

## 5 Gradual Collusion with Asymmetric Information on R&D Costs

### 5.1 Introduction

Cooperation gradually deepens in many social relationships. In many cases, it is because each party gradually knows about the partners' nature as they continue to cooperate and gradually trust the partners unless they have defected. In other words, they may increase the cooperation level as each of them becomes more confident that the partners are willing to keep the cooperation. Datta (1993), Ghosh and Ray (1996), and Kranton (1996) formalize this idea. Common to their model is that if each party can change the partner (especially after a deviation), cooperation has to be gradual to make a defection, which leads to building a new relationship from scratch, unattractive. It is the existence of the outside option of changing the partner that makes the cooperation gradual.

Firms in an oligopolistic industry may also collude gradually when each firm is not sure if other firms are willing to keep collusion. We argue in this chapter that the maximal collusion process can exhibit gradualism when each

of two firms is not sure if the other firm is engaging in R&D for a drastic process innovation.<sup>34</sup> As time goes by without a success of R&D, each firm puts more belief on that the other firm has the high R&D cost so that it never engages in R&D. The two price-setting firms cannot reach full collusion from the beginning if the original belief that the other firm has the high R&D costs is low. In that case, collusion level must go up gradually as the belief is updated. Unlike in the models mentioned above, the firms do not have an option of changing the partner, yet their collusion process is gradual. The cause of gradualism in our model is not the necessity to make the punishment severe, but is the gradual “trust” building on the other firm's willingness to keep cooperation perpetually.

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<sup>34</sup> Fershtman and Gandal (1994) and Martin (1995) also consider the semi-collusion (with complete information unlike our model) where two firms compete in cost-reducing R&D while they collude in the product market. Their research agendas are different from ours, however. Fershtman and Gandal (1994) argue that the possibility of product market collusion induces the firms to overinvest in R&D so that the net profits for each firm can be smaller under the semi-collusion than pure competition. Martin (1995) shows that compared with joint R&D activities, the competition in R&D makes more difficult to sustain collusion in the product market.

## 5.2 The Model

We consider an industry in which two firms produce a homogeneous good with a constant unit cost. The two firms individually set the prices in each period of infinitely long discrete time. They discount the future with a common discount factor of  $\delta \in (0, 1)$ . Each firm may engage in R&D for a process innovation whose success is so drastic that it would drive the rival firm out of the market.<sup>35</sup> For simplicity, we assume a linear R&D technology in which the R&D intensity in period  $t$  is measured by  $\alpha_t \in [0, 1]$ . If a firm spend  $\alpha_t c$  plus a fixed cost in period  $t$ , it succeeds in R&D with the probability  $\alpha_t q$ , where  $0 < q < 1$ , in period  $t + 1$ . There are two types of the firm differing in the fixed cost of R&D. Each firm is either of high-cost type in which case the firm can never profitably engage in R&D, or of low-cost type in which case the fixed cost of R&D is normalized to zero. It is common knowledge that each firm is either of the high-cost type with the probability  $p_1$  or the low-cost type with the probability  $1 - p_1$ . However, individual firms' actual types are private knowledge.

At the head of every period but the first, the result of each firm's R&D

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<sup>35</sup> It is sufficient to assume that the success of R&D reduces the unit cost so much that the monopoly price associated with the lower unit cost is less than the original unit cost.

conducted in the last period is revealed to the public. If only one firm succeeds in R&D, the other firm exits from the market and the game is over. Otherwise, the two firms simultaneously set the prices for their individual products, obtaining one-shot profits from the sales. Then, each firm can engage in R&D as long as neither firm has succeeded by then.

We shall derive a symmetric semi-collusive, sequential equilibrium with the following properties. The two firms seek maximal, self-enforcing collusion in the product market, while they non-cooperatively make R&D decisions in every period. The product market collusion will last until either a firm defects or a firm succeeds in R&D. If the collusion ends with a defection, we assume that both firms play the Bertrand-Nash equilibrium in the product market thereafter. If only one firm succeeds in R&D, the successful firm will continue to obtain a monopoly profits with a lower unit cost in every following period, while the other firm must be driven out of the market. If both firms simultaneously succeed in R&D, we assume that they also engage in the Bertrand-Nash equilibrium perpetually.<sup>36</sup>

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<sup>36</sup> We make this assumption mainly for simplifying the analysis. However, it may be justified on the ground that R&D is a hostile activity to the rival firm.

### 5.3 R&D Intensity and Evolution of Beliefs

The levels of R&D conducted by the two firms and the collusion level in the product market are closely related to each other. Since a success in R&D enables the firm to capture high monopoly profits instead of current shared profits, the amount of each firm's R&D investment varies with the current profits. But the current profits depend on the collusion level, which is affected by the belief on the other firm's type. What is more, the belief on the other firm's type evolves with the R&D intensities of the two firms.

The evolution of the belief obeys the Bayes rule. Let  $p_s$  denote the probability that a firm believes the other firm is the high-cost type in period  $s$ . Hence we have  $p_{t+1} = p_t / [p_t + (1 - p_t)(1 - \alpha_t q)]$  if no firms have succeeded in period  $t + 1$ . Notice that the belief that the other firm is the high-cost type increases if the low-cost type engages in R&D ( $0 < \alpha_t < 1$ ) in vain. To simplify the notation, we define the lag operator,  $L_{\alpha}$ , which maps  $p_{t+1}$  to  $p_t$ , i.e.,  $p_t = L_{\alpha_t}(p_{t+1})$ , with its inverse:

$$L_{\alpha}^{-1}(p_t) = \frac{p_t}{p_t + (1 - p_t)(1 - \alpha q)}. \quad (5.1)$$

Now, the probability that a firm believes the other firm does not succeed in R&D in period  $t + 1$  is the sum of the probability that the firm believes

the other firm is the high-cost type and the joint probability that the firm believes the other firm is the low-cost type and fails in R&D in period  $t$ . Since we use backward induction in the later analysis, it is convenient that we define this probability as a function of  $p_{t+1}$  and  $\alpha_t$ , i.e.,

$$P_{t+1}(p_{t+1}, \alpha_t) \equiv L_{\alpha_t}(p_{t+1}) + [1 - L_{\alpha_t}(p_{t+1})](1 - \alpha_t q). \quad (5.2)$$

Henceforth, we may suppress arguments of  $P_{t+1}$  if it would cause no confusion.

Let  $\pi^f$  denote a firm's per-period's profits under full collusion in the product market before a success of R&D happens. Since we focus on the symmetric equilibrium,  $\pi^f$  is a half of the joint monopoly profits. Moreover, we let  $\pi^{f*}$  denote a firm's per-period's monopoly profits with a low unit cost after a success in R&D. We assume that the process innovation is so drastic that  $\pi^{f*} > 2\pi^f$ .<sup>37</sup> Then, defining  $V \equiv \pi^f/(1 - \delta)$  and  $W \equiv \pi^{f*}/(1 - \delta)$ , we have  $V < W$ . While we have defined by  $V$  the present discounted value of the perpetual full collusion without R&D, we define by  $\tilde{V}$  the counterpart when the low-cost type makes the maximum R&D efforts in every period besides

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<sup>37</sup> The post-innovation price is the smallest of its rival firm's unit cost and the monopoly price in the total absence of its rival firm.

full collusion is sustained in the product market. Since the full collusion lasts until the firm succeeds in R&D, which occurs with the probability  $q$  per period, we have  $\tilde{V} = \pi^f - c + \delta qW + \delta(1 - q)\tilde{V}$ . Consequently, we obtain  $\tilde{V} = (\pi^f - c + \delta qW)/[1 - \delta(1 - q)]$ . Notice that the present discounted equilibrium value for the low-cost type in period  $t$ , denoted by  $V_t$ , must be smaller than or equal to the largest of  $V$  and  $\tilde{V}$ , i.e.,

$$V_t \leq \max\{V, \tilde{V}\}. \quad (5.3)$$

If a firm does R&D investment in this period, it pays the cost  $c$  and gets the revenue  $\delta\{qW + (1 - q)V\}$ . On the other sides, if the firm does not, then the profit is  $\delta V$ . Hence a firm have an incentive of R&D if  $\delta\{qW + (1 - q)V\} - c \geq \delta V$ , i.e.,  $\delta q(W - V) \geq c$ . Now, we have the following lemma about the incentive of R&D. This proof is relegated to the Appendix.

**Lemma 1:** *If  $\delta q(W - V) = c$ , then  $V = \tilde{V}$  and hence  $c = \delta q(W - \tilde{V}) = \delta q(W - V)$ . If  $\delta q(W - V) > c$ , then  $V < \tilde{V}$  and  $c < \delta q(W - \tilde{V}) < \delta q(W - V)$ . Finally, if  $\delta q(W - V) < c$ , then  $c > \delta q(W - \tilde{V}) > \delta q(W - V)$ .*

Now, we show that if  $\delta q(W - V) \leq c$  and  $\delta \geq 1/2$ , then neither firm engages in R&D until a firm deviates, and if  $\delta q(W - V) > c$ , then both low-cost type firms engage in R&D in every period.

We first consider the former case. If neither firm is supposed to engage in R&D and to deviate, the belief on the other firm's type is not updated so that the situation in the next period is exactly the same as that in the current period. Thus, the equilibrium payoff for a low-cost type firm satisfies  $V_t = \pi_t + \delta V_t$ . If a firm engages in R&D against its prescribed strategy, it obtains  $V'_t = \pi_t - \alpha_t c + \delta \alpha_t q W + \delta(1 - \alpha_t q)V_t$ . Notice that  $V_t \geq V'_t$  is reduced to  $\delta q(W - V_t) \leq c$ . Now suppose that for every period,  $\pi_t = \pi^f$  and neither firm engages in R&D. Then,  $V_t$  equals  $V$ , and hence  $\delta q(W - V) \leq c$  is satisfied for any  $t$  if  $\delta q(W - V) \leq c$ . Therefore, neither firm engages in R&D



on the equilibrium path, indeed.<sup>38</sup> Regardless of their types, the incentive constraint for both firms can be written as  $\pi^f/(1 - \delta) \geq 2\pi^f$ , where the left-hand side shows the equilibrium payoff while the right-hand side shows the payoff when it deviates optimally. Since the last inequality is reduced to  $\delta \geq 1/2$ , we have shown the following.

**Proposition 1:** *If  $\delta q(W - V) \leq c$  and  $\delta \geq 1/2$ , there is a sequential equi-*

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<sup>38</sup> Firms may come to do R&D after a deviation in the product market. Off the equilibrium path, if a firm engages in R&D, it obtains

$$-\alpha_t c + \delta \alpha_t q P_{t+1}(p_{t+1}, \alpha_t) W + \delta(1 - \alpha_t q) P_{t+1}(p_{t+1}, \alpha_t) V_{t+1}.$$

Otherwise it obtains  $\delta P_{t+1} V_{t+1}$ . Therefore a firm has an incentive to do R&D if and only if

$$-\alpha_t c + \delta \alpha_t q P_{t+1} W + \delta(1 - \alpha_t q) P_{t+1} V_{t+1} \geq \delta P_{t+1} V_{t+1}, \text{ i.e.,}$$

$$\delta q P_{t+1}(W - V_{t+1}) \geq c.$$

Note that the belief does not update when  $\alpha_t = 0$ . Thus if a firm does no R&D today, then no R&D tomorrow either. Hence when  $\alpha_t = 0$ , we get  $V_{t+1} = 0$  and  $P_{t+1} = 1$ . From the above discussion a firm has no incentives to do R&D even after a deviation if and only if

$$\delta q W < c.$$

*librium in which both firms perpetually continue to collude perfectly without making any effort on R&D.*

Next, we show how the sequence of R&D intensities are derived when  $\delta q(W - V) > c$ . The equilibrium payoff when both firms engage in R&D can be written as

$$V_t = \pi_t - \alpha_t c + \delta \alpha_t q P_{t+1} W + \delta (1 - \alpha_t q) P_{t+1} V_{t+1}.$$

On the other hand, the out-of-equilibrium payoff when the firm does not engage in R&D in only period  $t$  can be written as

$$V'_t = \pi_t + \delta P_{t+1} V_{t+1}.$$

Notice that each firm's belief on the other firm's type is updated according to the equilibrium R&D strategy regardless of the firm's actual R&D activities.

The equilibrium condition  $V_t \geq V'_t$  is reduced to

$$\delta q P_{t+1} (W - V_{t+1}) \geq c \tag{5.4}$$

Lemma 1 and (5.3) imply that  $\delta q(W - V_{t+1}) > c$  for any  $t$  under the assumption  $\delta q(W - V) > c$ . To find the equilibrium R&D intensities, we first derive a reduced form of  $P_{t+1}(p_{t+1}, \alpha_t)$ . By solving (5.1) for  $p_t$ , we find

$$L_{\alpha_t}(p_{t+1}) = \frac{(1 - \alpha_t q)p_{t+1}}{1 - \alpha_t q p_{t+1}}. \tag{5.5}$$

Substituting (5.5) into (5.2) yields

$$P_{t+1}(p_{t+1}, \alpha_t) = \frac{1 - \alpha_t q}{1 - \alpha_t q p_{t+1}}. \quad (5.6)$$

It follows directly from (5.6) that  $P_{t+1}$  decreases in  $\alpha_t$  from  $P_{t+1}(p_{t+1}, 0) = 1$  to  $P_{t+1}(p_{t+1}, 1) = (1 - q)/(1 - q p_{t+1})$ . The equilibrium R&D intensity in period  $t$  is the highest  $\alpha_t$  that satisfies (5.4) for given  $p_{t+1}$  and  $V_{t+1}$ . If  $\delta q P_{t+1}(p_{t+1}, 1)(W - V_{t+1}) \geq c$ , then engaging in R&D with  $\alpha_t = 1$  is obviously the equilibrium action. If  $\delta q P_{t+1}(p_{t+1}, 1)(W - V_{t+1}) < c$ , on the other hand, there exists  $\alpha_t \in (0, 1)$  such that  $\delta q P_{t+1}(p_{t+1}, \alpha_t)(W - V_{t+1}) = c$ . This is because  $\delta q(W - V_{t+1}) > c$  and  $P_{t+1}$  is monotonically decreasing in  $\alpha_t$  from  $P_{t+1}(p_{t+1}, 0) = 1$ . In this case, each firm is indifferent between engaging and not engaging in R&D in period  $t$  so that mixing these actions can be part of equilibrium.

**lemma 2:** *If  $\delta q(W - V) > c$ , each low-cost firm always engages in R&D.*

*For given  $p_{t+1}$  and  $V_{t+1}$ , R&D intensity  $\alpha_t$  is determined as the highest value*

*$\alpha_t \in [0, 1]$  satisfying  $\delta q P_{t+1}(p_{t+1}, \alpha_t)(W - V_{t+1}) \geq c$ .*

## 5.4 Collusion Process in the Product Market

Proposition 1 indicates that if the marginal R&D cost  $c$  is high and the discount factor is high enough, the two firms do not engage in R&D and perfectly collude in the product market from the beginning. If the marginal R&D cost is low, on the other hand, the low-cost firm would engage in R&D. Then each firm's belief on the other firm's type must be updated. The collusion level also varies with the belief. Indeed, the evolution of R&D intensities, belief on the other firm's type, and collusion level are closely and complicatedly related to one another so that it is difficult to derive the entire equilibrium in a general case. In what follows, therefore, we concentrate on the case where a low-cost firm fully invests in R&D in every period, i.e.,  $\alpha_t = 1$  for any  $t$ , in equilibrium. In order to concentrate our analysis on this case, we shall make the following assumption. Then we can find that the collusion level rises gradually under certain circumstances.

**Assumption 1**

$$c \leq \frac{\delta q(1-q)(\pi^{f^*} - \pi^f)}{1 - \delta(1-q^2)}.$$

This assumption is more likely to be satisfied as  $c$  is low and  $\delta$  is large. The next lemma shows that under the assumption  $\delta q P_{t+1}(p_{t+1}, 1)(W - V_{t+1}) \geq c$

and hence  $\alpha_t = 1$  for any  $t$ , both in the equilibrium path and in the out-of-equilibrium path following a firm's deviation from the collusive equilibrium. The proof is relegated to the Appendix.

**Lemma 3:** *Under Assumption 1, a low-cost firm fully invests in R&D in every period, i.e.,  $\alpha_t = 1$  for any  $t$  both in the equilibrium path and in the out-of-equilibrium path following a firm's deviation in selecting the price.*

Let  $V'_{t+1}$  denote the continuation payoff in period  $t + 1$  after a deviation occurred in period  $t$ . Since the profits from the product markets always equal to zero in the punishment phase,  $V'_{t+1}$  is nothing but the expected profits gained from only R&D activities. The incentive constraint for a low-cost type can be written as

$$\pi_t - c + \delta q P_{t+1} W + \delta(1 - q) P_{t+1} V_{t+1} \geq 2\pi_t - c + \delta q P_{t+1} W + \delta(1 - q) P_{t+1} V'_{t+1},$$

where the left-hand side shows the continuation payoff from cooperation while the right-hand side shows the continuation payoff when the firm deviates. Reflecting the fact that the firm's R&D intensity is maintained at the full level even after a deviation (Lemma 3), both sides of the inequality have the common terms regarding the expected R&D profits. Moreover, we find after

expanding  $V_{t+1}$  and  $V'_{t+1}$  that the terms regarding the expected R&D profits are cancelled out for every period.<sup>39</sup> Consequently, the above incentive constraint can be reduced to

$$\pi_t \leq \sum_{s=1}^{\infty} \delta^s (1-q)^s \prod_{i=1}^s P_{t+i} \pi_{t+s}, \quad (5.7)$$

where the left-hand side shows the gains from deviation while the right-hand side shows the loss from breaking the cooperation. Notice that  $(1-q)^s$  is the probability that the firm in question continuously fails in R&D for  $s$  periods, while  $\prod_{i=1}^s P_{t+i}$  is the conditional probability with which in period  $t$  the firm believes the other firm will not succeed in R&D for  $s$  more periods. Note that (5.1) can be written as  $p_{t+1} = p_t/P_{t+1}$ . Then if  $\alpha_{t+i} = 1$  for all  $i \geq 0$ , we repeatedly apply (5.1) and (5.2) to obtain

$$P_{t+s} = \frac{p_t + (1-p_t)(1-q)^s}{\prod_{i=1}^{s-1} P_{t+i}},$$

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<sup>39</sup> Concretely,  $V_{t+1}$  and  $V'_{t+1}$  are expanded as follows:

$$V_{t+1} = \pi_t - c + \delta q P_{t+1} W + \delta(1-q) P_{t+1} (\pi_{t+1} - c + \delta q P_{t+2} W + \delta^2(1-q) P_{t+2} (\pi_{t+2} - c + \delta q P_{t+3} W + \dots$$

and

$$V'_{t+1} = -c + \delta q P_{t+1} W + \delta(1-q) P_{t+1} (-c + \delta q P_{t+2} W + \delta^2(1-q) P_{t+2} (-c + \delta q P_{t+3} W + \dots$$

or equivalently,

$$\prod_{i=1}^s P_{t+i} = p_t + (1 - p_t)(1 - q)^s. \quad (5.8)$$

The conditional subjective probability in period  $t$  that the other firm will not succeed in R&D until period  $t + s$  is the sum of the subjective probability in  $t$  that the other is the high-cost type and the subjective probability in  $t$  that the other is the low-cost type and continues to fail in R&D until period  $t + s$ . Now substituting (5.8) into (5.7) gives us the reduced form of the incentive constraint for the low-cost type:

$$\pi_t \leq \sum_{s=1}^{\infty} \delta^s (1 - q)^s [p_t + (1 - p_t)(1 - q)^s] \pi_{t+s}. \quad (5.9)$$

For a high-cost firm, the subjective probability that neither firm will succeed in R&D until period  $t + s$  equals the subjective probability that the other firm will not succeed in R&D until  $t + s$ . Thus, the incentive constraint for the high-cost type can be written as

$$\pi_t \leq \sum_{s=1}^{\infty} \delta^s [p_t + (1 - p_t)(1 - q)^s] \pi_{t+s}. \quad (5.10)$$

Comparing (5.9) and (5.10), we immediately realize that (5.9) is tighter than (5.10). This is because when the firm in question is the low-cost type, the subjective probability that cooperation ends with a success in R&D is

higher and hence the loss from breaking cooperation by selecting a “wrong” price is lower. The sequence of maximal collusion profits must satisfy both (5.9) and (5.10) for every period. Due to this observation, however, we need only derive the sequence of the highest  $\pi_t$  that satisfies (5.9) for every  $t \geq 1$ .

In the rest of this section, we shall derive the maximal, self-enforcing collusion process, characterizing  $\pi_t$  backward in time. We find that the current maximal profits can be expressed as a function of the current belief on the other firm’s type. As the belief is updated, the collusion level changes together.

Since the repeated application of (5.1) yields  $p_t = p_1/[p_1 + (1 - p_1)(1 - q)^{t-1}]$ , we have the following lemma.

**Lemma 4:** *As  $t$  increases,  $p_t$  increases to 1 on the collusion path.*

If both firms will keep not succeeding in R&D, both firms may come to sustain the full collusion some other time. In order to see whether the full collusion is attained and sustained forever, we examine the low-cost firm’s incentive constraint in period  $t$  for the semi-perpetual (lasting until R&D



succeeds) full collusion:

$$\pi^f \leq \sum_{s=1}^{\infty} \delta^s (1-q)^s \pi^f [p_t + (1-p_t)(1-q)^s] \quad (5.11)$$

$$\sum_{s=1}^{\infty} \delta^s (1-q)^s - \sum_{s=1}^{\infty} \delta^s (1-q)^{2s} p_t \geq 1 - \sum_{s=1}^{\infty} \delta^s (1-q)^{2s} \quad (5.12)$$

$$p_t \geq \frac{[1 - \delta(1-q)][1 - 2\delta(1-q)^2]}{\delta q(1-q)} \quad (5.13)$$

It follows from Lemma 4 that if (5.13) is satisfied in period  $t$ , the full collusion is sustained thereafter unless R&D succeeds. Letting  $\hat{p} \equiv [1 - \delta(1-q)][1 - 2\delta(1-q)^2]/[\delta q(1-q)]$ , we can record this finding as follows.

**Lemma 5:** *If  $p_t \geq \hat{p}$ , the full collusion is sustained in period  $t$ .*

If  $\delta$  is very small, then  $\hat{p}$  is greater than one. In this case the full collusion can never be attained. On the other hand, as  $\delta$  increases,  $\hat{p}$  is more likely to be negative, in which case the full collusion can be attained from the very beginning. Indeed, we have the following lemma about the relation between  $\delta$  and  $\hat{p}$ . The proof is relegated to the Appendix.

**Lemma 6:**  *$0 < \hat{p} < 1$  if and only if  $\delta(1-q)^2 < 1/2 < \delta(1-q)$ .*

We make the assumption which assures that  $\hat{p}$  lies in the interval  $(0, 1)$ .

**Assumption 2**

$$\delta(1-q)^2 < 1/2 < \delta(1-q).$$

Let us define  $\hat{t}$  by  $\hat{t} = \min\{t : p_t \geq \hat{p}\}$  for a given  $p_1$ . Then, Lemmas 4, 5, and 6 imply the next proposition.

**Proposition 2:** *Unless R&D succeeds, the two firms eventually enter the full collusion phase, i.e.,  $\pi_t = \pi^f$  for  $t \geq \hat{t}$ , under Assumptions 1 and 2.*

If  $p_1$  is large enough that  $\hat{t} = 1$ , i.e.,  $p_1 \geq \hat{p}$ , then the two firms collude in the product market perfectly from period 1. However, if  $p_1$  is small enough that  $\hat{t} \geq 2$ , i.e.,  $p_1 < \hat{p}$ , then they cannot collude perfectly from the beginning. In such a case, they are obliged to raise the collusion level gradually until period  $\hat{t}$ , as we show below.

The maximal collusion profits in period  $\hat{t} - 1$  must satisfy the low-cost firm's incentive constraint with equality. It follows from (5.9) that the relevant incentive constraint can be written as

$$\pi_{\hat{t}-1} = \sum_{s=1}^{\infty} \delta^s (1-q)^s \pi^f [p_{\hat{t}-1} + (1-p_{\hat{t}-1})(1-q)^s]. \quad (5.14)$$

It is obvious that the right-hand side increases in  $p_{\hat{t}-1}$  and so does  $\pi_{\hat{t}-1}$ . Since (5.11) implies that the right-hand side of (5.14) equals  $\pi^f$  if  $p_{\hat{t}-1} = \hat{p}$ , we find that  $\pi_{\hat{t}-1} < \pi^f$ . Thus, we have shown the following lemma.

**Lemma 7:** *In period  $\hat{t} - 1$ , i.e.,  $p_{\hat{t}-1} \in [L(\hat{p}), \hat{p})$ , the collusion profits for*

each firm equal  $\pi_{t-1} = \sum_{s=1}^{\infty} \delta^s (1-q)^s \pi^f [p_{t-1} + (1-p_{t-1})(1-q)^s]$ , which is smaller than  $\pi^f$ .

Figure 1 shows the maximal collusion process; the upper diagram shows the maximal collusion profits for each firm as a function of the belief, while the lower diagram shows how the belief is updated. Thus far, we have derived the schedule in the lower diagram and the part of the schedule, corresponding to  $[L(\hat{p}), 1]$ , in the upper diagram.

We derive the rest of the schedule by backward induction. Suppose we have derived the maximal collusion schedule corresponding to period  $t+1$  onward, i.e., we have derived the schedule for  $[L^{t-t-1}(\hat{p}), 1]$ . For  $t$  strictly smaller than  $\hat{t}-1$ , we then derive  $\pi_t$  that satisfies the incentive constraint in period  $t$  with equality:

$$\pi_t = \sum_{s=1}^{\infty} \delta^s (1-q)^s \pi_{t+s} [p_t + (1-p_t)(1-q)^s]. \quad (5.15)$$

We simplify (5.15) using the fact that the incentive constraint is also binding in period  $t+1$ . The binding incentive constraint in period  $t+1$  can be written as

$$\pi_{t+1} = \sum_{s=2}^{\infty} \delta^{s-1} (1-q)^{s-1} \pi_{t+s} [p_{t+1} + (1-p_{t+1})(1-q)^{s-1}]. \quad (5.16)$$

Now, it is easily verified that

$$p_t + (1 - p_t)(1 - q)^s = [p_t + (1 - p_t)(1 - q)][p_{t+1} + (1 - p_{t+1})(1 - q)^{s-1}]. \quad (5.17)$$

This equation indicates that the subjective conditional probability in period  $t$  that the other firm will not succeed in R&D until  $t + s$  equals the subjective conditional probability in  $t + 1$  for the same event, multiplied by the subjective conditional probability in period  $t$  that the other firm will not succeed in R&D in  $t + 1$ . Using (5.17), and multiplying both sides by  $\delta(1 - q)$ , (5.16) can be reduced to

$$\delta(1 - q)[p_t + (1 - p_t)(1 - q)]\pi_{t+1} = \sum_{s=2}^{\infty} \delta^s (1 - q)^s \pi_{t+s} [p_t + (1 - p_t)(1 - q)^s].$$

Substituting this equality into (5.15), we obtain

$$\pi_t = 2\delta(1 - q)[p_t + (1 - p_t)(1 - q)]\pi_{t+1}. \quad (5.18)$$

Thus, having derived  $\pi_{t+1}$ , we can easily compute  $\pi_t$  from (5.18). The Appendix (Proof of Lemma 8) shows that  $2\delta(1 - q)[p_t + (1 - p_t)(1 - q)] < 1$  for any  $p_t < \hat{p}$ . Thus, we have the following.

**Lemma 8:** *In period  $t = 1, \dots, \hat{t} - 2$ ,  $\pi_t$  is determined by*

$$\pi_t = 2\delta(1 - q)[p_t + (1 - p_t)(1 - q)]\pi_{t+1},$$

for a given  $\pi_{t+1}$ . The coefficient  $2\delta(1-q)[p_t + (1-p_t)(1-q)]$  is strictly less than 1 for any  $p_t < \hat{p}$ .

From Lemmas 7 and 8 and Proposition 2, we can complete the maximal collusion schedule in Figure 1. Figure 1 also shows the collusion process when  $p_1$  lies in between  $L^2(\hat{p})$  and  $L(\hat{p})$ . The collusive level of output gradually increases from  $\pi_1$  to  $\pi^f$  in three periods as the belief is updated.

**Proposition 3:** *If the initial belief that the other firm is the high-cost type is low enough ( $p_1 < \hat{p}$ ), the maximal, self-enforcing collusion level increases gradually.*

## 5.5 Conclusion

In this chapter we consider that gradual collusion between firms in an oligopolistic industry may happen when they have incomplete information about their rivals' R&D costs. If each firm believes that the other firm has the low R&D costs with high probability, then each firm is afraid that with high probability the rival secretly engages in R&D investment and cheats and drives the firm out in the product market as soon as the rival succeeds in R&D. Thus firms cannot sustain the high collusion level in the product market. As time

goes by without a success of R&D, the belief that the rival's R&D costs is low becomes lower and the probability of cheating also becomes lower. Thus firms come to sustain the high collusion level.

As mentioned above the main point of this model is that as asymmetric information is gradually dissolved, the cooperation level also gradually increases. And we show this mechanism even among fixed players. These points are different from the preceded studies about gradual collusion.

However we have some remaining problems. First we only analyze the extreme two cases, i.e., neither firm engages in R&D and the low-cost type fully invest in R&D in every period. In the middle case, we guess that the level of of R&D investment and the collusion level dynamically change as they are closely related. It is interesting and important to consider such a dynamics.

More theoretically, when players are fixed and only one test makes asymmetric information between players dissolve, i.e., in our model  $q = 1$ , gradual collusion never happens. Why players gradually collude only if they gradually knows about the partners' nature may be an important problem. Because in reality we observe gradual collusion in the situation that it takes long time to dissolve asymmetric information while we do not observe when it takes short

time. We do not know why as yet. We would like to further investigation about these problems.

## 5.6 Appendix

### A. Proof of Lemma 1

Expressing  $\tilde{V}$  as a function of  $c$ , we first show that  $\tilde{V}(\delta q(W - V)) = V$ . Indeed, it follows directly from substituting  $\delta q(W - V)$  for  $c$  in  $\tilde{V}(c)$  and  $\pi^f/(1 - \delta)$  for  $V$ :

$$\begin{aligned}\tilde{V}(\delta q(W - V)) &= \frac{\pi^f - \delta q(W - V) + \delta qW}{1 - \delta(1 - q)} \\ &= \frac{\pi^f + \delta q\pi^f/(1 - \delta)}{1 - \delta(1 - q)} \\ &= \frac{\pi^f}{1 - \delta} \\ &= V.\end{aligned}$$

Next, let us define  $f(c) \equiv \delta q[W - \tilde{V}(c)]$ . Then,  $f'(c) = -\delta q\tilde{V}'(c) = \delta q/[1 - \delta(1 - q)]$ , which is positive. Moreover, since  $f'(c) < 1$  is equivalent to  $\delta < 1$  and the latter must hold, we also have  $f'(c) < 1$ . Together with  $f(\delta q(W - V)) = \delta q(W - V)$  as shown above, this observation implies that if  $c > \delta q(W - V)$ ,  $\delta q(W - \tilde{V})$  lies in between  $\delta q(W - V)$  and  $c$ , and that if  $c < \delta q(W - V)$ ,  $\delta q(W - \tilde{V})$  lies in between  $c$  and  $\delta q(W - V)$ . This ends the proof.

### B. Proof of Lemma 3

Both in the equilibrium path and in the out-of-equilibrium path following



a firm's deviation in selecting the price, the payoff is less than or equal to  $\tilde{V}$ , i.e.,  $V_{t+1} \leq \tilde{V}$ . Moreover, it is easy to see

$$P_{t+1}(p_{t+1}, 1) = \frac{1-q}{1-qp_{t+1}} \geq 1-q.$$

Therefore, we obtain

$$\begin{aligned} \delta q P_{t+1}(p_{t+1}, 1)(W - V_{t+1}) &\geq \delta q(1-q)(W - \tilde{V}) \\ &= \frac{\delta q(1-q)(\pi^{f*} - \pi^f + c)}{1 - \delta(1-q)}, \end{aligned}$$

after the substitution of  $\tilde{V} = (\pi^f - c + \delta q W) / [1 - \delta(1-q)]$  and  $W = \pi^{f*} / (1-\delta)$ . Thus, if  $\delta q(1-q)(\pi^{f*} - \pi^f + c) / [1 - \delta(1-q)] \geq c$ ,  $\delta q P_{t+1}(p_{t+1}, 1)(W - V_{t+1}) \geq c$  and hence  $\alpha_t = 1$  for any  $t$ . However, it is easy to show that this last inequality is equivalent to Assumption 1.

### C. Proof of Lemma 6

Let us first notice that  $\hat{p}$  satisfies (5.12) with equality. Since the left-hand side of (5.12) is positive,  $\hat{p} > 0$  if and only if  $\sum_{s=1}^{\infty} \delta^s (1-q)^{2s} < 1$ . This is equivalent to  $\delta(1-q)^2 < 1/2$ . Moreover, when  $\delta(1-q)^2 < 1/2$  is satisfied, it is easy to see from (5.12) that  $\hat{p} < 1$  if and only if  $\sum_{s=1}^{\infty} \delta^s (1-q)^s > 1$ . Since  $\sum_{s=1}^{\infty} \delta^s (1-q)^s > 1$  is equivalent to  $\delta(1-q) > 1/2$ , we have finished the proof.

#### D. Proof of Lemma 8

What remains to be shown is that  $2\delta(1-q)[p_t + (1-p_t)(1-q)] < 1$  for any  $p_t < \hat{p}$ . Since the left-hand side is increasing in  $p_t$ , we need only show  $2\delta(1-q)[\hat{p} + (1-\hat{p})(1-q)] < 1$ . It follows from the definition of  $\hat{p}$  that

$$\begin{aligned}\hat{p} + (1-\hat{p})(1-q) &= 1 - q + q\hat{p} \\ &= \frac{\delta(1-q)^2 + [1 - \delta(1-q)][1 - 2\delta(1-q)^2]}{\delta(1-q)}.\end{aligned}$$

Thus, we have

$$\begin{aligned}2\delta(1-q)[\hat{p} + (1-\hat{p})(1-q)] &< 1 \\ 2\delta(1-q)^2 + [2 - 2\delta(1-q)][1 - 2\delta(1-q)^2] &< 1 \\ [1 - 2\delta(1-q)][1 - 2\delta(1-q)^2] &< 0,\end{aligned}$$

which is satisfied by Assumption 2.

## 5.7 Figures

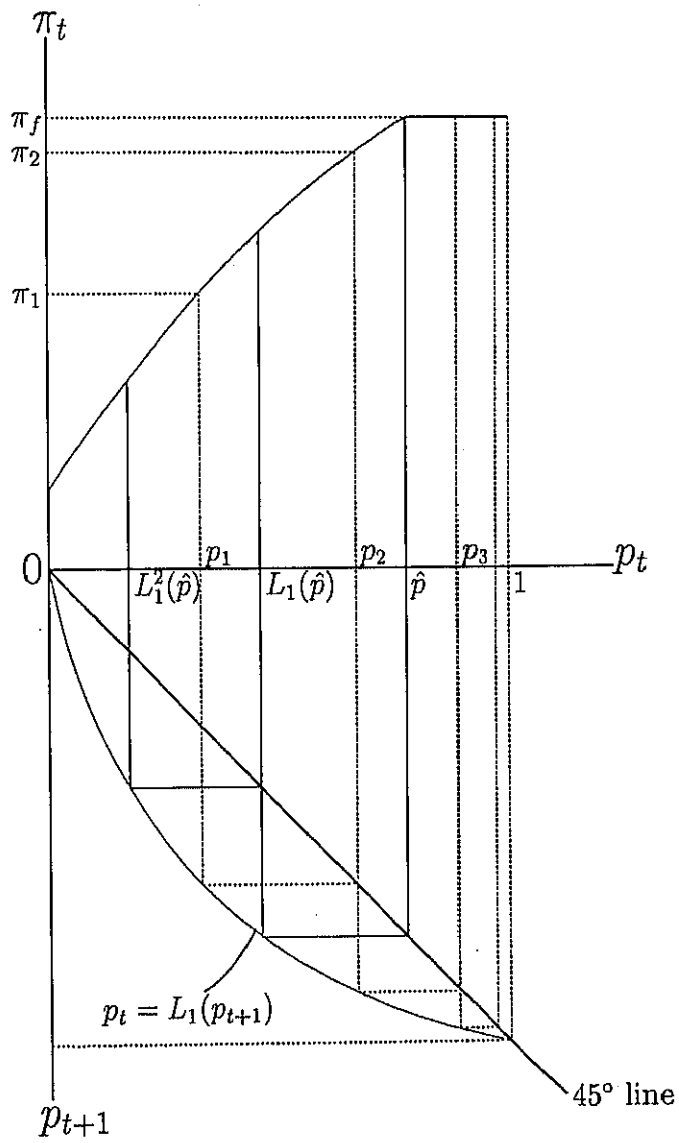


Figure 1: The Maximal Collusion Process