

## 4 Collusive Price Leadership under Repeated Games

### 4.1 Introduction

In this chapter we shall attempt to formulate a model to explain price leadership in the framework of a repeated game. Price leadership is known to have various types in practice. It is classified into three type: dominant firm, collusive, and barometric. Dominant firm price leadership is that there exists a single dominant firm which sets a price as the price leader. This type of price leadership is likely to occur when the leader has a large market share and other firms being too small to have a perceptible influence on price. Hence this can not be explained as a form of collusion between firms but by a sufficient cost advantage of the leader firm over rivals.<sup>22</sup> Collusive price leadership is one form that tacit collusion may take. The terminology loosely implies the phenomennon that a leader firm leads a price change and the other firms follow it cooperatively. Barometric price leadership means

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<sup>22</sup> For studies of dominant firm price leadership, see, e.g., Nichol (1930) or Deneckere and Kovenock (1992).

that the price leader does no more than act as a barometer of market conditions, setting prices approximating those that would emerge in any event under competition. Collusive and barometric price leadership are hard to be distinguished in actual cases.

We shall not explain all of them. The type we shall consider here is the one in which one firm, i.e., the leader sets a price which is followed by accommodating price settings by other firms. Moreover we shall confine our analysis to a case of collusive price leadership among firms with symmetric technology and information.

Price leadership is often observed in oligopolistic markets. It is traditionally discussed in the industrial organization literature that in such a market a price leader would emerge among firms "in lieu of overt collusion" and the leader's price is considered as a signal to communicate the collusive price to other firms.<sup>23</sup> Rotemberg and Saloner (1985,1990) also study collusive price leadership in the framework of a repeated game. Following the traditional consideration, their argument heavily depends on asymmetric information among firms. More concretely, they assume that firms face stochastic market

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<sup>23</sup> See Scherer and Ross (1990).

demand and one firm has better information on the demand state than the other. *If information is sufficiently asymmetric, the less informed firm prefers to follow the better informed firm, so the leader can emerge endogenously (1990, p.93).* Suppose that both firms agree to form such a leader-follower relationship in their model. In each period, at the time when the better informed firm makes its strategic decision, the less informed firm can observe the action chosen by the better informed firm but, as it cannot observe the demand state accurately, does not know whether the better informed firm deviates from its collusive strategy. Nevertheless their collusion will be sustained as long as the less informed firm believes that the better informed firm sets the collusive price and the less informed firm charges the same price set by the better informed firm. Then the leader (the better informed firm) will be able to secure more profit than the follower (the less informed firm). Albaek (1990) also explains price leadership under cost uncertainty. Similar to Rotemberg and Saloner, his study analyses it as a device to reveal information to the rivals.

By contrast to these papers we suppose symmetric information. Under the assumption we show that a leader-follower relationship may emerge. Moreover we assert that price leadership has another aspect on the collusion,

i.e., to reduce the firms' incentive of deviation from the collusion. In this chapter we regard a price leader as a firm which fixes its price and supply before other firms' choice of strategies. Price and supply are supposed to be fixed during the period once a firm chooses them. Therefore, if a leader firm deviates, the other firms, to the contrary to Rotemberg and Saloner, notice the deviation and can punish the leader in the period. Thus the leader firm gets no gain from deviation. Hence it will never deviate. The fact that the leader will not deviate in turn provide other firms incentive not to deviate.

In addition to providing a theoretical explanation on why price leadership emerges, our model enables us to explain interesting methods of implementing price leadership. They are rotating price leadership and divided price leadership. These types of price leadership have often been observed and have usually been considered to be the ways to camouflage cartels against the antitrust law.<sup>24</sup> However we shall explain them as schemes to stabilize and facilitate implicit collusion.

We shall set up the following model. The component game of the repeated game has three stages. In addition to the standard strategic variables (in this

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<sup>24</sup> See, e.g., Machlup (1952).

chapter, price and quantity), at stage 0 in each period firms are supposed to select the stage (1 or 2) in which they make strategic decisions about the standard strategic variables. In other words, the order of firms' play is endogenous. We call a firm choosing the first stage "the leader" and the other "the follower". Under this setting we shall consider two different types of strategy combinations. One type is that both firms select the same stage. The other is that one firm becomes the leader and the others followers. We call the former Simultaneous Strategy and the latter Leadership Strategy.<sup>25</sup> We shall argue that the collusive outcome can be sustained by means of the Leadership Strategy even if the discount factor is too low to collude in using the Simultaneous Strategy.

However this result is with a proviso that the allocation to be sustained is one such that the followers are more profitable than the leader. Hence it may be difficult to justify why a particular firm would willingly take the position of the leader. We shall consider rotating price leadership and divided price leadership as means to evade such difficulty. Rotating price leadership is a scheme in which firms become the leader in turn and divided price leadership

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<sup>25</sup> We shall give more precise definitions of these strategies in the text.

is one in which different firms act as the leader in different markets. We interpret both types of price leadership as schemes which equalize the leader's disadvantage among firms and stabilize their collusion. We shall give the detailed explanation of these phenomena in the text.

Our argument proceeds as follows. Section 2 provides a basic model. In section 3 we obtain the condition that Simultaneous Strategy is enforceable. In section 4 we obtain the condition that Leadership Strategy is enforceable and compare it to Simultaneous Strategy. We consider rotating price leadership and divided price leadership in section 5 and 6 respectively. We have concluding remarks in section 7.

## 4.2 Basic Model

There are  $n$  firms in the industry, where  $n \geq 2$ . They will be assumed to play an infinitely repeated game. They produce a homogenous good with a constant unit cost  $c$ . The market demand function of each period is  $q = D(p)$ . In each period firm  $i$  individually chooses both its price  $p_i$  and maximum quantity  $q_i$ .

Each period  $t$  of the repeated game is made up of the component game,

which has the following properties.

### The Construction of the Component Game

Each component game has three stages. At stage 0 each firm can decide whether to set its price and maximum quantity at stage 1 or 2. This decision is denoted by  $L$  and  $F$ .  $L(F)$  means that the firm sets its price and maximum quantity at stage 1 (stage 2) respectively. This determines a restriction on the actions available in the later period. If the action firm  $i$  chooses is  $L$ , then the available action at stage 1 is  $(p_i, q_i) \in \mathbb{R}_+^2$  and at stage 2 the action of stage 1 has to be maintained. If the action is  $F$ , then no action can be taken at stage 1 while at stage 2 the action is  $(p_i, q_i) \in \mathbb{R}_+^2$ . Purchases only take place after the end of stage 2.

As a result of this assumption, each firm has a chance to reveal its price and share to other firms. The underlined part of the construction may not be applicable to price competition, since it implies that price and quantity cannot be adjusted continuously. Theoretically price is usually considered to be adjustable but, in reality, firms incur costs in deciding price changes, sending new price lists and catalogs to retailers, changing price tags, advertising price cuts to consumers, and so on. If these costs are relatively high,

we may justify our assumption.

Next we specify the profit of firm  $i$ . To specify it, we have to specify individual demand and therefore give a specification of how industry demand is rationed relative to the supply. We assume the rationing rule as the following scheme:

$$D_i(p_1, q_i; p_{-i}, q_{-i}) = \left\{ \begin{array}{ll} \min\{D(p_i), q_i\} & \text{if } p_i < p_j \text{ for all } j \\ \frac{q_i}{\sum_{j=1}^n q_j} D(p_i) & \text{if all firms set } p_i \text{ and } \sum_{j=i}^n q_j \geq D(p_i) \\ q_i & \text{if all firms set } p_i \text{ and } \sum_{j=i}^n q_j < D(p_i) \\ \max\{0, \min\{q_i, D(p_i) - \sum_{j=1}^m q_j\}\} & \text{if } m \text{ firms set the price lower than } p_i, \end{array} \right.$$

where we denote a profile of prices and quantities for firm  $i$ 's opponents by

$p_{-i} = (p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_n)$  and  $q_{-i} = (q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_n)$  respectively.<sup>26</sup>

Using this individual demand, we define the per-period profit of firm  $i$  as follows:

$$\pi_i(p_i, q_i; p_{-i}, q_{-i}) = D_i(p_1, q_i; p_{-i}, q_{-i})(p_i - c).$$

Note that in this definition we assume that the firm has only to pay its

<sup>26</sup> The above equations do not include all cases. If we describe all cases, then it is only lengthy. Thus we write only the cases which we need for the following discussions.

production cost for the realized individual demand. Here we consider the situation that goods left unsold can be conserved as inventories at small cost.

We will focus on the case where firms discount future profits using discount factor  $\delta < 1$ . Let  $p_{it}$  or  $q_{it}$  denote the realized price or quantity of firm  $i$  at stage  $t$  respectively. If each firm's price and quantity of each period are determined, the firm  $i$ 's payoff of this repeated game is calculated as follows:

$$V_i(\delta) = \sum_{t=1}^{\infty} \delta^{t-1} \pi_i(p_{it}, q_{it}; p_{-it}, q_{-it}).$$

### 4.3 Simultaneous Strategy

In this section we consider a particular collusive strategy of the infinitely repeated game in which firms set their prices and quantities simultaneously in each period. Each firm will not choose  $L$  voluntarily since the follower has a chance to get the whole demand by under-cutting the leader's price. Hence the case that all firms choose  $F$  is natural. We concretely describe this strategy as follows:

#### Simultaneous Strategy (SS)

Let us define the collusive rule as follows. Each firm chooses  $F$  and sets a

collusive price at the level of the monopoly price  $p^m$  and quantity  $q_i$  such that  $\sum_{i=1}^n q_i = D(p^m)$ . All firms obey this rule in period 1. At stage 0 in period  $t$  firm  $i$  chooses  $F$  independently of actions of all firms from period 0 to  $t - 1$ . At stage 2 in period  $t$  firm  $i$  sets the collusive price and quantity if all firms obey the collusive rule in every period preceding  $t$  and choose  $F$  at stage 0 in period  $t$ . Otherwise firm  $i$  sets its price as  $p_i = c$  and its quantity as  $q_i = D(c)$  perpetually.

In this strategy firm  $i$ 's per-period collusive profit is  $(q_i/D(p^m))\pi(p^m)$ , where  $\pi(p^m) = (p^m - c)D(p^m)$ . As we define the collusive quantity as  $\sum_{i=1}^n q_i = D(p^m)$ ,  $q_i/D(p^m)$  is interpreted as the market share of firm  $i$  when firms collude. For convenience we shall define the collusive share  $s_i$  as  $s_i = q_i/D(p^m)$ . In using SS each firm has the possibility that it deviates at stages 0 or 2. However each firm has no incentive to deviation at stage 0, because it does not gain by doing so. Thus it suffices to check whether a firm deviates at stage 2 in any period to show the sustainability of SS. Therefore a necessary and sufficient condition that SS is self-enforceable becomes as follows:

for all  $i$ ,

$$(1 - s_i)\pi(p^m) \leq \frac{\delta}{1 - \delta} \{s_i\pi(p^m)\}. \quad (4.1)$$

The left-hand side is the gain from deviation. A deviating firm can get the whole demand in this market by under-cutting its price and setting its quantity at  $q_i = D(p^m)$ . In other words, its optimal deviation is to undercut its price slightly and get an additional demand  $(1 - s_i)D(p^m)$ .<sup>27</sup> In exchange, the firm gets zero profit in every period from the next period on.

On the other hand the right-hand side is the discounted sum of collusive payoffs for a firm when all firms continue to maintain the collusive price and quantity. We can reduce these inequalities to the following:

$$\delta \geq 1 - s_i. \quad (4.2)$$

The inequality (4.2) says that the collusion is sustained if and only if

$$\delta \geq \max_i \{1 - s_i\}.$$

Thus there are many combinations of firms' collusive shares if the discount factor is relatively high. It is clear that firms can collude at the lowest discount factor when  $s_i = 1/n$  for all  $i$ . Then the value of the discount factor

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<sup>27</sup> More precisely the profit gained by undercutting price is

$$\sup_{\epsilon} \{(p - \epsilon)D(p - \epsilon)\} - s_i\pi(p^m) = (1 - s_i)\pi(p^m).$$

is  $(n - 1)/n$ . Therefore if  $\delta < (n - 1)/n$ , then it is impossible for firms to collude as long as firms obey SS.

#### 4.4 Leadership Strategy

In this section we introduce a particular leadership strategy. Even in such a case as  $\delta < (n - 1)/n$  where the collusion under SS can not be sustained, we shall argue that firms could sustain their collusion provided that they use the following leadership strategy.

##### Leadership Strategy (LS)

(i) Let us define the collusive rule as follows. A firm as the leader chooses  $L$ , sets a collusive price  $p^m$ , and chooses a collusive leader's share  $s_L$ . Other firms choose  $F$ , set the collusive price  $p^m$  and a collusive share  $(1 - s_L)/(n - 1)$ .

All firms obey this collusive rule in period 1.

(ii) The firm which acts as the leader chooses  $L$  in every period. At stage 1 in period  $t$  it sets  $p^m$  and  $s_L$  if all firms obey the collusive rule in every period preceding  $t$  and besides at stage 0 in period  $t$ . Otherwise it sets its price at  $c$  and its quantity as  $q_i = D(c)$  from period  $t$  on.

(iii) Each firm which acts as the follower chooses  $F$  at stage 0 in every period.

On the premise that all firms continued to obey the collusive rule in every period preceding  $t$ , it chooses  $(p^m, \frac{1-s_L}{n-1})$  at stage 2 in period  $t$  only when the leader firm chooses  $(L, p^m, s_L)$  in period  $t$  and other firms choose  $F$  at stage 0 in period  $t$ . Otherwise it sets its price at  $c$  and its quantity as  $q_i = D(c)$  from period  $t$  on.

The reason why we analyze only the case where the follower's share is  $\frac{1-s_L}{n-1}$  is that firms can collude at the lowest discount factor as SS. If all firms follow LS, we can see that the leader will have no incentive to deviate this collusive state because it can get no gain from deviation. The reason is the following. If the leader does not choose the collusive price or the collusive share, then it is clear that the followers notice leader's deviation and set their prices as  $p = c$  at stage 2 in the period. Moreover choosing  $F$  by the leader also causes the followers to notice leader's deviation and leads to the same result.

It is not essential to analyze the case that a follower chooses  $L$ , because this deviation yield no gain, either. From the above discussion, in order to show that LS is enforceable, we need only check whether the follower deviates at stage 2 in each period. Therefore LS is enforceable if and only if

$$(1 - \frac{1-s_L}{n-1})\pi(p^m) \leq \frac{\delta}{1-\delta} \frac{1-s_L}{n-1} \pi(p^m). \quad (4.3)$$

It is clear that the left-hand side is the gain from deviation and the right-hand side is the long-run losses. From this inequality we directly get the next proposition.

**Proposition 1:** *Suppose that all firms follow the strategy LS. Then the collusion is sustainable if and only if*

$$\delta \geq \delta^{LS}(s_L, n) \equiv 1 - \frac{1 - s_L}{n - 1}. \quad (4.4)$$

Now we confirm that  $\delta^{LS}(s_L, n)$  has the following characteristics:

- (i) for any  $s_L^1$  and  $s_L^2$  such that  $1 \geq s_L^1 \geq s_L^2 \geq 0$ ,  $\delta^{LS}(s_L^1, n) \geq \delta^{LS}(s_L^2, n)$ ,
- (ii)  $\delta^{LS}(\frac{1}{n}, n) = \frac{n - 1}{n}$ .

LS as well as SS is enforceable when  $\delta \geq (n - 1)/n$ , though the use of LS would be more interesting when  $\delta < (n - 1)/n$ . If  $s_L$  is in the range  $[0, \frac{1}{n})$ , using LS makes firms collude when  $\delta < (n - 1)/n$ . Note that the value of  $\delta^{LS}(s_L, n)$  will lie in  $[1 - \frac{1}{n-1}, 1 - \frac{1}{n})$  if  $s_L \in [0, \frac{1}{n})$ . We rewrite this range as follows:

$$\delta^{LS} \in [\frac{n - 2}{n - 1}, \frac{n - 1}{n}). \quad (4.5)$$

Price leadership can emerge when  $\delta \geq \delta^{LS}$ . Thus we can state the followings.

1. Even if  $\delta < (n - 1)/n$ , using LS may make a collusion possible.
2. However  $s_L < 1/n$ . In other words, the leader has less share and less profit than followers in order to sustain the collusion.

We have shown that the introduction of LS enables firms to collude and get higher profits than using SS even if the discount factor is sufficiently low. However, finding a leader would be difficult, since the leader will have to expect less profit than followers. This motivates us consider a rotation system as a way to evade this difficulty in the next section.

Price leadership is more likely to be observed in concentrated industries. In our model we get the conclusion which is similar to this observation. In the inequality (4.4), if  $s_L$  is fixed, the range of the discount factor which LS is enforceable shrinks as the number of firms  $n$  increases. Moreover the range (4.5) shrinks as  $n$  increases.<sup>28</sup> The range (4.5) is the additional one in which collusion becomes to be sustained by switching from SS to LS.

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<sup>28</sup> From (4.5) we get

$$\frac{n-2}{n-1} - \frac{n-1}{n} = \frac{1}{n(n-1)}.$$

## 4.5 Rotating Price Leadership

Here we consider the rotating price leadership. This rotation system is often observed in practice when firms in an industry, as we assume in this chapter, have similar scales or technologies. As examples we may mention the cases of the cigarette industry in USA during 1920s and 1930s or the current beer industry in Japan.

Now we will consider the following strategy including a rotation system.

### Rotating Leadership Strategy (RS)

(i) Let us define the collusive rule as follows. A firm, which is selected *randomly* at the beginning of each period, chooses  $L$ , sets the collusive price  $p^m$ , and the collusive share  $s_L$ , where  $s_L$  is assumed to be identical among leaders in all periods.<sup>29</sup> Other firms choose  $F$ , set  $p^m$ , and choose the share  $(1 - s_L)/(n - 1)$ . All firms obey this collusive rule in period 1.

(ii) In period  $t$ , a firm, which is selected *randomly* at the beginning of the period, behaves the same way as in part (i) of the definition of LS. Other firms behave the same way as in part (iii) of the definition of LS.

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<sup>29</sup> "randomly" means drawing lots in which each firm gets a winning number with probability  $1/n$ .

The distinctive feature of this strategy is that all firms potentially assume the leadership in turn. On this ground, the rotation system may be considered as a rule to evade the disadvantage of leader firms.

Like LS it suffices to check whether the follower deviates at stage 2 in every period in order to show that firms collude in using RS. Thus RS is enforceable if and only if

$$\left(1 - \frac{1 - s_L}{n - 1}\right)\pi(p^m) \leq \frac{\delta}{1 - \delta} \left\{ \frac{1}{n} s_L \pi(p^m) + \frac{n - 1}{n} \frac{1 - s_L}{n - 1} \pi(p^m) \right\}. \quad (4.6)$$

The left-hand side is the gain from deviation as in the inequality (4.3). In RS each firm becomes the leader, in every period, with the probability  $1/n$  and the follower with the probability  $n - 1/n$ . Thus the right-hand side is the long-run loss.

From the inequality (4.6) we get the next proposition directly.

**Proposition 2:** *Suppose that all firms follow the strategy RS. Then collusion is sustainable if and only if*

$$\delta \geq \delta^{RS}(s_L, n) \equiv \frac{1}{1 + \frac{n-1}{n(n+s_L-2)}}. \quad (4.7)$$

The derivation of this inequality is as follows:

$$(4.6) \Leftrightarrow \left(1 - \frac{1 - s_L}{n - 1}\right) \leq \frac{\delta}{1 - \delta} \left(\frac{1}{n} s_L + \frac{n - 1}{n} \frac{1 - s_L}{n - 1}\right) \quad (4.9)$$

$$\Leftrightarrow n(n + s_L - 2) \leq \frac{\delta}{1 - \delta} (n - 1) \quad (4.10)$$

$$\Leftrightarrow n(n + s_L - 2) \leq \delta \{n - 1 + n(n + s_L - 2)\} \quad (4.11)$$

$$\Leftrightarrow \delta \geq \frac{n(n + s_L - 2)}{n - 1 + n(n + s_L - 2)}. \quad (4.12)$$

$\delta^{RS}(s_L, n)$  also has the same characteristics as  $\delta^{LS}(s_L, n)$  in section 4. Thus  $s_L$  needs to be in the range  $[0, \frac{1}{n})$  so that using RS makes firms collude when  $\delta < (n - 1)/n$ . If  $s_L \in [0, \frac{1}{n})$ , the value of  $\delta^{RS}(s_L, n)$  will lie in the range as follows:

$$\delta^{RS} \in \left[\frac{n(n - 2)}{n^2 - n - 1}, \frac{n - 1}{n}\right). \quad (4.12)$$

The lower bounds of the ranges (4.5) and (4.12) are different and we confirm from simple calculations that for any  $n \geq 2$ ,

$$\frac{n - 2}{n - 1} \leq \frac{n(n - 2)}{n^2 - n - 1}. \quad (4.13)$$

In other words, in using LS firms can collude when the discount factor is lower than in using RS. The reason is as follows. From the inequality (4.3) we see that decreasing  $s_L$  not only decreases the gain from deviation but also increases the per-period collusive profit for the followers when firms obey

LS. On the other hand from the inequality (4.9) we see that the ratio of the per-period collusive profit to the gain from deviation is  $n - 1$  to  $n(n + s_L - 2)$ , which is the second term of the denominator of the right-hand side in the inequality (4.7). Thus we see that decreasing  $s_L$  has the only effect decreasing the gain from deviation when firms obey RS. Hence the collusive range in using RS is smaller than in using LS.

We may state the following economic implications about proposition 2.

1. Collusion is sustained even if the discount factor  $\delta$  is lower than  $(n - 1)/n$ .
2.  $s_L < 1/n$  if  $\delta < (n - 1)/n$ .
3. Each firm's *ex-ante* profit is the same among all firms.

The implication 1 means that firms can collude by obeying RS even under a situation where their collusion can not be sustained by obeying SS. This is similar to the case for firms obeying LS. However the range of the discount factor where price leadership is sustainable is smaller in using RS than in using LS. Implications 2 and 3 mean that while adopting RS insures the industry-wide profit of the monopoly level and the equality of each firm's expected profit, each leader is required to reduce its share to sustain the

collusion. That is, a leader will incur short-run loss of assuming leadership, but the loss of being a leader can be compensated by being followers.

The rotation system has been considered to be a method in which firms camouflage a cartel. However this phenomenon seems to be observed independent of the existence of a cartel. For example, in the Japanese beer industry, the Fair Trade Commission has never disclosed any cartel, though the rotating price leadership by Supporo and Asahi has been observed. Thus it may be more natural to consider the rotating price leadership as one form of implicit collusion. Proposition 2 seems to provide a satisfactory explanation of these observed phenomena. We shall consider another way to evade the difficulty in which the leader firm has less profit than others in the next section.

## 4.6 Divided Price Leadership

Divided price leadership is a case which firms in an industry sell in multiple markets and different firms act as leaders in different markets. For example, in American glass container industry this type of price leadership has been

observed.<sup>30</sup> Another example may be the cigarette industry in USA after World War II.<sup>31</sup>

To explain these facts we may use a model of multimarket contact.<sup>32</sup> In a model of multimarket contact markets do not have interdependency each other. Now we shall modify the per-period demand and cost function in our model as follows. There are two firms, 1 and 2, in this industry. Both firms produce two kinds of goods  $a$  and  $b$ . We assume that the demands for goods  $a$  and  $b$  are not related to each other. Thus the demand curves for good  $a$  and  $b$  are respectively denoted by  $D^a(p^a)$  and  $D^b(p^b)$  where  $p^a$  and  $p^b$  denote respectively prices of goods  $a$  and  $b$ . And we use the superscripts with respect to the order of decision and the share in the same way. For simplicity we suppose that both demand functions are symmetrical, that is,  $D^a(p) = D^b(p) = D(p)$  for any  $p$ . Both firms have identical cost functions. The cost functions of both goods obey constant returns to scale and the marginal costs of producing both goods are the same. The marginal cost is

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<sup>30</sup> See Machlup (1952).

<sup>31</sup> See the detail in Schere and Ross (1990).

<sup>32</sup> Bernheim and Whinston (1986) and Matsushima (1993) study the relationship between multimarket contact and implicit collusion.

denoted by  $c$ .

Under the above setting, a necessary and sufficient condition that SS is self-enforceable is as follows:

for both  $i \in \{1, 2\}$ ,

$$(1 - s_i^a)\pi(p^m) + (1 - s_i^b)\pi(p^m) \leq \frac{\delta}{1 - \delta} \{s_i^a\pi(p^m) + s_i^b\pi(p^m)\}, \quad (4.14)$$

where  $\pi(p) \equiv (p - c)D(p)$ . Again, by the symmetry, we shall assume that  $s_1^a = s_2^a$  and  $s_1^b = s_2^b$ . Then we reduce the inequality (4.14) to:

$$\delta \geq \frac{1}{2}.$$

SS is not enforceable when  $\delta < \frac{1}{2}$ . Then firms may use LS or RS enabling firms collude even when  $\delta < \frac{1}{2}$ . However in this setting we can find a better strategy. Consider the following strategy.

#### **Divided Leadership Strategy (DS)**

(i) Let us define the collusive rule as follows. At stage 0 firm 1 chooses  $L^a$  and  $F^b$  and firm 2 chooses  $F^a$  and  $L^b$ . At stage 1 firm 1 sets the collusive price  $p^m$  and the collusive share  $s_L^a$  of the market  $a$  and firm 2 sets the collusive price  $p^m$  and the collusive share  $s_L^b$  of the market  $b$ . At stage 2 firm 1 sets the price  $p^m$  and the share  $1 - s_L^b$  of the market  $b$  and firm 2 sets the price  $p^m$  and the share  $1 - s_L^a$  of the market  $a$ . All firms obey this collusive rule

in period 1.

(ii) At stage 0 in period  $t$  firm 1 chooses  $L^a$  and  $F^b$  and firm 2 chooses  $F^a$  and  $L^b$  if all firms obey the collusive rule in every period proceeding  $t$ . Otherwise both firms set their price at  $c$  and quantity at  $D(c)$  in both markets from stage 1 in period  $t$  on.

(iii) At stage 1 of period  $t$  firm 1 sets  $p^m$  and  $s_L^a$  in the market  $a$  and firm 2 sets  $p^m$  and  $s_L^b$  in the market  $b$  if all firms continued to obey the collusive rule in every period proceeding  $t$  and besides at stage 0 in period  $t$ . Otherwise both firms sets their price at  $c$  and their quantity at  $D(c)$  in both markets from this stage 1 on.

(iv) At stage 2 in period  $t$  firm 1 sets  $p^m$  and  $1 - s_L^b$  in the market  $b$  and firm 2 sets  $p^m$  and  $1 - s_L^a$  in the market  $a$  if all firms obey the collusive rule in every period proceeding  $t$  and besides at stage 0 and 1 in period  $t$ . Otherwise both firms sets their prices as  $p = c$  and their quantity at  $D(c)$  from this stage on.

In part (i) of DS, the actions which both firms choose in each period as long as they collude are stated. Part (i) says that firm 1 acts as the leader in the market  $a$  and the follower in the market  $b$  while firm 2 acts as the leader in

the market  $b$  and the follower in the market  $a$ . From part (iii) and (iv), it is clear that firms can not gain from deviation when they deviate either at stage 0 or 1. Thus we need only find a condition firms do not deviate just at stage 2.

For simplicity we assume that  $s_L^a = s_L^b = s_L$ . Then a necessary and sufficient condition that DS is self-enforceable is as follows:

for all  $i$ ,

$$(1 - s_L)\pi(p^m) \leq \frac{\delta}{1 - \delta} \{s_L\pi(p^m) + (1 - s_L)\pi(p^m)\}. \quad (4.15)$$

The left-hand side is the gain from deviation. When using DS, each firm does not gain from deviation in the market where the firm acts as the leader because the leader is punished by the follower at stage 2. Hence each firm gets the gain from deviation only in the market where it acts as the follower. The right-hand side is the long-run loss when each firm is punished in both markets in the long-run. From the inequality (4.15) we get the following proposition.

**Proposition 3:** *Suppose that both firms follow the strategy DS. Then their collusion is sustainable if and only if*

$$\delta \geq \delta^{DS}(s_L) \equiv \frac{s_L}{1 + s_L}. \quad (4.16)$$

$\delta^{DS}(s_L)$  has the following characteristics:

- (i) for any  $s_L^1$  and  $s_L^2$  such that  $1 \geq s_L^1 \geq s_L^2 \geq 0$ ,  $\delta^{DS}(s_L^1) \geq \delta^{DS}(s_L^2)$ ,
- (ii)  $\delta^{DS}(1) = \frac{1}{2}$ .

Noting these characteristics, we may state the following economic implications of proposition 3.

1. Collusion is sustainable even if the discount factor is lower than  $\frac{1}{2}$ .
2. Each firm can have the same profit.
3. A particular firm continues to be the leader in a particular market.
4. The leader's share may be bigger than the follower's share when  $\delta^{DS} \in (\frac{1}{3}, \frac{1}{2})$ .

The implication 1 is the same as propositions 1 and 2. The implication 2 says that being the leader does not make firms disadvantageous either in the long-run or in the short-run while firms incur some losses of being the leader in LS and RS.

The word "price leadership" often invokes an impression that a special firm continues to be the leader and gain more profit than others. Implication 3 and 4 may explain part of this impression. However our model do not

explain the type of price leadership that a firm acts as the leader *of a whole industry*. This case may typify the image of “price leadership”.

Nevertheless our model appears to explain the case of the cigarette industry in USA after World War II when American Tobacco acted as the leader in nonfilter cigarettes while Reynolds was the leader in filter tips. Moreover in the market where one company was the leader, it had a wider share than the other and each acted as the follower in the other market.<sup>33</sup> This observation fits well the characterization of the DS equilibrium in our model.

#### 4.7 Concluding Remarks

In this chapter we have considered the effect of price leadership on firms' collusive behavior. We have shown that price leadership enables firms collude even if the discount factor is too low to sustain their collusion without price leadership and firms gain more collusive profits. However at the collusive equilibrium, the leader is less advantageous than followers. To avoid this problem we have investigated cases of rotating price leadership and divided price leadership.

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<sup>33</sup> See in detail Schere and Ross.

In practice the phenomena that a special firm continues to be the leader of a whole industry are often observed. Probably the leader firm is considered to have special power which other firms do not have. Asymmetric information may provide a firm with such special power, as Rotemberg and Saloner or Albaek have shown. Their assumption that a firm has more information on the market demand or cost may not be persuasive in justifying collusive price leadership because such asymmetric information may disappear soon among firms which have similar technology and scale. Hence it remains a problem whether or not we can justify the presence of persistent asymmetric information in the long-run in an oligopolistic market.

The origin of leader's power may include technical advantage. However we have limited technology to be symmetric in this chapter. If we allow asymmetric technologies, then we run into a difficulty in specifying a plausible asymmetric equilibrium point in the collusion. We would like to pursue further investigation to allow the presence of asymmetry in our model.