbol{r},\boldsymbol{r})$ are pure imaginary, properties under time-reversal are derived from Eqs. (3.17)

$$\rho_{tt_3}^{T}(\boldsymbol{r}) = \rho_{tt_3}(\boldsymbol{r}), \quad \tau_{tt_3}^{T}(\boldsymbol{r}) = \tau_{tt_3}(\boldsymbol{r}), \quad \overleftrightarrow{J}_{tt_3}^{T}(\boldsymbol{r}) = \overleftrightarrow{J}_{tt_3}(\boldsymbol{r}), \tag{3.23a}$$

$$\boldsymbol{s}_{tt_3}^{T}(\boldsymbol{r}) = -\boldsymbol{s}_{tt_3}(\boldsymbol{r}), \quad \boldsymbol{T}_{tt_3}^{T}(\boldsymbol{r}) = -\boldsymbol{T}_{tt_3}(\boldsymbol{r}), \quad \boldsymbol{j}_{tt_3}^{T}(\boldsymbol{r}) = -\boldsymbol{j}_{tt_3}(\boldsymbol{r}), \tag{3.23b}$$

and from Eqs. (3.19)

$$\rho_{tt_3}^{(\pm)\lambda T}(\boldsymbol{r}) = \pm\rho_{tt_3}^{(\pm)\lambda}(\boldsymbol{r}), \; \tau_{tt_3}^{(\pm)\lambda T}(\boldsymbol{r}) = \pm\tau_{tt_3}^{(\pm)\lambda}(\boldsymbol{r}), \quad \overleftrightarrow{J}_{tt_3}^{(\pm)\lambda T}(\boldsymbol{r}) = \pm\overleftrightarrow{J}_{tt_3}^{(\pm)\lambda}(\boldsymbol{r}), \tag{3.24a}$$

$$\boldsymbol{s}_{tt_3}^{(\pm)\lambda T}(\boldsymbol{r}) = \mp\boldsymbol{s}_{tt_3}^{(\pm)\lambda}(\boldsymbol{r}), \; \boldsymbol{T}_{tt_3}^{(\pm)\lambda T}(\boldsymbol{r}) = \mp\boldsymbol{T}_{tt_3}^{(\pm)\lambda}(\boldsymbol{r}), \; \boldsymbol{j}_{tt_3}^{(\pm)\lambda T}(\boldsymbol{r}) = \mp\boldsymbol{j}_{tt_3}^{(\pm)\lambda}(\boldsymbol{r}). \tag{3.24b}$$

Therefore, because of $\rho^{(0)}(x,x') = \rho^{(0)T}(x,x')$ and Eq. (2.94), the time odd component of the density $\rho^{(0)}$ and $\rho^{(+)}$ and the time even component of the density $\rho^{(-)}$ are vanishing:

$$\boldsymbol{s}_{tt_3}(\boldsymbol{r}) = \boldsymbol{T}_{tt_3}(\boldsymbol{r}) = \boldsymbol{j}_{tt_3}(\boldsymbol{r}) = 0, \tag{3.25a}$$

$$\boldsymbol{s}_{tt_3}^{(+)\lambda}(\boldsymbol{r}) = \boldsymbol{T}_{tt_3}^{(+)\lambda}(\boldsymbol{r}) = \boldsymbol{j}_{tt_3}^{(+)\lambda}(\boldsymbol{r}) = 0, \tag{3.25b}$$

$$\rho_{tt_3}^{(-)\lambda}(\boldsymbol{r}) = \tau_{tt_3}^{(-)\lambda}(\boldsymbol{r}) = \overleftrightarrow{J}_{tt_3}^{(-)\lambda}(\boldsymbol{r}) = 0. \tag{3.25c}$$

Then, the local transition densities and currents are given as

$$\rho_{tt_3}^{\lambda}(\boldsymbol{r}) = \rho_{tt_3}^{(+)\lambda}(\boldsymbol{r}), \quad \tau_{tt_3}^{\lambda}(\boldsymbol{r}) = \tau_{tt_3}^{(+)\lambda}(\boldsymbol{r}), \quad \overleftrightarrow{J}_{tt_3}^{\lambda}(\boldsymbol{r}) = \overleftrightarrow{J}_{tt_3}^{(+)\lambda}(\boldsymbol{r}), \tag{3.26a}$$

$$\boldsymbol{s}_{tt_3}^{\lambda}(\boldsymbol{r}) = \boldsymbol{s}_{tt_3}^{(-)\lambda}(\boldsymbol{r}), \quad \boldsymbol{T}_{tt_3}^{\lambda}(\boldsymbol{r}) = \boldsymbol{T}_{tt_3}^{(-)\lambda}(\boldsymbol{r}), \quad \boldsymbol{j}_{tt_3}^{\lambda}(\boldsymbol{r}) = \boldsymbol{j}_{tt_3}^{(-)\lambda}(\boldsymbol{r}). \tag{3.26b}$$

For example, the time evolution of the local transition density is given as

$$\delta\rho(\boldsymbol{r}, t) = \rho^{\lambda}_{tt_3}(\boldsymbol{r}) \cos(\omega_\lambda t). \tag{3.27}$$

Similarly, for transition current,

$$\delta\boldsymbol{j}(\boldsymbol{r}, t) = i\boldsymbol{j}^{\lambda}_{tt_3}(\boldsymbol{r}) \sin(\omega_\lambda t). \tag{3.28}$$

3.2 Most general representation

3.2.1 Skyrme energy functional

The Skyrme energy functional is derived from the expectation value of the Skyrme interaction in Eq. (3.1) for a Slater determinant constructed by the occupied single-particle wave functions (see appendix). Then, the Skyrme energy functional is given in the form as [8, 23]

$$\begin{aligned}\mathcal{E}_{\mathrm{Sk}} = \sum_{t=0,1} \sum_{t_3=-t}^{t} & C_t^{\rho} \rho_{tt_3}^2 + C_t^{s} \boldsymbol{s}_{tt_3}^2 + C_t^{\Delta\rho} \rho_{tt_3} \Delta\rho_{tt_3} + C_t^{\Delta s} \boldsymbol{s}_{tt_3} \cdot \Delta \boldsymbol{s}_{tt_3} \\ & + C_t^{\tau}\left(\rho_{tt_3}\tau_{tt_3} - \boldsymbol{j}_{tt_3}^2\right) + C_t^{T}\left(\boldsymbol{s}_{tt_3} \cdot \boldsymbol{T}_{tt_3} - \overleftrightarrow{J}_{tt_3}^2\right) \\ & + C_t^{\nabla J}\left(\rho_{tt_3} \nabla \cdot \boldsymbol{J}_{tt_3} + \boldsymbol{s}_{tt_3} \cdot \nabla \times \boldsymbol{j}_{tt_3}\right).\end{aligned} \tag{3.29}$$

Only the $t_3 = 0$ component of the isovector $t = 1$ terms contribute to nuclear ground states and excitation without charge-exchange, while the $t_3 = \pm 1$ components contribute only to charge-exchange excitation (see next section). The relations between the parameters C_t^x and the Skyrme parameters t_i, x_i,

W_0 and α are

$$
\begin{aligned}
C_0^\rho &= \tfrac{3}{8}t_0 + \tfrac{3}{48}t_3\rho_{00}^\alpha, & (3.30a)\\
C_1^\rho &= -\tfrac{1}{4}t_0\left[\tfrac{1}{2}+x_0\right] - \tfrac{1}{24}t_3\left[\tfrac{1}{2}+x_3\right]\rho_{00}^\alpha, & (3.30b)\\
C_0^s &= -\tfrac{1}{4}t_0\left[\tfrac{1}{2}-x_0\right] - \tfrac{1}{24}t_3\left[\tfrac{1}{2}-x_3\right]\rho_{00}^\alpha, & (3.30c)\\
C_1^s &= -\tfrac{1}{8}t_0 - \tfrac{1}{48}t_3\rho_{00}^\alpha, & (3.30d)\\
C_0^{\Delta\rho} &= -\tfrac{9}{64}t_1 + \tfrac{1}{16}t_2\left[\tfrac{5}{4}+x_2\right], & (3.30e)\\
C_1^{\Delta\rho} &= \tfrac{3}{32}t_1\left[\tfrac{1}{2}+x_1\right] + \tfrac{1}{32}t_2\left[\tfrac{1}{2}+x_2\right], & (3.30f)\\
C_0^{\Delta s} &= \tfrac{3}{32}t_1\left[\tfrac{1}{2}-x_1\right] + \tfrac{1}{32}t_2\left[\tfrac{1}{2}+x_2\right], & (3.30g)\\
C_1^{\Delta s} &= \tfrac{3}{64}t_1 + \tfrac{1}{64}t_2, & (3.30h)\\
C_0^\tau &= \tfrac{3}{16}t_1 + \tfrac{1}{4}t_2\left[\tfrac{5}{4}+x_2\right], & (3.30i)\\
C_1^\tau &= -\tfrac{1}{8}t_1\left[\tfrac{1}{2}+x_1\right] + \tfrac{1}{8}t_2\left[\tfrac{1}{2}+x_2\right], & (3.30j)\\
C_0^T &= -\tfrac{1}{8}t_1\left[\tfrac{1}{2}-x_1\right] + \tfrac{1}{8}t_2\left[\tfrac{1}{2}+x_2\right], & (3.30k)\\
C_1^T &= -\tfrac{1}{16}t_1 + \tfrac{1}{16}t_2, & (3.30l)\\
C_0^{\nabla J} &= -\tfrac{3}{4}W_0, & (3.30m)\\
C_1^{\nabla J} &= -\tfrac{1}{4}W_0. & (3.30n)
\end{aligned}
$$

Note that the parameters C_t^ρ and C_t^s depend on the density $\rho_{00}(\boldsymbol{r})$. We can divid the Skyrme energy functional into time-even and time-odd part:

$$
\mathcal{E}_{\mathrm{Sk}} = \sum_{t=0,1} \mathcal{E}_t^{\mathrm{even}} + \mathcal{E}_t^{\mathrm{odd}}, \tag{3.31}
$$

where

$$
\begin{aligned}
\mathcal{E}_t^{\mathrm{even}} &= \sum_{t_3=-t}^{t} C_t^\rho \rho_{tt_3}^2 + C_t^{\Delta\rho}\rho_{tt_3}\Delta\rho_{tt_3} + C_t^\tau\left(\rho_{tt_3}\tau_{tt_3} - \boldsymbol{j}_{tt_3}^2\right)\\
&\quad + C_t^{\nabla J}\left(\rho_{tt_3}\boldsymbol{\nabla}\cdot\boldsymbol{J}_{tt_3}\right),\\
\mathcal{E}_t^{\mathrm{odd}} &= \sum_{t_3=-t}^{t} C_t^s \boldsymbol{s}_{tt_3}^2 + C_t^{\Delta s}\boldsymbol{s}_{tt_3}\cdot\Delta\boldsymbol{s}_{tt_3} + C_t^T\left(\boldsymbol{s}_{tt_3}\cdot\boldsymbol{T}_{tt_3} - \overleftrightarrow{J}_{tt_3}^2\right)\\
&\quad + C_t^{\nabla J}\left(\boldsymbol{s}_{tt_3}\cdot\boldsymbol{\nabla}\times\boldsymbol{j}_{tt_3}\right).
\end{aligned} \tag{3.32a}
$$

3.2.2 Hartree-Fock equation

In most general representation, the Hartree-Fock equation (2.52) with Skyrme interaction is written as

$$
\sum_{\sigma'\tau'} h(\boldsymbol{r}\sigma\tau, \boldsymbol{r}\sigma'\tau')\phi_k(\boldsymbol{r}\sigma'\tau') = e_k\phi_k(\boldsymbol{r}\sigma\tau) \tag{3.33}
$$

with a single-particle hamiltonian

$$h(\boldsymbol{r}\sigma\tau, \boldsymbol{r}\sigma'\tau') = \left[-\frac{\hbar^2}{2m}\triangle\delta_{\tau\tau'} + F_{00}(\boldsymbol{r})\delta_{\tau\tau'} + \sum_{t_3} F_{1t_3}(\boldsymbol{r})\tau^{t_3}_{\tau\tau'}\right]\delta_{\sigma\sigma'}$$
$$+ \left[\boldsymbol{G}_{00}(\boldsymbol{r})\delta_{\tau\tau'} + \sum_{t_3} \boldsymbol{G}_{1t_3}(\boldsymbol{r})\tau^{t_3}_{\tau\tau'}\right]\cdot\boldsymbol{\sigma}_{\sigma\sigma'}, \tag{3.34}$$

where

$$F_{tt_3} = -\boldsymbol{\nabla}\cdot[M_{tt_3}(\boldsymbol{r})\boldsymbol{\nabla}] + U_{tt_3}(\boldsymbol{r})$$
$$+ \frac{1}{2i}\big(\boldsymbol{\nabla}\cdot\boldsymbol{I}_{tt_3}(\boldsymbol{r}) + \boldsymbol{I}_{tt_3}(\boldsymbol{r})\cdot\boldsymbol{\nabla}\big) + U_{\mathrm{C},tt_3}(\boldsymbol{r}) \tag{3.35a}$$
$$\boldsymbol{G}_{tt_3} = -\sum_{\mu}\nabla_\mu\cdot[(\boldsymbol{C}_{tt_3}(\boldsymbol{r}))\nabla_\mu] + \boldsymbol{\Sigma}_{tt_3}(\boldsymbol{r})$$
$$+ \frac{1}{2i}\sum_{\mu\nu}[\nabla_\mu B_{tt_3,\mu\nu}(\boldsymbol{r}) + B_{tt_3,\mu\nu}(\boldsymbol{r})\nabla_\mu]\,\boldsymbol{e}_\nu. \tag{3.35b}$$

$\boldsymbol{e}_\nu$ is unit vector. In terms of the energy functionals in Eqs. (3.32) Various potentials in F_{tt_3} and G_{tt_3} are defined as

$$M_{tt_3} = \frac{\partial\mathcal{E}_0^{\text{even}}}{\partial\tau_{tt_3}} = C_t^\tau\rho_{tt_3}, \tag{3.36a}$$

$$U_{00} = \frac{\partial\mathcal{E}_0^{\text{even}}}{\partial\rho_{00}} + \frac{\partial\mathcal{E}_0^{\text{odd}}}{\partial\rho_{00}} + \frac{\partial\mathcal{E}_1^{\text{even}}}{\partial\rho_{00}} + \frac{\partial\mathcal{E}_1^{\text{odd}}}{\partial\rho_{00}}$$
$$= 2C_0^\rho\rho_{00} + 2C_0^{\Delta\rho}\Delta\rho_{00} + C_0^\tau\tau_{00} + C_0^{\nabla J}\boldsymbol{\nabla}\cdot\boldsymbol{J}_{00}$$
$$+ \sum_{tt_3}\left[\frac{\partial C_t^\rho}{\partial\rho_{00}}\rho_{tt_3}^2 + \frac{\partial C_t^s}{\partial\rho_{00}}\boldsymbol{s}_{tt_3}^2\right], \tag{3.36b}$$

$$U_{1t_3} = \frac{\partial\mathcal{E}_1^{\text{even}}}{\partial\rho_{1t_3}} = 2C_1^\rho\rho_{1t_3} + 2C_1^{\Delta\rho}\Delta\rho_{1t_3} + C_1^\tau\tau_{1t_3} + C_1^{\nabla J}\boldsymbol{\nabla}\cdot\boldsymbol{J}_{1t_3}, \tag{3.36c}$$

$$B_{tt_3,\mu\nu} = \frac{\partial\mathcal{E}_t^{\text{even}}}{\partial J_{tt_3,\mu\nu}} = -2C_t^\tau J_{tt_3,\mu\nu} - 2C_t^{\nabla J}\sum_\omega\epsilon_{\mu\nu\omega}\nabla_\omega\rho_{tt_3}, \tag{3.36d}$$

$$\boldsymbol{C}_{tt_3} = \frac{\partial\mathcal{E}_t^{\text{odd}}}{\partial\boldsymbol{T}_{tt_3}} = C_t^T\boldsymbol{s}_{tt_3}, \tag{3.36e}$$

$$\boldsymbol{\Sigma}_{tt_3} = \frac{\partial \mathcal{E}_t^{\text{odd}}}{\partial \boldsymbol{s}_{tt_3}} = 2C_t^s \boldsymbol{s}_{tt_3} + 2C_t^{\Delta s}\Delta \boldsymbol{s}_{tt_3} + C_t^T \boldsymbol{T}_{tt_3} + C_t^{\nabla J}\boldsymbol{\nabla}\times \boldsymbol{j}_{tt_3}, \quad (3.36\text{f})$$

$$\boldsymbol{I}_{tt_3} = \frac{\partial \mathcal{E}_t^{\text{odd}}}{\partial \boldsymbol{j}_{tt_3}(\boldsymbol{r})} = -2C_t^T \boldsymbol{j}_{tt_3} + C_t^{\nabla j}\boldsymbol{\nabla}\times \boldsymbol{s}_{tt_3}, \quad (3.36\text{g})$$

$$U_{\text{C},tt_3} = \frac{\partial \mathcal{E}_{\text{Coul}}}{\partial \rho_{tt_3}} = (-1)^t \delta_{t_3 0}\left[\frac{e}{2}\Phi_{\text{C}} - \frac{e^2}{2}\left(\frac{3}{2\pi}\right)^{1/3}\rho_{\text{p}}(\boldsymbol{r})^{1/3}\right]. \quad (3.36\text{h})$$

3.2.3 RPA equation

Here, we present the RPA equation made use of in the actual numerical calculation. The RPA equation (2.48) with the Skyrme interaction is written as

$$\begin{aligned}\hbar\omega_\lambda \phi_i^{(\mp)\nu}(\boldsymbol{r}\sigma\tau) = &\sum_{\sigma'\tau'}[h(\boldsymbol{r}\sigma\tau, \boldsymbol{r}\sigma'\tau') - e_i \delta_{\sigma\sigma'}\delta_{\tau\tau'}]\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma'\tau') \\ &+ \sum_{\sigma'\tau'\sigma''\tau''}\int d\boldsymbol{r}' P(\boldsymbol{r}\sigma\tau, \boldsymbol{r}'\sigma'\tau') h^{(\pm)\lambda}(\boldsymbol{r}'\sigma'\tau', \boldsymbol{r}'\sigma''\tau'')\phi_i(\boldsymbol{r}'\sigma''\tau'')\end{aligned} \quad (3.37)$$

with the hamiltonian

$$\begin{aligned}h^{(\pm)\lambda}(\boldsymbol{r}\sigma\tau, \boldsymbol{r}\sigma'\tau') = &\left[F_{00}^{(\pm)\lambda}(\boldsymbol{r})\delta_{\tau\tau'} + \sum_{t_3} F_{1t_3}^{(\pm)\lambda}(\boldsymbol{r})\tau_{\tau\tau'}^{t_3}\right]\delta_{\sigma\sigma'} \\ &+ \left[\boldsymbol{G}_{00}^{(\pm)\lambda}(\boldsymbol{r})\delta_{\tau\tau'} + \sum_{t_3}\boldsymbol{G}_{1t_3}^{(\pm)\lambda}(\boldsymbol{r})\tau_{\tau\tau'}^{t_3}\right]\cdot \boldsymbol{\sigma}_{\sigma\sigma'},\end{aligned} \quad (3.38)$$

where

$$\begin{aligned}F_{tt_3}^{(\pm)\lambda}(\boldsymbol{r}) = &-\boldsymbol{\nabla}\cdot[M_{tt_3}^{(\pm)\lambda}(\boldsymbol{r})\boldsymbol{\nabla}] + U_{tt_3}^{(\pm)\lambda}(\boldsymbol{r}) \\ &+ \frac{1}{2i}(\boldsymbol{\nabla}\cdot \boldsymbol{I}_{tt_3}^{(\pm)\lambda}(\boldsymbol{r}) + \boldsymbol{I}_{tt_3}^{(\pm)\lambda}(\boldsymbol{r})\cdot\boldsymbol{\nabla}) + U_{\text{C},tt_3}^{(\pm)\lambda}(\boldsymbol{r}) \quad (3.39\text{a}) \\ \boldsymbol{G}_{tt_3}^{(\pm)\lambda}(\boldsymbol{r}) = &-\sum_{\mu}\nabla_\mu \cdot \left[\boldsymbol{C}_{tt_3}^{(\pm)\lambda}(\boldsymbol{r})\nabla_\mu\right] + \boldsymbol{\Sigma}_{tt_3}^{(\pm)\lambda}(\boldsymbol{r}) \\ &+ \frac{1}{2i}\sum_{\mu\nu}\left[\nabla_\mu B_{tt_3,\mu\nu}^{(\pm)\lambda}(\boldsymbol{r}) + B_{tt_3,\mu\nu}^{(\pm)\lambda}(\boldsymbol{r})\nabla_\mu\right]\boldsymbol{e}_\nu \quad (3.39\text{b})\end{aligned}$$

Various transition potentials in $F_{tt_3}^{(\pm)\lambda}$ and $G_{tt_3}^{(\pm)\lambda}$ are defined as

$$M_{tt_3}^{(\pm)\lambda} = \frac{\partial^2 \mathcal{E}_t^{\mathrm{even}}}{\partial \tau_{tt_3} \partial \rho_{tt_3}} \rho_{tt_3}^{(\pm)\lambda} = C_t^\tau \rho_{tt_3}^{(\pm)\lambda}, \tag{3.40a}$$

$$\begin{aligned} U_{00}^{(\pm)\lambda} &= \frac{\partial}{\partial \rho_{00}} \left[\left(\sum_{tt_3} \frac{\partial \mathcal{E}_{\mathrm{Sk}}}{\partial \rho_{tt_3}} \rho_{tt_3}^{(\pm)\lambda} + \frac{\partial \mathcal{E}_t^{\mathrm{odd}}}{\partial \boldsymbol{s}_{tt_3}} \cdot \boldsymbol{s}_{tt_3}^{(\pm)\lambda} \right) \right. \\ &\left. + \frac{\partial \mathcal{E}_0^{\mathrm{even}}}{\partial \tau_{00}} \tau_{00}^{(\pm)\lambda} + \frac{\partial \mathcal{E}_0^{\mathrm{even}}}{\partial \boldsymbol{J}_{00}} \cdot \boldsymbol{J}_{00}^{(\pm)\lambda} \right] \\ &= 2C_0^\rho \rho_{00}^{(\pm)\lambda} + 2C_0^{\Delta\rho} \Delta \rho_{00}^{(\pm)\lambda} + C_0^\tau \tau_{00}^{(\pm)\lambda} + C_0^{\nabla J} \boldsymbol{\nabla} \cdot \boldsymbol{J}_{00}^{(\pm)\lambda} \\ &+ 2 \frac{\partial C_0^\rho}{\partial \rho_{00}} \rho_{00} \rho_{00}^{(\pm)\lambda} + 2 \sum_{tt_3} \left[\frac{\partial C_t^\rho}{\partial \rho_{00}} \rho_{tt_3} \rho_{tt_3}^{(\pm)\lambda} + \frac{\partial C_t^s}{\partial \rho_{00}} \boldsymbol{s}_{tt_3} \cdot \boldsymbol{s}_{tt_3}^{(\pm)\lambda} \right. \\ &\left. + \left(\frac{\partial^2 C_t^\rho}{\partial \rho_{00} \partial \rho_{00}} \rho_{tt_3}^2 + \frac{\partial^2 C_t^s}{\partial \rho_{00} \partial \rho_{00}} \boldsymbol{s}_{tt_3}^2 \right) \rho_{00}^{(\pm)\lambda} \right], \end{aligned} \tag{3.40b}$$

$$\begin{aligned} U_{1t_3}^{(\pm)\lambda} &= \frac{\partial}{\partial \rho_{1t_3}} \left[\frac{\partial \mathcal{E}_1^{\mathrm{even}}}{\partial \rho_{1t_3}} \rho_{1t_3}^{(\pm)\lambda} + \frac{\partial \mathcal{E}_1^{\mathrm{even}}}{\partial \tau_{1t_3}} \tau_{1t_3}^{(\pm)\lambda} + \frac{\partial \mathcal{E}_1^{\mathrm{even}}}{\partial \boldsymbol{J}_{1t_3}} \cdot \boldsymbol{J}_{1t_3}^{(\pm)\lambda} + \frac{\partial \mathcal{E}_1^{\mathrm{even}}}{\partial \rho_{00}} \rho_{00}^{(\pm)\lambda} \right] \\ &= 2C_1^\rho \rho_{1t_3} + 2C_1^{\Delta\rho} \Delta \rho_{1t_3} + C_1^\tau \tau_{1t_3} + C_1^{\nabla J} \boldsymbol{\nabla} \cdot \boldsymbol{J}_{1t_3} + 2 \frac{\partial C_1^\rho}{\partial \rho_{00}} \rho_{1t_3} \rho_{00}^{(\pm)\lambda}, \end{aligned} \tag{3.40c}$$

$$\begin{aligned} B_{tt_3,\mu\nu}^{(\pm)\lambda} &= \frac{\partial}{\partial J_{tt_3,\mu\nu}} \left[\frac{\partial \mathcal{E}_t^{\mathrm{even}}}{\partial J_{tt_3,\mu\nu}} J_{tt_3,\mu\nu}^{(\pm)\lambda} + \frac{\partial \mathcal{E}_t^{\mathrm{even}}}{\partial \rho_{tt_3}} \rho_{tt_3}^{(\pm)\lambda} \right] \\ &= -2C_t^\tau J_{tt_3,\mu\nu}^{(\pm)\lambda} - 2C_t^{\nabla J} \sum_\omega \epsilon_{\mu\nu\omega} \nabla_\omega \rho_{tt_3}^{(\pm)\lambda}, \end{aligned} \tag{3.40d}$$

$$\boldsymbol{C}_{tt_3}^{(\pm)\lambda} = \frac{\partial^2 \mathcal{E}_t^{\mathrm{odd}}}{\partial \boldsymbol{T}_{tt_3} \partial \boldsymbol{s}_{tt_3}} \cdot \boldsymbol{s}_{tt_3}^{(\pm)\lambda} = C_t^T \boldsymbol{s}_{tt_3}^{(\pm)\lambda}, \tag{3.40e}$$

$$\begin{aligned} \boldsymbol{\Sigma}_{tt_3}^{(\pm)\lambda} &= \frac{\partial}{\partial \boldsymbol{s}_{tt_3}} \left[\frac{\partial \mathcal{E}_t^{\mathrm{odd}}}{\partial \boldsymbol{s}_{tt_3}} \cdot \boldsymbol{s}_{tt_3}^{(\pm)\lambda} + \frac{\partial^2 \mathcal{E}_t^{\mathrm{odd}}}{\partial \boldsymbol{T}_{tt_3}} \cdot \boldsymbol{T}_{tt_3}^{(\pm)\lambda} \right. \\ &\left. + \frac{\partial^2 \mathcal{E}_t^{\mathrm{odd}}}{\partial \boldsymbol{j}_{tt_3}} \cdot \boldsymbol{j}_{tt_3}^{(\pm)\lambda} + \frac{\partial^2 \mathcal{E}_t^{\mathrm{odd}}}{\partial \rho_{00}} \rho_{00}^{(\pm)\lambda} \right] \\ &= 2C_t^s \boldsymbol{s}_{tt_3}^{(\pm)\lambda} + 2C_t^{\Delta s} \Delta \boldsymbol{s}_{tt_3}^{(\pm)\lambda} + C_t^T \boldsymbol{T}_{tt_3}^{(\pm)\lambda} \\ &+ C_t^{\nabla J} \boldsymbol{\nabla} \times \boldsymbol{j}_{tt_3}^{(\pm)\lambda} + 2 \frac{\partial C_t^s}{\partial \rho_{00}} \boldsymbol{s}_{tt_3} \rho_{00}^{(\pm)\lambda}, \end{aligned} \tag{3.40f}$$

$$
\begin{aligned}
\boldsymbol{I}_{tt_3}^{(\pm)\lambda} &= \frac{\partial}{\partial \boldsymbol{j}_{tt_3}} \left[\frac{\partial \mathcal{E}_t^{\mathrm{odd}}}{\partial \boldsymbol{j}_{tt_3}} \cdot \boldsymbol{j}_{tt_3}^{(\pm)\lambda} + \frac{\partial \mathcal{E}_t^{\mathrm{odd}}}{\partial \boldsymbol{s}_{tt_3}} \cdot \boldsymbol{s}_{tt_3}^{(\pm)\lambda} \right] \\
&= -2C_t^T \boldsymbol{j}_{tt_3}^{(\pm)\lambda} + C_t^{\nabla j} \boldsymbol{\nabla} \times \boldsymbol{s}_{tt_3}^{(\pm)\lambda},
\end{aligned} \tag{3.40g}
$$

$$
\begin{aligned}
U_{\mathrm{C},tt_3} &= \frac{\partial}{\partial \rho_{tt_3}} \left[\frac{\partial \mathcal{E}_{\mathrm{Coul}}}{\partial \rho_{00}} \rho_{00}^{(\pm)\lambda} + \frac{\partial \mathcal{E}_{\mathrm{Coul}}}{\partial \rho_{10}} \rho_{10}^{(\pm)\lambda} \right] \\
&= (-1)^t \delta_{t_3 0} \left\{ \frac{e}{2} \Phi_{\mathrm{C}}^{(\pm)\lambda} - \frac{e^2}{6} \left(\frac{3}{2\pi} \right)^{1/3} \rho_{\mathrm{p}}^{-2/3} \rho_{\mathrm{p}}^{(\pm)\lambda} \right\},
\end{aligned} \tag{3.40h}
$$

where a Coulomb potential $\Phi_{\mathrm{C}}^{(\pm)\lambda}$ for the transition proton density $\rho_{\mathrm{p}}^{(\pm)\lambda}$ is defined as

$$
\Phi_{\mathrm{C}}^{(\pm)\lambda}(\boldsymbol{r}) = e \int \mathrm{d}\boldsymbol{r}' \frac{\rho_{\mathrm{p}}^{(\pm)\lambda}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|} = \frac{e}{2} \int \mathrm{d}\boldsymbol{r}' \frac{\rho_{00}^{(\pm)\lambda}(\boldsymbol{r}') - \rho_{10}^{(\pm)\lambda}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}. \tag{3.41}
$$

We call $\Phi_{\mathrm{C}}^{(\pm)\lambda}$ the transition Coulomb potential. We write down the RPA equation (3.37) as the abbreviation form:

$$
(\hat{h} - e_i)\phi_i^{(\pm)\lambda} + \hat{P}\hat{h}^{(\pm)\lambda}\phi_i = \hbar\omega_\lambda \phi_i^{(\mp)\nu}. \tag{3.42}
$$

3.3 No mixing of isospin component

We assume that the proton and neutron is not mixed in each of the single-particle wave function $\phi_i(x)$ of the ground state:

$$
\phi_i(\boldsymbol{r}\sigma\mathrm{n}) \neq 0, \qquad \phi_i(\boldsymbol{r}\sigma\mathrm{p}) = 0, \qquad i = 1, \ldots, N, \tag{3.43a}
$$

$$
\phi_i(\boldsymbol{r}\sigma\mathrm{n}) = 0, \qquad \phi_i(\boldsymbol{r}\sigma\mathrm{p}) \neq 0, \qquad i = N+1, \ldots, A, \tag{3.43b}
$$

where A, N and Z represent nucleon, neutron and proton number respectively. Then, the densities and currents in Eq. (3.17) is divided neutron and proton part

$$
\rho_{00} = \rho = \rho_{\mathrm{n}} + \rho_{\mathrm{p}}, \qquad \rho_{10} = \rho_{\mathrm{n}} - \rho_{\mathrm{p}}, \qquad \rho_{1\pm1} = 0, \tag{3.44a}
$$

$$
\boldsymbol{s}_{00} = \boldsymbol{s} = \boldsymbol{s}_{\mathrm{n}} + \boldsymbol{s}_{\mathrm{p}}, \qquad \boldsymbol{s}_{10} = \boldsymbol{s}_{\mathrm{n}} - \boldsymbol{s}_{\mathrm{p}}, \qquad \boldsymbol{s}_{1\pm1} = 0, \tag{3.44b}
$$

$$
\tau_{00} = \tau = \tau_{\mathrm{n}} + \tau_{\mathrm{p}}, \qquad \tau_{10} = \tau_{\mathrm{n}} - \tau_{\mathrm{p}}, \qquad \tau_{1\pm1} = 0, \tag{3.44c}
$$

$$
\boldsymbol{T}_{00} = \boldsymbol{T} = \boldsymbol{T}_{\mathrm{n}} + \boldsymbol{T}_{\mathrm{p}}, \qquad \boldsymbol{T}_{10} = \boldsymbol{T}_{\mathrm{n}} - \boldsymbol{T}_{\mathrm{p}}, \qquad \boldsymbol{T}_{1\pm1} = 0, \tag{3.44d}
$$

$$
\boldsymbol{j}_{00} = \boldsymbol{j} = \boldsymbol{j}_{\mathrm{n}} + \boldsymbol{j}_{\mathrm{p}}, \qquad \boldsymbol{j}_{10} = \boldsymbol{j}_{\mathrm{n}} - \boldsymbol{j}_{\mathrm{p}}, \qquad \boldsymbol{j}_{1\pm1} = 0, \tag{3.44e}
$$

$$
\overleftrightarrow{J}_{00} = \overleftrightarrow{J} = \overleftrightarrow{J}_{\mathrm{n}} + \overleftrightarrow{J}_{\mathrm{p}}, \qquad \overleftrightarrow{J}_{10} = \overleftrightarrow{J}_{\mathrm{n}} - \overleftrightarrow{J}_{\mathrm{p}}, \qquad \overleftrightarrow{J}_{1\pm1} = 0, \tag{3.44f}
$$

where densities ρ, ρ_{n} and ρ_{p} represent a total, neutron and proton density respectively.

3.3.1 Skyrme energy functional

Putting Eq. (3.44) into (3.29), the Skyrme energy density is rewritten into [56]

$$\begin{aligned}
\mathcal{E}_{\mathrm{Sk}} = & C^{\rho}_{\mathrm{tot}}\rho^2 + C^{\rho}_{\mathrm{sum}}(\rho_{\mathrm{n}}^2 + \rho_{p}^2) \\
& + C^{s}_{\mathrm{tot}}\boldsymbol{s}^2 + C^{s}_{\mathrm{sum}}(\boldsymbol{s}_{\mathrm{n}}^2 + \boldsymbol{s}_{\mathrm{p}}^2) \\
& + C^{\Delta\rho}_{\mathrm{tot}}\rho\Delta\rho + C^{\Delta\rho}_{\mathrm{sum}}(\rho_{\mathrm{n}}\Delta\rho_{\mathrm{n}} + \rho_{\mathrm{p}}\Delta\rho_{\mathrm{p}}) \\
& + C^{\Delta s}_{\mathrm{tot}}\boldsymbol{s}\cdot\Delta\boldsymbol{s} + C^{\Delta s}_{\mathrm{sum}}(\boldsymbol{s}_{\mathrm{n}}\cdot\Delta\boldsymbol{s}_{\mathrm{n}} + \boldsymbol{s}_{\mathrm{p}}\cdot\Delta\boldsymbol{s}_{\mathrm{p}}) \\
& + C^{\tau}_{\mathrm{tot}}\big(\rho\tau - \boldsymbol{j}^2\big) + C^{\tau}_{\mathrm{sum}}\big(\rho_{\mathrm{n}}\tau_{\mathrm{n}} - \boldsymbol{j}_{\mathrm{n}}^2 + \rho_{\mathrm{p}}\tau_{\mathrm{p}} - \boldsymbol{j}_{\mathrm{p}}^2\big) \\
& + C^{T}_{\mathrm{tot}}\big(\boldsymbol{s}\cdot\boldsymbol{T} - \overleftrightarrow{J}^2\big) + C^{T}_{\mathrm{sum}}\big(\boldsymbol{s}_{\mathrm{n}}\cdot\boldsymbol{T}_{\mathrm{n}} - \overleftrightarrow{J}^2_{\mathrm{n}} + \boldsymbol{s}_{\mathrm{p}}\cdot\boldsymbol{T}_{\mathrm{p}} - \overleftrightarrow{J}^2_{\mathrm{p}}\big) \\
& + C^{\nabla J}_{\mathrm{tot}}\big(\rho\boldsymbol{\nabla}\cdot\boldsymbol{J} + \boldsymbol{s}\cdot\boldsymbol{\nabla}\times\boldsymbol{j}\big) \\
& + C^{\nabla J}_{\mathrm{sum}}\big(\rho_{\mathrm{n}}\boldsymbol{\nabla}\cdot\boldsymbol{J}_{\mathrm{n}} + \rho_{\mathrm{p}}\boldsymbol{\nabla}\cdot\boldsymbol{J}_{\mathrm{p}} + \boldsymbol{s}_{\mathrm{n}}\cdot\boldsymbol{\nabla}\times\boldsymbol{j}_{\mathrm{n}} + \boldsymbol{s}_{\mathrm{p}}\cdot\boldsymbol{\nabla}\times\boldsymbol{j}_{\mathrm{p}}\big),
\end{aligned} \tag{3.45}$$

where

$$C^{x}_{\mathrm{tot}} = C^{x}_{0} - C^{x}_{1}, \qquad C^{x}_{\mathrm{sum}} = 2C^{x}_{1}. \tag{3.46}$$

We can divid the Skyrme energy functional in Eq. (3.45) into a time-even and a time-odd part:

$$\mathcal{E}_{\mathrm{Sk}} = \mathcal{E}^{\mathrm{even}}_{\mathrm{Sk}} + \mathcal{E}^{\mathrm{odd}}_{\mathrm{Sk}}, \tag{3.47}$$

where

$$\begin{aligned}
\mathcal{E}^{\mathrm{even}}_{\mathrm{Sk}} = & C^{\rho}_{\mathrm{tot}}\rho^2 + C^{\rho}_{\mathrm{sum}}(\rho_{\mathrm{n}}^2 + \rho_{p}^2) \\
& + C^{\Delta\rho}_{\mathrm{tot}}\rho\Delta\rho + C^{\Delta\rho}_{\mathrm{sum}}(\rho_{\mathrm{n}}\Delta\rho_{\mathrm{n}} + \rho_{\mathrm{p}}\Delta\rho_{\mathrm{p}}) \\
& + C^{\tau}_{\mathrm{tot}}\rho\tau + C^{\tau}_{\mathrm{sum}}\big(\rho_{\mathrm{n}}\tau_{\mathrm{n}} + \rho_{\mathrm{p}}\tau_{\mathrm{p}}\big) - C^{T}_{\mathrm{tot}}\overleftrightarrow{J}^2 - C^{T}_{\mathrm{sum}}\big(\overleftrightarrow{J}^2_{\mathrm{n}} + \overleftrightarrow{J}^2_{\mathrm{p}}\big) \\
& + C^{\nabla J}_{\mathrm{tot}}\big(\rho\boldsymbol{\nabla}\cdot\boldsymbol{J}\big) + C^{\nabla J}_{\mathrm{sum}}\big(\rho_{\mathrm{n}}\boldsymbol{\nabla}\cdot\boldsymbol{J}_{\mathrm{n}} + \rho_{\mathrm{p}}\boldsymbol{\nabla}\cdot\boldsymbol{J}_{\mathrm{p}}\big),
\end{aligned} \tag{3.48a}$$

$$\begin{aligned}
\mathcal{E}^{\mathrm{odd}}_{\mathrm{Sk}} = & C^{s}_{\mathrm{tot}}\boldsymbol{s}^2 + C^{s}_{\mathrm{sum}}(\boldsymbol{s}_{\mathrm{n}}^2 + \boldsymbol{s}_{\mathrm{p}}^2) \\
& + C^{\Delta s}_{\mathrm{tot}}\boldsymbol{s}\cdot\Delta\boldsymbol{s} + C^{\Delta s}_{\mathrm{sum}}(\boldsymbol{s}_{\mathrm{n}}\cdot\Delta\boldsymbol{s}_{\mathrm{n}} + \boldsymbol{s}_{\mathrm{p}}\cdot\Delta\boldsymbol{s}_{\mathrm{p}}) \\
& - C^{\tau}_{\mathrm{tot}}\boldsymbol{j}^2 - C^{\tau}_{\mathrm{sum}}\big(\boldsymbol{j}_{\mathrm{n}}^2 + \boldsymbol{j}_{\mathrm{p}}^2\big) + C^{T}_{\mathrm{tot}}\boldsymbol{s}\cdot\boldsymbol{T} + C^{T}_{\mathrm{sum}}\big(\boldsymbol{s}_{\mathrm{n}}\cdot\boldsymbol{T}_{\mathrm{n}} + \boldsymbol{s}_{\mathrm{p}}\cdot\boldsymbol{T}_{\mathrm{p}}\big) \\
& + C^{\nabla J}_{\mathrm{tot}}\big(\boldsymbol{s}\cdot\boldsymbol{\nabla}\times\boldsymbol{j}\big) + C^{\nabla J}_{\mathrm{sum}}\big(\boldsymbol{s}_{\mathrm{n}}\cdot\boldsymbol{\nabla}\times\boldsymbol{j}_{\mathrm{n}} + \boldsymbol{s}_{\mathrm{p}}\cdot\boldsymbol{\nabla}\times\boldsymbol{j}_{\mathrm{p}}\big).
\end{aligned} \tag{3.48b}$$

3.3.2 Hartree-Fock equation

Putting Eq. (3.44) into (3.34), the single-particle hamiltonian is rewritten into

$$h(\boldsymbol{r}\sigma\tau, \boldsymbol{r}\sigma'\tau') = \left[-\frac{\hbar^2}{2m}\Delta\delta_{\tau\tau'} + F_{00}(\boldsymbol{r})\delta_{\tau\tau'} + F_{10}(\boldsymbol{r})\tau^{0}_{\tau\tau'}\right]\delta_{\sigma\sigma'} \tag{3.49}$$

$$+ \left[\boldsymbol{G}_{00}(\boldsymbol{r})\delta_{\tau\tau'} + \boldsymbol{G}_{10}(\boldsymbol{r})\tau^{0}_{\tau\tau'}\right]\cdot\boldsymbol{\sigma}_{\sigma\sigma'}. \tag{3.50}$$

Therefore, the HF equation (3.33) is divided into those for the neutron and the proton single-particle wave function:

$$\sum_{\sigma'} h_{\mathrm{n}}(\boldsymbol{r}\sigma\sigma')\phi_i(\boldsymbol{r}\sigma'\mathrm{n}) = e_i\phi_i(\boldsymbol{r}\sigma\mathrm{n}), \qquad i = 1, \ldots, N, \tag{3.51a}$$

$$\sum_{\sigma'} h_{\mathrm{p}}(\boldsymbol{r}\sigma\sigma')\phi_i(\boldsymbol{r}\sigma'\mathrm{p}) = e_i\phi_i(\boldsymbol{r}\sigma\mathrm{p}), \qquad i = N+1, \ldots, A, \tag{3.51b}$$

with the neutron single-particle hamiltonian h_{n} and the proton single-particle hamiltonian h_{p} defined as

$$\begin{aligned} h_{\mathrm{n}}(\boldsymbol{r}\sigma\sigma') &= h(\boldsymbol{r}\sigma\mathrm{n}, \boldsymbol{r}\sigma'\mathrm{n}) \\ &= \left[-\frac{\hbar^2}{2m}\Delta + F_{00}(\boldsymbol{r}) + F_{10}(\boldsymbol{r})\right]\delta_{\sigma\sigma'} + [G_{00}(\boldsymbol{r}) + G_{10}(\boldsymbol{r})]\cdot\boldsymbol{\sigma}_{\sigma\sigma'}, \end{aligned} \tag{3.52a}$$

$$\begin{aligned} h_{\mathrm{p}}(\boldsymbol{r}\sigma\sigma') &= h(\boldsymbol{r}\sigma\mathrm{p}, \boldsymbol{r}\sigma'\mathrm{p}) \\ &= \left[-\frac{\hbar^2}{2m}\Delta + F_{00}(\boldsymbol{r}) - F_{10}(\boldsymbol{r})\right]\delta_{\sigma\sigma'} + [G_{00}(\boldsymbol{r}) - G_{10}(\boldsymbol{r})]\cdot\boldsymbol{\sigma}_{\sigma\sigma'}, \end{aligned} \tag{3.52b}$$

where the follwing relations are used:

$$\phi_i(\boldsymbol{r}\sigma\mathrm{p}) = 0, \qquad i = 1, \ldots, N, \tag{3.53a}$$

$$\phi_i(\boldsymbol{r}\sigma\mathrm{n}) = 0, \qquad i = N+1, \ldots, A, \tag{3.53b}$$

$$h(\boldsymbol{r}\sigma\mathrm{n}, \boldsymbol{r}\sigma'\mathrm{p}) = h(\boldsymbol{r}\sigma\mathrm{p}, \boldsymbol{r}\sigma'\mathrm{n}) = 0 \tag{3.53c}$$

3.3.3 PRA equation

Putting Eqs. (3.53) into Eq. (2.35), the projection operator removing the occupied states is given as

$$\begin{aligned} P(\boldsymbol{r}\sigma\tau, \boldsymbol{r}'\sigma'\tau') = \delta(\boldsymbol{r} - \boldsymbol{r}')\delta_{\sigma\sigma'}\delta_{\tau\tau'} &- \delta_{\tau\mathrm{n}}\delta_{\tau'\mathrm{n}}\sum_{i=1}^{N}\phi_i(\boldsymbol{r}\sigma\mathrm{n})\phi_i^*(\boldsymbol{r}'\sigma'\mathrm{n}) \\ &- \delta_{\tau\mathrm{p}}\delta_{\tau'\mathrm{p}}\sum_{i=N+1}^{A}\phi_i(\boldsymbol{r}\sigma\mathrm{p})\phi_i^*(\boldsymbol{r}'\sigma'\mathrm{p}). \end{aligned} \tag{3.54}$$

Then, the projection operators removing occupied states for the neutron and proton are defined as

$$
\begin{aligned}
P_{\rm n}(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma') &\equiv P(\boldsymbol{r}\sigma{\rm n},\boldsymbol{r}'\sigma'{\rm n}) \\
&= \delta(\boldsymbol{r}-\boldsymbol{r}')\delta_{\sigma\sigma'} - \sum_{i=1}^{N}\phi_i(\boldsymbol{r}\sigma{\rm n})\phi_i^*(\boldsymbol{r}'\sigma'{\rm n}), \qquad (3.55a)\\
P_{\rm p}(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma') &\equiv P(\boldsymbol{r}\sigma{\rm p},\boldsymbol{r}'\sigma'{\rm p}) \\
&= \delta(\boldsymbol{r}-\boldsymbol{r}')\delta_{\sigma\sigma'} - \sum_{i=N+1}^{A}\phi_i(\boldsymbol{r}\sigma{\rm p})\phi_i^*(\boldsymbol{r}'\sigma'{\rm p}). \qquad (3.55b)
\end{aligned}
$$

Putting Eqs. (3.53) and (3.55) into (3.37), the RPA equations are written down as follows: for the case of $i = 1, \ldots, N$,

$$
\begin{aligned}
\hbar\omega_\lambda\phi_i^{(\mp)\nu}(\boldsymbol{r}\sigma{\rm n}) &= \sum_{\sigma'}[h_{\rm n}(\boldsymbol{r}\sigma\sigma') - e_i\delta_{\sigma\sigma'}]\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma'{\rm n}) \\
&+ \sum_{\sigma'\sigma''}\int d\boldsymbol{r}' P_{\rm n}(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma')h^{(\pm)\lambda}(\boldsymbol{r}'\sigma'{\rm n},\boldsymbol{r}'\sigma''{\rm n})\phi_i(\boldsymbol{r}'\sigma''{\rm n}), \qquad (3.56a)\\
\hbar\omega_\lambda\phi_i^{(\mp)\nu}(\boldsymbol{r}\sigma{\rm p}) &= \sum_{\sigma'}[h_{\rm p}(\boldsymbol{r}\sigma\sigma') - e_i\delta_{\sigma\sigma'}]\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma'{\rm p}) \\
&+ \sum_{\sigma'\sigma''}\int d\boldsymbol{r}' P_{\rm p}(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma')h^{(\pm)\lambda}(\boldsymbol{r}'\sigma'{\rm p},\boldsymbol{r}'\sigma''{\rm n})\phi_i(\boldsymbol{r}'\sigma''{\rm n}), \qquad (3.56b)
\end{aligned}
$$

and for the case of $i = N+1, \ldots, A$,

$$
\begin{aligned}
\hbar\omega_\lambda\phi_i^{(\mp)\nu}(\boldsymbol{r}\sigma{\rm n}) &= \sum_{\sigma'}[h_{\rm n}(\boldsymbol{r}\sigma\sigma') - e_i\delta_{\sigma\sigma'}]\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma'{\rm n}) \\
&+ \sum_{\sigma'\sigma''}\int d\boldsymbol{r}' P_{\rm n}(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma')h^{(\pm)\lambda}(\boldsymbol{r}'\sigma'{\rm n},\boldsymbol{r}'\sigma''{\rm p})\phi_i(\boldsymbol{r}'\sigma''{\rm p}), \qquad (3.56c)\\
\hbar\omega_\lambda\phi_i^{(\mp)\nu}(\boldsymbol{r}\sigma{\rm p}) &= \sum_{\sigma'}[h_{\rm p}(\boldsymbol{r}\sigma\sigma') - e_i\delta_{\sigma\sigma'}]\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma'{\rm p}) \\
&+ \sum_{\sigma'\sigma''}\int d\boldsymbol{r}' P_{\rm p}(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma')h^{(\pm)\lambda}(\boldsymbol{r}'\sigma'{\rm p},\boldsymbol{r}'\sigma''{\rm p})\phi_i(\boldsymbol{r}'\sigma''{\rm p}). \qquad (3.56d)
\end{aligned}
$$

If we select the single-particle wave functions as

$$\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{n}) \neq 0, \qquad \phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{p}) = 0, \qquad i = 1, \ldots, N, \tag{3.57a}$$

$$\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{n}) = 0, \qquad \phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{p}) \neq 0, \qquad i = N+1, \ldots, A, \tag{3.57b}$$

then the wave functions $\phi_i^{(\pm)\lambda}$ is the solution of the excitation without charge-exchange. In this case, the RPA equation is given as

$$\begin{aligned}\hbar\omega_\lambda \phi_i^{(\mp)\nu}(\boldsymbol{r}\sigma\mathrm{n}) = &\sum_{\sigma'}[h_\mathrm{n}(\boldsymbol{r}\sigma\sigma') - e_i\delta_{\sigma\sigma'}]\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma'\mathrm{n}) \\ &+ \sum_{\sigma'\sigma''}\int \mathrm{d}\boldsymbol{r}' P_\mathrm{n}(\boldsymbol{r}\sigma, \boldsymbol{r}'\sigma') h_\mathrm{n}^{(\pm)\lambda}(\boldsymbol{r}'\sigma'\sigma'')\phi_i(\boldsymbol{r}'\sigma''\mathrm{n}), \\ &\qquad i = 1, \ldots, N,\end{aligned} \tag{3.58a}$$

$$\begin{aligned}\hbar\omega_\lambda \phi_i^{(\mp)\nu}(\boldsymbol{r}\sigma\mathrm{p}) = &\sum_{\sigma'}[h_\mathrm{p}(\boldsymbol{r}\sigma\sigma') - e_i\delta_{\sigma\sigma'}]\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma'\mathrm{p}) \\ &+ \sum_{\sigma'\sigma''}\int \mathrm{d}\boldsymbol{r}' P_\mathrm{p}(\boldsymbol{r}\sigma, \boldsymbol{r}'\sigma') h_\mathrm{p}^{(\pm)\lambda}(\boldsymbol{r}'\sigma'\sigma'')\phi_i(\boldsymbol{r}'\sigma''\mathrm{p}), \\ &\qquad i = N+1, \ldots, A,\end{aligned} \tag{3.58b}$$

with the transition hamiltonian hamiltonians for neutron and proton defined as

$$\begin{aligned}&h_\mathrm{n}^{(\pm)\lambda}(\boldsymbol{r}\sigma\sigma') \equiv h^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{n}, \boldsymbol{r}\sigma'\mathrm{n}) \\ &= \left[F_{00}^{(\pm)\lambda}(\boldsymbol{r}) + F_{10}^{(\pm)\lambda}(\boldsymbol{r})\right]\delta_{\sigma\sigma'} + \left[\boldsymbol{G}_{00}^{(\pm)\lambda}(\boldsymbol{r}) + \boldsymbol{G}_{10}^{(\pm)\lambda}(\boldsymbol{r})\right]\cdot \boldsymbol{\sigma}_{\sigma\sigma'},\end{aligned} \tag{3.59a}$$

$$\begin{aligned}&h_\mathrm{p}^{(\pm)\lambda}(\boldsymbol{r}\sigma\sigma') \equiv h^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{p}, \boldsymbol{r}\sigma'\mathrm{p}) \\ &= \left[F_{00}^{(\pm)\lambda}(\boldsymbol{r}) - F_{10}^{(\pm)\lambda}(\boldsymbol{r})\right]\delta_{\sigma\sigma'} + \left[\boldsymbol{G}_{00}^{(\pm)\lambda}(\boldsymbol{r}) - \boldsymbol{G}_{10}^{(\pm)\lambda}(\boldsymbol{r})\right]\cdot \boldsymbol{\sigma}_{\sigma\sigma'}.\end{aligned} \tag{3.59b}$$

While, if we select the single-particle wave functions as

$$\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{n}) = 0, \qquad \phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{p}) \neq 0, \qquad i = 1, \ldots, N, \tag{3.60a}$$

$$\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{n}) \neq 0, \qquad \phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{p}) = 0, \qquad i = N+1, \ldots, A, \tag{3.60b}$$

then the wave functions $\phi_i^{(\pm)\lambda}$ is the solution of the charge-exchange excitation. In this case, the RPA equation is given as

$$\hbar\omega_\lambda \phi_i^{(\mp)\nu}(\boldsymbol{r}\sigma\mathrm{p}) = \sum_{\sigma'}[h_\mathrm{p}(\boldsymbol{r}\sigma\sigma') - e_i\delta_{\sigma\sigma'}]\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma'\mathrm{p})$$
$$+ \sum_{\sigma'\sigma''}\int \mathrm{d}\boldsymbol{r}' P_\mathrm{p}(\boldsymbol{r}\sigma, \boldsymbol{r}'\sigma') h^{(\pm)\lambda}(\boldsymbol{r}'\sigma'\mathrm{p}, \boldsymbol{r}'\sigma''\mathrm{n})\phi_i(\boldsymbol{r}'\sigma''\mathrm{n}),$$
$$i = 1, \ldots, N, \tag{3.61a}$$

$$\hbar\omega_\lambda \phi_i^{(\mp)\nu}(\boldsymbol{r}\sigma\mathrm{n}) = \sum_{\sigma'}[h_\mathrm{n}(\boldsymbol{r}\sigma\sigma') - e_i\delta_{\sigma\sigma'}]\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma'\mathrm{n})$$
$$+ \sum_{\sigma'\sigma''}\int \mathrm{d}\boldsymbol{r}' P_\mathrm{n}(\boldsymbol{r}\sigma, \boldsymbol{r}'\sigma') h^{(\pm)\lambda}(\boldsymbol{r}'\sigma'\mathrm{n}, \boldsymbol{r}'\sigma''\mathrm{p})\phi_i(\boldsymbol{r}'\sigma''\mathrm{p}),$$
$$i = N+1, \ldots, A. \tag{3.61b}$$

In this paper, we only deal with the excitations without charge-exchange.

3.4 Case of Even-Even nuclei

In the case of the even-even nuclei, the single-particle wave functions of HF ground state are composed by the time reversal pair of the single-particle wave function $\{\phi_i, \phi_{\bar{i}}; i = 1, \ldots, A/2\}$:

$$\phi_i(\boldsymbol{r}\sigma\mathrm{n}) \neq 0, \qquad \phi_i(\boldsymbol{r}\sigma\mathrm{p}) = 0, \qquad i = 1, \ldots, \tfrac{N}{2}, \tag{3.62a}$$
$$\phi_i(\boldsymbol{r}\sigma\mathrm{n}) = 0, \qquad \phi_i(\boldsymbol{r}\sigma\mathrm{p}) \neq 0, \qquad i = \tfrac{N}{2}+1, \ldots, \tfrac{A}{2}, \tag{3.62b}$$
$$\phi_i(\boldsymbol{r}\sigma\tau) = \mathcal{T}\phi_{i-A/2}(\boldsymbol{r}\sigma\tau), \qquad i = \tfrac{A}{2}+1, \ldots, \tfrac{A}{2}+\tfrac{N}{2}, \tag{3.62c}$$
$$\phi_i(\boldsymbol{r}\sigma\tau) = \mathcal{T}\phi_{i-A/2-N/2}(\boldsymbol{r}\sigma\tau), \qquad i = \tfrac{A}{2}+\tfrac{N}{2}+1, \ldots, A, \tag{3.62d}$$

where operator $\mathcal{T}$ is time-reversal operator. Then the local density, kinetic density and spin-orbit tensor for HF ground state, which are time-even components, are given as

$$\rho_q(\boldsymbol{r}) = 2\sum_{i=1}^{\frac{A}{2}}\sum_{\sigma}\phi_i^*(\boldsymbol{r}\sigma q)\phi_i(\boldsymbol{r}\sigma q), \tag{3.63a}$$

$$\tau_q(\boldsymbol{r}) = 2\sum_{i=1}^{\frac{A}{2}}\sum_{\sigma}\boldsymbol{\nabla}\phi_i^*(\boldsymbol{r}\sigma q)\cdot\boldsymbol{\nabla}\phi_i(\boldsymbol{r}\sigma q), \tag{3.63b}$$

$$J_{q,\mu\nu}(\boldsymbol{r}) = \frac{1}{i}\sum_{i=1}^{\frac{A}{2}}\sum_{\sigma\sigma'}\left[\phi_i^*(\boldsymbol{r}\sigma q)\nabla_\mu\phi_i(\boldsymbol{r}\sigma' q) - \nabla\mu\phi_i^*(\boldsymbol{r}\sigma q)\phi_i(\boldsymbol{r}\sigma' q)\right]\sigma_{\nu,\sigma\sigma'}. \tag{3.63c}$$

The spin density $\boldsymbol{s}_q$, spin kinetic density $\boldsymbol{T}_q$ and current $\boldsymbol{j}_q$, which are time-odd components, are vanishing (see Eq (3.25a)).

Furthermore, the wave functions $\phi_i^{(\pm)\lambda}$ can be written as (see Eq. (2.99))

$$\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{n}) \neq 0, \qquad \phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{p}) = 0, \qquad i = 1,\ldots,\tfrac{N}{2}, \tag{3.64a}$$

$$\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{n}) = 0, \qquad \phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\mathrm{p}) \neq 0, \qquad i = \tfrac{N}{2}+1,\ldots,\tfrac{A}{2}, \tag{3.64b}$$

$$\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\tau) = \mathcal{T}\phi_{i-A/2}^{(\pm)\lambda}(\boldsymbol{r}\sigma\tau), \qquad i = \tfrac{A}{2}+1,\ldots,\tfrac{A}{2}+\tfrac{N}{2}, \tag{3.64c}$$

$$\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma\tau) = \mathcal{T}\phi_{i-A/2-N/2}^{(\pm)\lambda}(\boldsymbol{r}\sigma\tau), \qquad i = \tfrac{A}{2}+\tfrac{N}{2}+1,\ldots,A. \tag{3.64d}$$

Then, transition densities and currents used in RPA calculation are given as

$$\rho_q^{(+)\nu}(\boldsymbol{r}) = 4\sum_{i=1}^{A/2}\sum_{\sigma}\mathrm{Re}\left[\phi_i^*(\boldsymbol{r}\sigma q)\phi_i^{(+)\nu}(\boldsymbol{r}\sigma q)\right] \tag{3.65a}$$

$$\tau_q^{(+)\nu}(\boldsymbol{r}) = 4\sum_{i=1}^{A/2}\sum_{\sigma}\mathrm{Re}\left[\boldsymbol{\nabla}\phi_i^*(\boldsymbol{r}\sigma q)\cdot\boldsymbol{\nabla}\phi_i^{(+)\nu}(\boldsymbol{r}\sigma q)\right] \tag{3.65b}$$

$$\begin{aligned} J_{00,\mu\nu}^{(+)\nu}(\boldsymbol{r}) = 2\sum_{i=1}^{A/2}\sum_{\sigma\sigma'}\mathrm{Im}\Big[\Big\{&\phi_i^*(\boldsymbol{r}\sigma q)\nabla_\mu\phi_i^{(+)\nu}(\boldsymbol{r}\sigma'\tau) \\ &-\nabla_\mu\phi_i^*(\boldsymbol{r}\sigma q)\phi_i^{(+)\nu}(\boldsymbol{r}\sigma'\tau)\Big\}\sigma_{\nu,\sigma\sigma'}\Big] \end{aligned} \tag{3.65c}$$

$$\boldsymbol{s}_q^{(-)\nu}(\boldsymbol{r}) = 4i\sum_{i=1}^{A/2}\sum_{\sigma\sigma'}\mathrm{Im}\left[\phi_i^*(\boldsymbol{r}\sigma q)\phi_i^{(-)\nu}(\boldsymbol{r}\sigma'\tau)\boldsymbol{\sigma}_{\sigma\sigma'}\right] \tag{3.65d}$$

$$\boldsymbol{T}_q^{(-)\nu}(\boldsymbol{r}) = 4i\sum_{i=1}^{A/2}\sum_{\sigma\sigma'}\mathrm{Im}\left[\boldsymbol{\nabla}\phi_i^*(\boldsymbol{r}\sigma q)\cdot\boldsymbol{\nabla}\phi_i^{(-)\nu}(\boldsymbol{r}\sigma'\tau)\boldsymbol{\sigma}_{\sigma\sigma'}\right] \tag{3.65e}$$

$$\begin{aligned} \boldsymbol{j}_q^{(-)\nu}(\boldsymbol{r}) = -2i\sum_{i=1}^{A/2}\sum_{\sigma}\mathrm{Re}\Big[&\phi_i^*(\boldsymbol{r}\sigma q)\boldsymbol{\nabla}\phi_i^{(-)\nu}(\boldsymbol{r}\sigma q) \\ &-\boldsymbol{\nabla}\phi_i^*(\boldsymbol{r}\sigma q)\phi_i^{(-)\nu}(\boldsymbol{r}\sigma q)\Big] \end{aligned} \tag{3.65f}$$

The densities and currents $\boldsymbol{s}_{tt_3}^{(+)}$, $\boldsymbol{T}_{tt_3}^{(+)}$, $\boldsymbol{j}_{tt_3}^{(+)}$, $\rho_{tt_3}^{(-)}$, $\tau_{tt_3}^{(-)}$, $J_{tt_3,\mu\nu}^{(-)}$ are vanishing (see Eq (3.25b) and Eq (3.25c)).

The transition Coulomb potential in Eq. (3.41) is given by

$$\Phi_{\mathrm{C}}^{(+)\lambda}(\boldsymbol{r}) = e\int \mathrm{d}\boldsymbol{r}'\frac{\rho_{\mathrm{P}}^{(+)\lambda}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|} \tag{3.66}$$

3.4.1 Skyrme energy functional

For even-even nuclei, because time-odd properties of densities are vanishing, the time-odd part of the Skyrme energy functional does not contribute to the total binding energy. Then, the total energy is given by

$$E = \int \mathrm{d}\boldsymbol{r}\,\mathcal{E}^{\mathrm{even}} - E_{\mathrm{corr}}, \tag{3.67}$$

with the energy functional $\mathcal{E}$ is defined as

$$\mathcal{E}^{\mathrm{even}} = \mathcal{E}_{\mathrm{kin}} + \mathcal{E}_{\mathrm{Sk}}^{\mathrm{even}} + \mathcal{E}_{\mathrm{Coul}}. \tag{3.68}$$

Thus, the single-particle hamiltonian in the Hartree-Fock equation is derived from the first derivatives of the time-even part of the total energy functional, $\mathcal{E}^{\mathrm{even}}$, with respect to the local densities in Eq. (3.63). However, the transition hamiltonian in the RPA equation is not derived from the second derivatives of the time-even part of total energy functional, $\mathcal{E}^{\mathrm{even}}$, but derived from the second derivatives of the total energy functional $\mathcal{E}$ in Eq. 3.45 with respect to the local densities in Eq. (3.44).

3.4.2 HF equation

In this subsubsection, we give the HF equation used in the actual numerical calculation.

The HF equation (3.51) is change into that for the single-particle wave function of Eq. (3.62):

$$\sum_{\sigma'} h_{\mathrm{n}}(\boldsymbol{r}\sigma\sigma')\phi_i(\boldsymbol{r}\sigma'\mathrm{n}) = e_i\phi_i(\boldsymbol{r}\sigma\mathrm{n}), \qquad i = 1, \ldots, \tfrac{N}{2} \tag{3.69a}$$

$$\sum_{\sigma'} h_{\mathrm{p}}(\boldsymbol{r}\sigma\sigma')\phi_i(\boldsymbol{r}\sigma'\mathrm{p}) = e_i\phi_i(\boldsymbol{r}\sigma\mathrm{p}), \qquad i = \tfrac{N}{2}+1, \ldots, \tfrac{A}{2}, \tag{3.69b}$$

with the single-particle hamiltonian defined as

$$\begin{aligned} h_q(\boldsymbol{r}\sigma\sigma') = {} & \left[-\frac{\hbar^2}{2m}\Delta - \boldsymbol{\nabla}\cdot[M_q(\boldsymbol{r})\boldsymbol{\nabla}] + U_q(\boldsymbol{r}) + U_C(\boldsymbol{r})\delta_{q\mathrm{p}}\right]\delta_{\sigma\sigma'} \\ & + \frac{1}{2i}\left[\nabla_\mu B_{q,\mu\nu}(\boldsymbol{r}) + B_{q,\mu\nu}(\boldsymbol{r})\nabla_\mu\right]\sigma_{\nu,\sigma\sigma'}, \end{aligned} \tag{3.70}$$

where various potentials are given as

$$M_q = C^{\tau}_{\mathrm{tot}}\rho + C^{\tau}_{\mathrm{sum}}\rho_q, \tag{3.71a}$$

$$\begin{aligned} U_q = {} & 2C^{\rho}_{\mathrm{tot}}\rho + 2C^{\rho}_{\mathrm{sum}}\rho_q + 2C^{\Delta\rho}_{\mathrm{tot}}\Delta\rho + 2C^{\Delta\rho}_{\mathrm{sum}}\Delta\rho_q + C^{\tau}_{\mathrm{tot}}\tau + C^{\tau}_{\mathrm{sum}}\tau_q \\ & + C^{\nabla J}_{\mathrm{tot}}\boldsymbol{\nabla}\cdot\boldsymbol{J} + C^{\nabla J}_{\mathrm{sum}}\boldsymbol{\nabla}\cdot\boldsymbol{J}_q + \frac{\partial C^{\rho}_{\mathrm{tot}}}{\partial\rho}\rho^2 + \frac{\partial C^{\rho}_{\mathrm{sum}}}{\partial\rho}(\rho_{\mathrm{n}}^2 + \rho_{\mathrm{p}}^2), \end{aligned} \tag{3.71b}$$

$$B_{q,\mu\nu} = -2C^{T}_{\mathrm{tot}}J_{\mu\nu} - 2C^{T}_{\mathrm{sum}}J_{q,\mu\nu} - 2\sum_{\omega}\epsilon_{\mu\nu\omega}\left[C^{\nabla J}_{\mathrm{tot}}\nabla_\omega\rho + C^{\nabla J}_{\mathrm{sum}}\nabla_\omega\rho_q\right], \tag{3.71c}$$

$$U_C = e\Phi_C - e^2\left(\frac{3}{\pi}\right)^{1/3}\rho_{\mathrm{p}}^{1/3}. \tag{3.71d}$$

3.4.3 RPA equation

In this subsubsection, we give the HF equation used in the actual numerical calculation.

The projection operators removing occupied states for the neutron and proton are defined as

$$
\begin{aligned}
P_{\mathrm{n}}(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma') &= \delta(\boldsymbol{r}-\boldsymbol{r}')\delta_{\sigma\sigma'} \\
&\quad - \sum_{i=1}^{N/2} \phi_i(\boldsymbol{r}\sigma\mathrm{n})\phi_i^*(\boldsymbol{r}'\sigma'\mathrm{n}) + \phi_{\bar{i}}(\boldsymbol{r}\sigma\mathrm{n})\phi_{\bar{i}}^*(\boldsymbol{r}'\sigma'\mathrm{n}), \qquad (3.72\mathrm{a}) \\
P_{\mathrm{p}}(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma') &= \delta(\boldsymbol{r}-\boldsymbol{r}')\delta_{\sigma\sigma'} \\
&\quad - \sum_{i=N/2+1}^{A/2} \phi_i(\boldsymbol{r}\sigma\mathrm{p})\phi_i^*(\boldsymbol{r}'\sigma'\mathrm{p}) + \phi_{\bar{i}}(\boldsymbol{r}\sigma\mathrm{p})\phi_{\bar{i}}^*(\boldsymbol{r}'\sigma'\mathrm{p}). \qquad (3.72\mathrm{b})
\end{aligned}
$$

The RPA equation (3.58) is change into that for the wave function of Eq. (3.64):

$$
\begin{aligned}
\hbar\omega_\lambda \phi_i^{(\mp)\nu}(\boldsymbol{r}\sigma\mathrm{n}) &= \sum_{\sigma'} [h_{\mathrm{n}}(\boldsymbol{r}\sigma\sigma') - e_i\delta_{\sigma\sigma'}]\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma'\mathrm{n}) \\
&\quad + \sum_{\sigma'\sigma''} \int d\boldsymbol{r}' P_{\mathrm{n}}(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma') h_{\mathrm{n}}^{(\pm)\lambda}(\boldsymbol{r}'\sigma'\sigma'')\phi_i(\boldsymbol{r}'\sigma''\mathrm{n}), \\
&\qquad\qquad i = 1,\ldots,\tfrac{N}{2}, \qquad (3.73\mathrm{a}) \\
\hbar\omega_\lambda \phi_i^{(\mp)\nu}(\boldsymbol{r}\sigma\mathrm{p}) &= \sum_{\sigma'} [h_{\mathrm{p}}(\boldsymbol{r}\sigma\sigma') - e_i\delta_{\sigma\sigma'}]\phi_i^{(\pm)\lambda}(\boldsymbol{r}\sigma'\mathrm{p}) \\
&\quad + \sum_{\sigma'\sigma''} \int d\boldsymbol{r}' P_{\mathrm{p}}(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma') h_{\mathrm{p}}^{(\pm)\lambda}(\boldsymbol{r}'\sigma'\sigma'')\phi_i(\boldsymbol{r}'\sigma''\mathrm{p}), \\
&\qquad\qquad i = \tfrac{N}{2}+1,\ldots,\tfrac{A}{2}. \qquad (3.73\mathrm{b})
\end{aligned}
$$

with the transition hamiltonian defined as

$$
\begin{aligned}
h_q^{(+)\lambda}(\boldsymbol{r}\sigma\sigma') &= \left[-\boldsymbol{\nabla}\cdot[M_q^{(+)\lambda}(\boldsymbol{r})\boldsymbol{\nabla}] + U_q^{(+)\lambda}(\boldsymbol{r}) + U_{\mathrm{C}}^{(+)\lambda}(\boldsymbol{r})\delta_{q\mathrm{p}}\right]\delta_{\sigma\sigma'} \\
&\quad + \frac{1}{2i}\sum_{\mu\nu}\left[\nabla_\mu B_{q,\mu\nu}^{(+)\lambda}(\boldsymbol{r}) + B_{q,\mu\nu}^{(+)\lambda}(\boldsymbol{r})\nabla_\mu\right]\sigma_{\nu,\sigma\sigma'} \qquad (3.74) \\
h_q^{(-)\lambda}(\boldsymbol{r}\sigma\sigma') &= \sum_\nu \left[-\boldsymbol{\nabla}\cdot[C_{q,\nu}^{(-)\lambda}(\boldsymbol{r})\boldsymbol{\nabla}] + \Sigma_{q,\nu}^{(-)\lambda}(\boldsymbol{r})\right]\sigma_{\nu,\sigma\sigma'} \\
&\quad + \frac{1}{2i}\left[\boldsymbol{\nabla}\cdot\boldsymbol{I}_q^{(-)\lambda}(\boldsymbol{r}) + \boldsymbol{I}_q^{(-)\lambda}(\boldsymbol{r})\cdot\boldsymbol{\nabla}\right]\delta_{\sigma\sigma'}, \qquad (3.75)
\end{aligned}
$$

where various potentials are given as

$$
\begin{aligned}
M_q^{(+)\lambda} &= C_{\text{tot}}^{\tau}\rho^{(+)\lambda} + C_{\text{sum}}^{\tau}\rho_q^{(+)\lambda}, &(3.76\text{a})\\
U_q^{(+)\lambda} &= 2C_{\text{tot}}^{\rho}\rho^{(+)\lambda} + 2C_{\text{sum}}^{\rho}\rho_q^{(+)\lambda} + 2C_{\text{tot}}^{\Delta\rho}\Delta\rho^{(+)\lambda} + 2C_{\text{sum}}^{\Delta\rho}\Delta\rho_q^{(+)\lambda}\\
&\quad + C_{\text{tot}}^{\tau}\tau^{(+)\lambda} + C_{\text{sum}}^{\tau}\tau_q^{(+)\lambda} + C_{\text{tot}}^{\nabla J}\boldsymbol{\nabla}\cdot\boldsymbol{J}^{(+)\lambda} + C_{\text{sum}}^{\nabla J}\boldsymbol{\nabla}\cdot\boldsymbol{J}_q^{(+)\lambda}\\
&\quad + 2\frac{\partial C_{\text{tot}}^{\rho}}{\partial\rho}\rho\rho^{(+)\lambda} + 2\frac{\partial C_{\text{sum}}^{\rho}}{\partial\rho}\rho_q\rho^{(+)\lambda}\\
&\quad + 2\frac{\partial C_{\text{tot}}^{\rho}}{\partial\rho}\rho\rho^{(+)\lambda} + 2\frac{\partial C_{\text{sum}}^{\rho}}{\partial\rho}(\rho_{\text{n}}\rho_{\text{n}}^{(+)\lambda} + \rho_{\text{p}}\rho_{\text{p}}^{(+)\lambda})\\
&\quad + 2\frac{\partial^2 C_{\text{tot}}^{\rho}}{\partial\rho\partial\rho}\rho^2\rho^{(+)\lambda} + 2\frac{\partial^2 C_{\text{sum}}^{\rho}}{\partial\rho\partial\rho}(\rho_{\text{n}} + \rho_{\text{p}})^2\rho^{(+)\lambda}, &(3.76\text{b})\\
B_{q,\mu\nu}^{(+)\lambda} &= -2C_{\text{tot}}^{\tau}J_{\mu\nu}^{(+)\lambda} - 2C_{\text{sum}}^{\tau}J_{q,\mu\nu}^{(+)\lambda}\\
&\quad - 2C_{\text{tot}}^{\nabla J}\epsilon_{\mu\nu\omega}\nabla_\omega\rho^{(+)\lambda} - 2C_{\text{sum}}^{\nabla J}\epsilon_{\mu\nu\omega}\nabla_\omega\rho_q^{(+)\lambda}, &(3.76\text{c})\\
U_{\text{C}}^{(+)\lambda} &= \frac{e}{2}\Phi_{\text{C}}^{(+)\lambda} - \frac{e^2}{6}\left(\frac{3}{2\pi}\right)^{1/3}\rho_{\text{p}}^{-2/3}(\boldsymbol{r})\rho_{\text{p}}^{(+)\lambda}, &(3.76\text{d})\\
\boldsymbol{C}_{\text{sum}}^{(-)\lambda} &= C_{\text{tot}}^{T}\boldsymbol{s}^{(-)\lambda} + C_{\text{sum}}^{T}\boldsymbol{s}_q^{(-)\lambda}, &(3.76\text{e})\\
\boldsymbol{\Sigma}_q^{(-)\lambda} &= 2C_{\text{tot}}^{s}\boldsymbol{s}^{(-)\lambda} + 2C_{\text{sum}}^{s}\boldsymbol{s}_q^{(-)\lambda} + 2C_{\text{tot}}^{\Delta s}\Delta\boldsymbol{s}^{(-)\lambda} + 2C_{\text{sum}}^{\Delta s}\Delta\boldsymbol{s}_q^{(-)\lambda}\\
&\quad + C_{\text{tot}}^{T}\boldsymbol{T}^{(-)\lambda} + C_{\text{sum}}^{T}\boldsymbol{T}_q^{(-)\lambda}\\
&\quad + C_{\text{tot}}^{\nabla J}\boldsymbol{\nabla}\times\boldsymbol{j}^{(-)\lambda} + C_{\text{sum}}^{\nabla J}\boldsymbol{\nabla}\times\boldsymbol{j}_q^{(-)\lambda}, &(3.76\text{f})\\
\boldsymbol{I}_q^{(-)\lambda} &= -2C_{\text{tot}}^{T}\boldsymbol{j}_q^{(-)\lambda} - 2C_{\text{sum}}^{T}\boldsymbol{j}_q^{(-)\lambda}\\
&\quad + C_{\text{tot}}^{\nabla j}\boldsymbol{\nabla}\times\boldsymbol{s}_q^{(-)\lambda} + C_{\text{sum}}^{\nabla j}\boldsymbol{\nabla}\times\boldsymbol{s}_q^{(-)\lambda}. &(3.76\text{g})
\end{aligned}
$$