

Chapter 1

General Introduction

1.1 Introduction

Studies of liquid helium have been a rich source of ideas and inspirations for physics for a long time. Since bulk liquid helium is an isotropic and uniform system free from randomness, it is an ideal model system of theoretical condensed matter physics. The ideas that were clarified through studies of helium have had a great influence on various fields in condensed matter physics.

In 1908, Kamerlingh Onnes succeeded in liquefying ^4He gas at an extremely low temperature, $T = 4.2\text{K}$. In 1911, he discovered the phenomenon of superconductivity — the resistance of mercury suddenly became zero at $T = 4.2\text{K}$ — by using liquid ^4He to cool mercury. This became a breakthrough of low temperature physics. One then searched for a sign of the solidification of liquid ^4He at lower temperatures, but a sign of the solidification was not observed. Instead some anomalies were discovered around $T = 2.2\text{K}$. In 1932, Keeson, a coworker of Kamerlingh Onnes, observed a peak in specific heat around $T = 2.2\text{K}$. Ehrenfest called it λ transition afterwards. Viscosity of liquid ^4He was measured by several methods and was found to become zero below the λ transition. Kapitza first called this fluid the superfluid (1938).

On the other hand, ^3He , which is isotope of ^4He , rarely exists in nature compared with ^4He . A liquefaction of ^3He gas, which can be provided enormously by a nuclear reaction, succeeded in 1949. The specific heat of liquid ^3He was then measured at temperatures down to 1 K, but there was no anomaly and any transition was not observed. This fact implied that the quantum statistics is essentially important for the properties of matter at low temperatures. It implied that superfluidity of ^4He was a consequence of the bose condensation, and that liquid ^3He could not undergo a phase transition into a superfluid state as it is, because ^3He atoms follow the fermi statistics.

In 1957, the mechanism of superconductivity of electrons in a metal was clarified by Bardeen, Cooper and Schrieffer (BCS) [1]; electrons form Cooper pairs owing to an effective attractive interaction and one can consider the superconductivity to be caused by the Bose condensation of Cooper pairs. It is then natural to expect the same mechanism — the formation of Cooper pairs and their condensation — works in liquid ^3He and liquid ^3He can also exhibit a transition into a superfluid state as liquid ^4He does albeit at a much lower temperature. Indeed, a phase transition into a superfluid state in liquid ^3He was predicted shortly after the discovery of the BCS theory by extending the BCS theory to liquid ^3He [2]. Because of the hard core repulsion between ^3He atoms, the Cooper pairs in superfluid ^3He was predicted to be anisotropic; in ref. 2 it was predicted to be of the d -wave symmetry.

Superfluidity of liquid ^3He was discovered much later, because much progress in low temperature technology was needed to reach the low temperatures of the order of mK, where Osheroff *et al.* finally discovered superfluidity of ^3He in 1972. As had been expected, the Cooper pairs were found to be anisotropic, but of the p -wave symmetry in disagreement with the theoretical prediction [2]. It was clarified that strong ferromagnetic spin fluctuations were essential to cause the p -wave superfluidity in ^3He . After discovery, superfluidity of bulk liquid ^3He has been intensively studied and is now well understood. The lessons we can learn from the study of the superfluidity of bulk liquid ^3He are (1) the richness and the variety of the phenomena exhibited by an anisotropic superfluid, and, more importantly, (2) a possibility of superfluidity (or superconductivity) caused by many-body processes such as spin fluctuations. Those lessons have indeed helped to shape our understanding of exotic superconductors such as the heavy-fermion systems and the high- T_c superconductors, where anisotropic Cooper pairs are likely to be formed and the effect of the (antiferromagnetic) spin fluctuations is considered to be important.

The forefront of the research of liquid ^3He is now the study of liquid ^3He in restricted geometries, in particular, in two dimensions. This is partly motivated by recent discoveries of many interesting phenomena observed in two-dimensional (2D) fermion systems: the quantum Hall effect [3, 4], various phenomena in mesoscopic systems [5] and, in particular, high- T_c superconductivity [6], to name a few. These discoveries have shown that the physics in spatially two dimensions is unexpectedly rich and, at the same time, our understanding of it is unsatisfactory. It is then natural to expect that we can obtain a new insight from the study of 2D liquid ^3He as we have done from the study of bulk liquid ^3He .

The most interesting phenomenon in 2D liquid ^3He would be a transition into a superfluid state. The purpose of this thesis is to study the effective interaction between ^3He atoms that are adsorbed on a flat substrate and whose motions are restricted in two dimensions and to determine the symmetry of the Cooper pairs in

the possible superfluid state.

1.2 Review of experimental results of ^3He films on graphite

In this section we review important experimental results of ^3He films adsorbed on graphite. The system of ^3He adsorbed on graphite consists of multiple layers and one can change the areal density layer by layer. Helium atoms in the first layer are tightly bound to the graphite, but can freely move along the graphite surface. They thus make an ideal 2D fermion system. Similarly, ^3He atoms in the second layer can also be considered to be ideal 2D fermions, because ^3He atoms in the first layer are so tightly bound to the substrate that exchange between ^3He atoms in the second layer and those in the first layer hardly occurs. As ^3He atoms in the n th ($n \geq 3$) layer can be frequently exchanged with those in the $(n - 1)$ layer, however, they are not purely two-dimensional. In this thesis, we focus on the system of the monolayer of ^3He atoms.

The phase diagram of the first layer of ^3He on graphite was obtained by Greywall [7] and is shown in Fig. 1.1. In Fig. 1.1, F refers to a liquid state, R to commensurate solid phases and S to an incommensurate solid phase. Greywall determined the phase boundary between the liquid state and the solid phase by measuring the heat capacity. In the liquid state, the heat capacity varied linearly with temperature. When a solid phase mixed, additional contribution to the heat capacity was observed. From this observation he determined the phase boundary. A similar phase diagram was also obtained for the second layer.

It can be seen that the areal density of the liquid phase can be varied considerably. Helium atoms are in a liquid state for $\rho_{2D} \lesssim 0.045 \text{\AA}^{-2}$ and no liquid-gas separation was observed down to $\rho_{2D} \simeq 0.01 \text{\AA}^{-2}$. This is in marked contrast to bulk liquid ^3He , whose density can be hardly changed (See below.).

Greywall estimated the effective mass m_3^* from the heat capacity data (See Fig. 1.2.). In the dense region, the effective mass is strongly enhanced as in bulk liquid ^3He , where the mass enhancement $m_3^*/m_3 = 3 \simeq 5$ is observed depending on pressure. On the other hand, the mass enhancement vanishes in the dilute limit. This shows that one can change the strength of the correlation effect in 2D liquid ^3He by changing the areal density.

The magnetic susceptibility was measured by Lusher *et al.* [8]. They measured the magnetic susceptibility of monolayer liquid ^3He adsorbed on the surface of graphite preplated with a monolayer of solid ^4He at temperatures down to 6 mK. As can be seen from Fig. 1.3, the susceptibility is constant in a wide temperature range,

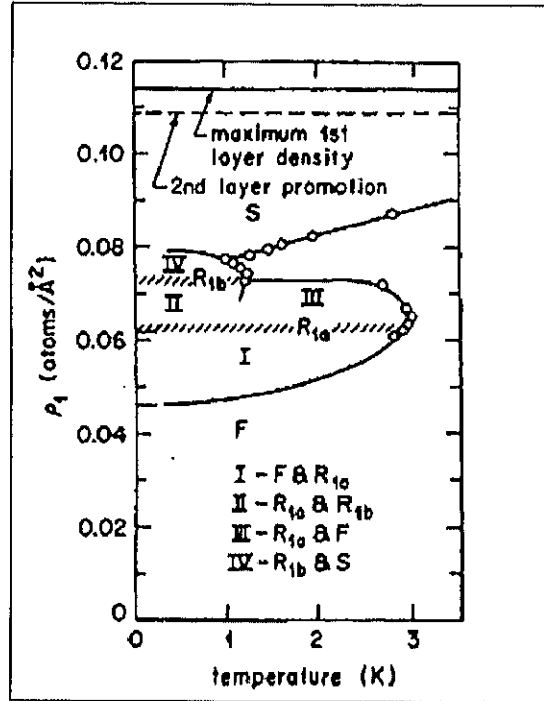


Figure 1.1: The first layer phase diagram proposed for ^3He on graphite. F refers to a liquid state, R to commensurate solid phases and S to an incommensurate solid phase. Here subscripts of R, a or b, stand for the structure of commensurate solid. ρ_1 is the first layer areal density. (After Greywall [7].).

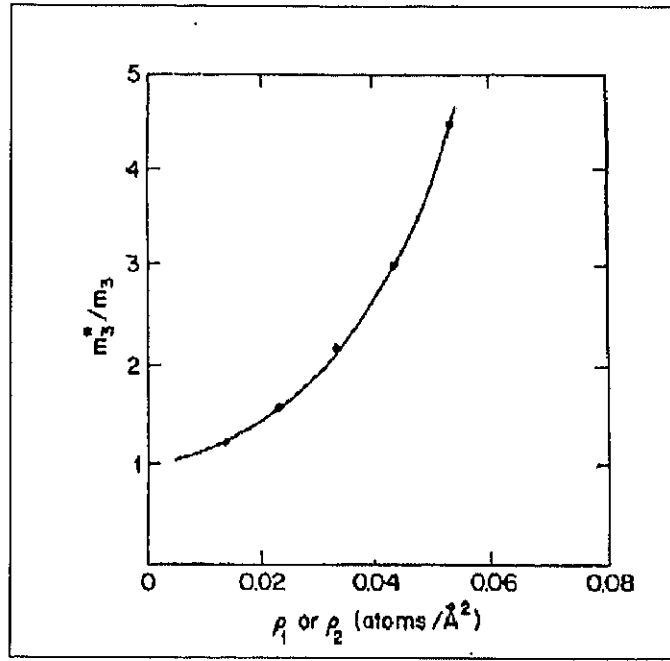


Figure 1.2: The effective mass m_3^* for the first and second layer fluids extracted from heat capacity data. Here m_3 is the atomic mass of ^3He . (After Greywall [7].).

which implies that the system is indeed a 2D fermi liquid state (Strictly speaking, the susceptibility should vary linearly with temperature at low temperatures because of the correlation effect [9]. However, the magnitude of the linear coefficient can be too small to be observed in experiments, in particular, in the dilute cases.). Moreover, the susceptibility is enhanced with the density as the effective mass is.

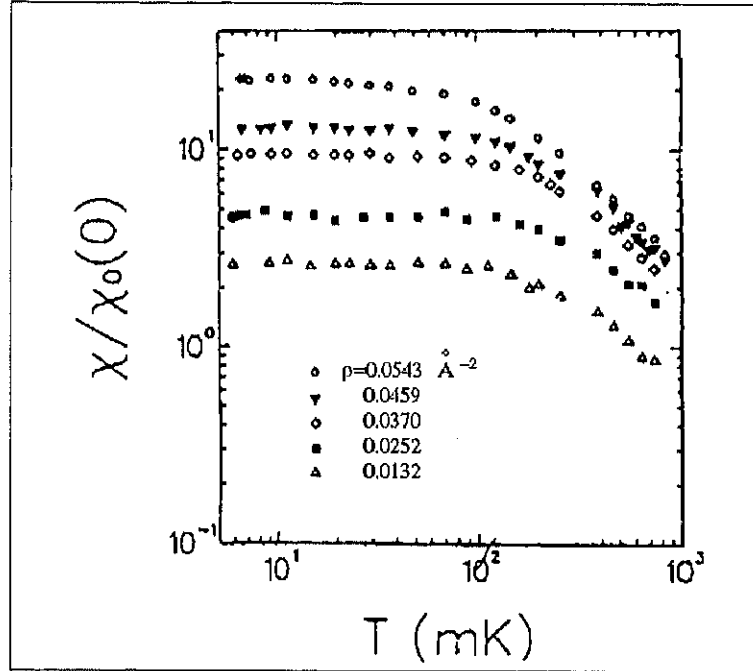


Figure 1.3: The susceptibility normalized by zero-temperature ideal-gas susceptibility as a function of temperature for the second layer at the areal densities of 0.0132 \AA^{-2} , 0.0252 \AA^{-2} , 0.0370 \AA^{-2} , 0.0459 \AA^{-2} , and 0.0543 \AA^{-2} . (After Lusher *et al.* [8].).

This means that spin fluctuations are enhanced in the dense region. In Fig. 1.4, the inverse of the uniform susceptibility is shown as a function of the interatomic spacing. For comparison, the data for bulk liquid ^3He is also shown. In the dense region, the enhancement of the susceptibility in 2D is approximately equal to that in

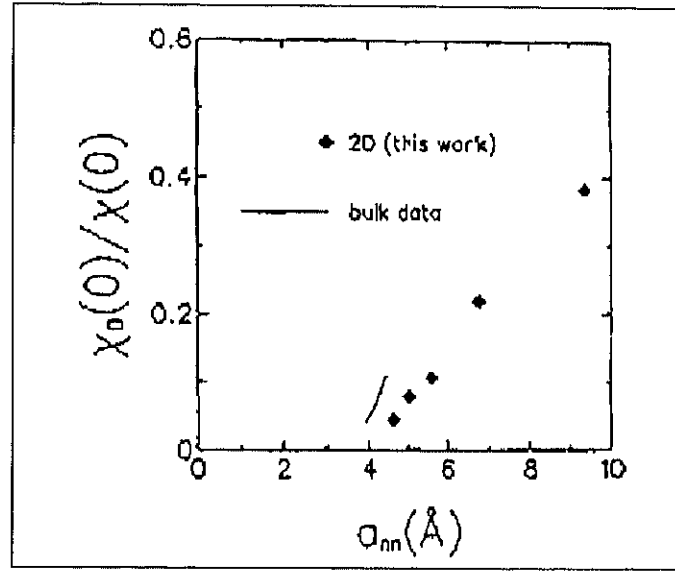


Figure 1.4: A comparison of inverse susceptibility enhancement as a function of interatomic separation a_{nn} in 2D and bulk. (After Lusher *et al.* [8].).

3D. On the other hand, as the density decreases, the enhancement diminishes. This shows that one can change the enhancement of spin fluctuations in 2D by changing the areal density. This is again in marked contrast to the case in 3D, where the density is hardly changed and the susceptibility is always strongly enhanced.

Lastly, we comment on the recent experimental search for superfluidity in 2D liquid ^3He on graphite. Greywall and Busch measured the heat capacity of liquid ^3He in the first and second layer on graphite down to 2 mK [10]. They observed an anomaly in the heat capacity independent of the areal density at 3.2 mK. They suggested that this might indicate a superfluid transition of 2D liquid ^3He . At present, however, their suggestion is not widely accepted. For example, Morishita *et al.* measured the heat capacity of ^3He films on graphite down to 100 μK [11], but did not find any anomaly. It is fair to say that superfluidity of 2D liquid ^3He has not been detected so far.

1.3 Theoretical approaches

There are generally two approaches to liquid ^3He : (A) a microscopic theory which starts from a microscopic Hamiltonian taking account of the interaction between ^3He atoms [2] and (B) a phenomenological theory which focuses on spin fluctuations [12, 13, 14, 15, 16]. An example of the first approach is the reaction (R-) matrix theory developed by Brueckner *et al.* [2]. They applied the R-matrix theory to 3D liquid ^3He . It is known that their results of the symmetry of the superfluid state turned out to be incorrect [17]. The main reason for the failure is that the effect of spin fluctuations, which is strongly enhanced and is considered to be the origin of the effective attractive interaction in 3D ^3He , were not taken into account in the R-matrix theory. On the other hand, in dilute 2D ^3He , spin fluctuations are not so strongly enhanced as in 3D ^3He [8] as was discussed in the previous section. Therefore, it is possible that the first approach, i.e., the microscopic approach, is valid for 2D ^3He at least in the dilute region. Spin fluctuations are also strongly enhanced in 2D as the density increases. In the dense region, therefore, the second approach, i.e., a paramagnon-type theory will be more suitable. In 2D, it is not a trivial problem what type of spin fluctuations, i.e., ferromagnetic or antiferromagnetic, develops as the correlation sets in, in contrast to 3D, where strong ferromagnetic spin fluctuations develop favoring a spin triplet pairing. In this thesis we adopt both approaches; they are complementary to each other.

Here, two remarks are in order. First, we concentrate on 2D liquid ^3He adsorbed on graphite (or on flat and solid substrate) in this thesis. Another interesting 2D liquid ^3He is formed on the surface of liquid ^4He [18]. In this system, the indirect

interaction mediated by ^4He atoms is important, and the present theory is only partly applicable. The second remark is that, by a two-dimensional system, we mean a system whose length z in the third direction is of the order of atomic scale. In fact, we consider a monolayer ^3He system in this thesis. If the length z of the third direction is smaller than the coherence length ξ of a superfluid state, the system can be considered to be two-dimensional in the superfluid state. However, if z is much larger than the atomic scale d , e.g., the interparticle distance, $d \ll z \lesssim \xi$, the normal phase is still considered to be three-dimensional, and the effective interaction may be calculated by assuming the system to be three-dimensional. In the thesis, the normal phase is also (strictly) two-dimensional, and therefore the effective interaction can be completely different from that in 3D.

This thesis is organized as follows. In Chapter 2, we apply the R-matrix theory to 2D liquid ^3He . In Section 2.1, we introduce the Lennard-Jones potential [19, 20] as the interaction potential between ^3He atoms and give the formalism for calculating the effective interaction using the R-matrix theory. Numerical results of the effective interaction and the superfluid transition temperature are given in Section 2.2. A summary of Chapter 2 is given in Section 2.3. In Chapter 3, we investigate the effective interaction mediated by spin fluctuations. We introduce the model and give the formalism for calculating the effective interaction in Section 3.1. Numerical results are given in Section 3.2. A summary of Chapter 3 is given in Section 3.3. Chapter 4 contains the concluding remarks.

References

- [1] J. Bardeen, L. N. Cooper and J. R. Schrieffer: *Phys. Rev.* **108** (1957) 1175.
- [2] K. A. Brueckner, T. Soda, P. W. Anderson, and P. Morel: *Phys. Rev.* **118** (1960) 1442.
- [3] *The Quantum Hall Effect* (Springer Verlag, New York, 1997) eds. R. E. Prange and S. M. Girvin.
- [4] *Perspectives in Quantum Hall Effects* (John Wiley & Sons, New York, 1998) eds. S. Das Sarma and A. Pinczuk.
- [5] Y. Imry: *Introduction to Mesoscopic Physics* (Oxford, New York, 1997)
- [6] J. G. Bednorz and K. A. Müller: *Z. Phys. B* **64** (1986) 189.
- [7] D. S. Greywall: *Phys. Rev. B* **41** (1990) 1842.
- [8] C. P. Lusher, B. P. Cowan and J. Saunders: *Phys. Rev. Lett.* **67** (1991) 2497.
- [9] D. S. Hirashima and H. Takahashi: *J. Phys. Soc. Jpn.* **67** (1998) 3816.
- [10] D. S. Greywall and P. A. Busch: *Phys. Rev. Lett* **65** (1990) 64.
- [11] M. Morishita, K. Ishida, Y. Yawata, H. Nagatani and H. Fukuyama: *J. Low. Temp. Phys.* **110** (1998) 351.
- [12] S. Nakajima: *Prog. Theor. Phys.* **50** (1973) 1101.
- [13] P. W. Anderson and W. F. Brinkman: *Phys. Rev. Lett.* **30** (1973) 1108.
- [14] W. F. Brinkman and P. W. Anderson: *Phys. Rev. A* **8** (1973) 2732.
- [15] W. F. Brinkman, J. W. Serene and P. W. Anderson: *Phys. Rev. A* **10** (1974) 2786.

- [16] Y. Kuroda: Prog. Theor. Phys. **51** (1974) 1269.
- [17] A. J. Leggett: Rev. Mod. Phys. **47** (1975) 331.
- [18] R. B. Hallock: Physics Today **39** (1998) 51.
- [19] J. De Boer and A. Michels: Physica **5** (1938) 945.
- [20] J. De Boer and L. J. Lunbeck: Physica **14** (1948) 510.