2 Collusion under Financial Constraints:

Collusion or Predation When the Discount Factor is near One?

2.1 Introduction

Folk theorems claim that firms are likely to cooperate when the discount factor is sufficiently high (e.g., Friedman, 1971; Rubinstein, 1979; Fudenberg and Maskin, 1986). On the contrary, established firms with high discount factors are known to have the incentive to predation (e.g., Milgrom and Roberts, 1982; Benoit, 1984). In an attempt to explain this apparent contradiction, we shall incorporate predation into the repeated "prisoner's dilemma" game. One way to accomplish this attempt is to assume that all firms in the market face some financial constraints.[12] In this framework we consider the possibility of collusion and find that financially constrained firms cannot collude when the discount factor is sufficiently close to one but collusion emerges at lower discount factors.

[12] McGee (1958, 1980), Bork (1978), and Easterbrook (1981) suggest that predatory pricing is unlikely to occur when firms do not face financial constraints.

2.2 The Model

First we define the one-shot simultaneous game $g(s_1, s_2) = (\{1,2\}, \{A(s_i)\}_{i=1,2}, \pi)$, in which actions available for players depend on state variables s_1 and s_2. $A(s_i)$ is the set of the available actions a_i for firm i defined as follows:

$$A(s_i) = \begin{cases} \{C, F\} & \text{if } s_i \leq T_i \\ \{E\} & \text{if } s_i \geq T_i + 1, \end{cases}$$

for a given $T_i \in \mathbb{N}$.[13] The actions "C" and "F" mean "cooperating" and "finking" in the market respectively while "E" means "exiting" from the market. When $s_i \leq T_i$ for $i = 1, 2$, two firms compete in the market and the payoff has the following prisoner's dilemma structure.

Firm 2

Firm 1		C	F
	C	π^c, π^c	$0, 2\pi^c$
	F	$2\pi^c, 0$	$0, 0$

When $s_i \geq T_i + 1$ and $s_j \leq T_j$, firm i is out of the market and earns zero profit, while firm j enjoys the monopoly profit $2\pi^c$, independently of j's chosen action. Finally, when $s_i \geq T_i + 1$ for $i = 1, 2$, both firms are out of

[13] $\mathbb{N}$ and $\mathbb{R}$ stand for the sets of natural numbers and real numbers respectively.

business, each earning zero profit.

We next define the extensive game $G(T_1, T_2; \delta)$ where at each stage $t = 1, 2, \cdots$, firms play $g(s_{1t}, s_{2t})$ defined according to the levels of state variables. The motion of the state variable s_i is as follows: At stage 1, $s_{i1} = 0$. If firm i obtains zero profit at stage t, $s_{i,t+1} = s_{it} + 1$; and otherwise $s_{i,t+1} = s_{it}$. A strategy of firm i, σ_i, is a function which associates an available action to every possible information set. Let $a_{it}(\sigma)$ denote the realized action of firm i at stage t when both firms act according to $\sigma = (\sigma_1, \sigma_2)$. Thus the payoff function of $G(T_1, T_2; \delta)$ is as follows:

$$V_i(\sigma; \delta) = \sum_{t=1}^{\infty} \delta^{t-1} \pi_i(a_{1t}(\sigma), a_{2t}(\sigma)).$$

Our formulation described above intends to capture the situation that firms in the market face financial constraints. Specifically in our model, if firm i has zero profit $T_i + 1$ times, it must exit. Thus T_i means the maximum length of stages which firm i can withstand a predation or a price war in the market.[14] We shall assume that $T_1 \leq T_2$.

[14] We can imagine that the manager is obliged to exit from the market by the shareholders if they don't receive dividends for a certain period.

2.3 The Possibility of Collusion

In the literature of repeated games we can find that collusion is more likely to occur as the discount factor becomes higher. However in this model, where firms face financial constraints, we can show that their collusion does not emerge when the discount factor is near one.

Proposition 1: *Let $2\pi^c\theta$ be the average per-period profit of firm 2 under collusion for $\theta \in (0,1)$. Then for any $\theta \in (0,1)$ there exists $\delta' < 1$ such that if $\delta > \delta'$ firm 2 has an incentive to predation.*

Proof: If firm 2 continues to choose F for $T_1 + 1$ successive periods, firm 1 must exit from the market and thereby firm 2 will enjoy monopoly power from period $T_2 + 2$. The minimum payoff which firm 2 gets through such a predatory behavior is:

$$0 + \delta 0 + \cdots + \delta^{T_1}(0) + \delta^{T_1+1}(2\pi^c) + \delta^{T_1+2}(2\pi^c) + \cdots = \frac{\delta^{T_1+1}}{1-\delta}(2\pi^c).$$

Hence firm 2 has an incentive to predation if

$$\frac{\delta^{T_1+1}}{1-\delta}(2\pi^c) > \frac{2\pi^c\theta}{1-\delta}.$$

Thus firm 2 has an incentive to predation when $\delta > \theta^{1/(T_1+1)}$. □

Taking this result into consideration, we next consider the possibility of collusion or find the condition of collusion, using trigger strategy.

Trigger Strategy: *σ_i^{TR} is a sequence of σ_{it}^{TR} such that $\sigma_{i1}^{TR} = C$ and for $t = 2, 3, \cdots$,*

$$\sigma_{it}^{TR} = \begin{cases} C & \text{if } (s_1, s_2) = (0,0) \\ F & \text{if } (s_1, s_2) \neq (0,0) \text{ and } s_i \leq T_i \\ E & \text{if } s_i \geq T_i + 1. \end{cases}$$

Proposition 2: *Trigger strategies $(\sigma_1^{TR}, \sigma_2^{TR})$ is a subgame perfect Nash equilibrium of $G(T_1, T_2; \delta)$ if and only if $\delta \in [\underline{\delta}(T_1), \overline{\delta}(T_1)]$, where $1/2 < \underline{\delta}(T_1)$ and $\overline{\delta}(T_1) < 1$.*

Proof: The incentive conditions for firm 1 and 2 are respectively given by

$$\pi^c + \delta\pi^c + \delta^2\pi^c + \cdots \geq 2\pi^c + \delta \cdot 0 + \delta^2 \cdot 0 + \cdots \tag{2.1}$$

$$\pi^c + \delta\pi^c + \delta^2\pi^c + \cdots \geq 2\pi^c + \delta \cdot 0 + \cdots + \delta^{T_1} \cdot 0 + \delta^{T_1+1}(2\pi^c) + \cdots \tag{2.2}$$

Condition (2.2), which automatically implies condition (2.1), can be reduced to:

$$2\delta^{T_1+1} - 2\delta + 1 \leq 0. \tag{2.3}$$

We can easily show that, provided $T_1 \geq 4$, this inequality has a solution $\delta \in [\underline{\delta}(T_1), \overline{\delta}(T_1)]$, where $1/2 < \underline{\delta}(T_1)$ and $\overline{\delta}(T_1) < 1$. □

When firms are myopic, they cannot cooperate because the incentive to cheat is overriding in such situations. But this result, as is well known in the study of repeated games, is only a part of what Proposition 2 implies. The other is that when firms are hypermetropic, collusion is again impossible, now because of the predatory behavior of the financially advantageous firm. Thus cooperation emerges when firms view the future in the medium perspective.

2.4 Conclusion

In Proposition 1 we claim that when the discount factor is near one i.e., firms are patient, the firm with financial advantage (firm 2) surely forces a price war and therefore the firms cannot collude.

Then in Proposition 2 we consider the possibility of collusion when we restrict our attention to trigger strategies. We found that collusion between them will emerge only when the discount factor is in the medium range.