Chapter 3

The Response of the Internal Prices in the Agricultural Household Model with Two Missing Markets

3.1 Introduction

De Janvry, Fafchamps, and Sadoulet (1991) highlighted two anomalous features commonly observed for agricultural households, called "internal instability" and "external stability", and investigated them using an agricultural household model with missing markets for food and labor. In case the markets for some commodities are missing, changes in exogenous variables such as market prices make the household adjust its demand and supply of the commodities within the household, which causes their virtual or "internal" prices to fluctuate. Since the fluctuation of the internal prices is unobservable from outside the household, it may be referred to as internal instability. On the other hand, changes in exogenous variables influence the household's factors and commodities in two ways. One is their direct effect with the internal prices being fixed, and the other is their indirect effect via changes in the internal prices. Since the indirect effect often makes the observable response of the factors and commodities inelastic, the inelastic response may be referred to as external stability.

The simulation analysis by de Janvry et al. succeeded in producing the marked internal instability and external stability for some cases, but they did not show how their results are derived from the assumptions in their simulation. In other words, their analysis is ambiguous about which assumptions are crucial to produce the two anomalous features. Search for such assumptions requires a detailed analysis of the response of the internal prices as well as the factors and commodities. Most studies applying agricultural household models, however, have limited their analysis to the case of one market failure. One reason for this limitation may be their lack of interest in dealing with two or more market failures, but there are some other important cases as well as the case of missing markets for food and labor. For example, Ellis (1993) mentions for peasant households in some regions that males are engaged in more productive tasks such as cash crop production while females are engaged in less productive tasks such as food crop production, which is endorsed by a number of studies reviewed by Quisumbing (1996). Then, the behavior of self-employed peasant households may be better described by a model with two market failures for male and female labor. Another more basic reason for the limitation may be the absence of intuitive interpretations for the response of the internal prices in the case of two market failures.[1] Since the indirect effect of changes in the internal prices is responsible for the anomalous response of quantity variables, or the external stability, we will have a better grasp of the anomalous features of agricultural households by understanding how the internal prices respond.

This chapter offers a two-step interpretation of how the internal prices respond in an agricultural household model with missing markets for both food and labor. First, we examine the response of the internal prices in the case of one missing market for either food or labor, which gives the "initial effects" of changes in exogenous variables. More specifically, we examine the shifts and slopes of the demand and supply functions in the "internal market" for each commodity just as in the traditional studies applying a model with one missing market. Starting from the initial effects, cross price effects between food and labor cause the interactions of the two internal markets to change the internal prices continually. Then, the response of the internal prices in the case of two missing markets is thought of as the sum of the changes in the internal prices in the continual interactions. For illustration, we use the values of elasticities and shares assumed by de Janvry, Fafchamps, and Sadoulet (1991) to evaluate various elasticities required for the interpretation. It is found from this analysis that the intuitive interpretation above will reveal not only how the internal instability arises but also which assumptions are crucial to produce it.

[1] Besley (1988) makes a comparative statics analysis of agricultural household models with two or three market failures and directly examines the response of the virtual prices expressed in form similar to equation (3.9) of this chapter.

The following section introduces an agricultural household model with missing markets for both food and labor. The third section gives a detailed account of the initial effects as well as the interactions between the two internal markets to illustrate how the internal prices respond. The final section offers some concluding remarks.

3.2 The Model

It is assumed that the peasant household uses labor, x_l, and other variable inputs, x_v, to produce a cash crop, x_c, and a food crop, x_f.[2] Its production technology is described by

$$g(x) \leq 0, \quad x = (x_c, x_f, x_l, x_v). \tag{3.1}$$

The household consumes food, c_f, leisure, c_l, and manufactured goods, c_m, and its utility function is of the form $u(c)$, $c = (c_f, c_l, c_m)$. The implicit production function $g(\cdot)$ and the utility function $u(\cdot)$ are assumed to be well-behaved in the usual sense.

The household sells cash crop, and buys other variable inputs and manufactured goods at their market prices p_k ($k = c, v, m$), while it is self-sufficient in food crop and labor, so that the demands for them are equal to their respective supplies.

$$c_i = x_i^a + T_i, \quad i \in NT \equiv \{f, l\}, \tag{3.2}$$

where T_i denotes the endowment of commodity i ($T_l > 0$ and $T_f = 0$) and NT denotes the set of nontradable commodities. Quantity x_i^a is defined to represent x_i for i = c, f; $-x_i$ for i = l, v; and 0 for i = m.

The household is assumed to maximize its utility function $u(c)$ with respect to c_i and x_j ($i \in C \equiv \{f, l, m\}$, $j \in P \equiv \{c, f, l, v\}$) subject to the budget constraint (3.3) below as well as the constraints (3.1) and (3.2).

$$\sum_{i \in T} p_i c_i = \sum_{i \in T} p_i (x_i^a + T_i), \tag{3.3}$$

where $c_c = c_v = x_m = 0$ by assumption and $T \equiv \{c, v, m\}$ denotes the set of tradable commodities. The endowment T_i for $i \in T$ is assumed to be nil. Define $p_i^* = \mu_i/\lambda$ for $i \in NT$ and $p_i^* = p_i$ for $i \in T$, where μ_i and λ denote the Lagrange multipliers associated with the constraints (3.2) and (3.3), respectively. Then, the optimality conditions for an

[2] Unlike de Janvry et al., factor inputs are defined to be positive quantities.

interior solution to this problem are written as

$$\phi g_i = \lambda p_i^*, \quad i \in P, \tag{3.4.1}$$

$$g(x) = 0, \tag{3.4.2}$$

$$u_i = \lambda p_i^*, \quad i \in C, \tag{3.4.3}$$

$$\textstyle\sum_{i \in C} p_i^* c_i = Y, \tag{3.4.4}$$

$$c_i = x_i^a + T_i, \quad i \in NT, \tag{3.4.5}$$

where g_i and u_i respectively denote the first derivatives of the functions $g(\cdot)$ and $u(\cdot)$, and ϕ denotes the Lagrange multiplier associated with the technological constraint (3.1). Expression $Y \equiv \sum_{i \in T \cup NT} p_i^*(x_i^a + T_i)$ represents the full income evaluated at the price p_i^*. The prices p_f^* and p_l^* are endogenous variables which equilibrate the demands for food and labor with their respective supplies in the internal markets within the household and may be referred to as their internal prices. If the household participates in the competitive markets for food and labor, their internal prices p_i^* are identical to their exogenous market prices p_i and the agricultural household model becomes separable (Singh, Squire, and Strauss, 1986; Sadoulet and de Janvry, 1995).

The optimality conditions (3.4.1) and (3.4.2) are formally equivalent to those in the corresponding separable model. Hence, they yield the factor demand and output supply functions $x_i = x_i(p^*)$ $(i \in P)$, $p^* = (p_i^*)$ $(i \in T \cup NT)$, with the two endogenous internal prices being included in their arguments.[3] Similarly, the optimality conditions (3.4.3) and (3.4.4) yield the uncompensated commodity demand functions $c_i = c_i(p^*, Y)$ $(i \in C)$.

3.3 Comparative Statics Analysis under Two Missing Markets

This section illustrates how the internal prices of food and labor respond to changes in the market prices using the assumptions made by de Janvry, Fafchamps, and Sadoulet (1991). Before starting the analysis, we need to obtain some price elasticities from Table 3.1 of their paper which presents the values of full income shares and various elasticities. Since the price elasticities of production factors and outputs in the

[3] For convenience of expression, all prices associated with both production organization and consumption choice are included in the price vector p^*.

table correspond to $\partial \ln x_i/\partial \ln p_j$ ($i, j \in P$), the following matrix of elasticities is obtained directly from the table:

$$[\partial \ln x_i / \partial \ln p_j] = \begin{bmatrix} 0.80 & -0.54 & -0.04 & -0.22 \\ -0.77 & 1.00 & -0.08 & -0.15 \\ 0.15 & 0.22 & -0.50 & 0.13 \\ 0.37 & 0.17 & 0.05 & -0.60 \end{bmatrix}, \tag{3.5}$$

where i and j denote f (food crop), c (cash crop), v (other variable inputs), and l (labor) in this order. On the other hand, the price elasticities of consumption commodities in the table correspond to $\partial \ln h_i/\partial \ln p_j - (p_j c_j/Y)(\partial \ln c_i/\partial \ln Y)$ ($i, j \in C$), where h_i denotes the compensated demand for commodity i defined in a way similar to Strauss (1986). The values of these price elasticities, full income shares, and full income elasticities are substituted into a Slutsky-type equation in our model,

$\partial \ln c_i/\partial \ln p_j = \partial \ln h_i/\partial \ln p_j + \{p_j(T_j + x_j^a - c_j)/Y\}(\partial \ln c_i/\partial \ln Y)$,

to yield

$$[\partial \ln c_i / \partial \ln p_j] = \begin{bmatrix} -0.50 & 0.20 & 0.17 & 0.21 & -0.07 \\ 0.22 & -0.46 & 0.06 & 0.27 & -0.10 \\ 0.68 & 0.47 & -1.50 & 0.56 & -0.20 \end{bmatrix}, \tag{3.6}$$

where i denotes f (food), l (leisure), and m (manufactured goods), and j denotes f, l, m, c, and v in this order. Note that changes in the internal prices of food and labor have no income effects because of the condition $T_j + x_j^a - c_j = 0$ for j = f, l. Then, the values in the matrices (3.5) and (3.6) are substituted into equations (3.8) and (3.9) below to yield the values of the elasticities of the internal prices with respect to the prices of cash crop, other variable inputs, and manufactured goods, which are presented in Table 3.1.

Now, we first examine the elasticity $(\partial \ln p_i^*/\partial \ln p_k)_1$ ($i \in NT$; $k \in T$) of the internal price p_i^* in the case of one missing market for either food or labor. Expression $(\partial \ln z/\partial \ln p)_n$ denotes the elasticity of z with respect to p when the number of missing markets is equal to n (=1, 2), or when n internal prices are allowed to change. For convenience of expression, rewrite equation (3.4.5) as

$$q_i^D = q_i^S \quad (i \in NT), \tag{3.7}$$

where $q_f^D = c_f$ and $q_l^D = x_l$ represent the demands for food and labor respectively, while $q_f^S = x_f$ and $q_l^S = T_l - c_l$ represent their respective supplies. If we substitute the demand and supply functions $c_i = c_i(p^*, Y)$ and $x_i = x_i(p^*)$ ($i \in NT$) into equation (3.7) and differentiate both sides of it with respect to p_k with the other internal price p_j ($j \neq i$; $j \in NT$) being fixed, then we obtain

$$(\partial \ln p_i^*/\partial \ln p_k)_1 = -D_{ik}/D_{ii}, \tag{3.8}$$

where $D_{ij} \equiv \partial \ln q_i^D/\partial \ln p_j - \partial \ln q_i^S/\partial \ln p_j$.

The response of the internal price in equation (3.8) results from interactions between the demand and supply functions in the internal market for commodity i. Table 3.2 presents the slopes and shifts of the demand and supply functions obtained from the matrices (3.5) and (3.6).[4] The slope of the demand function, $\partial \ln q_i^D/\partial \ln p_i$, is evaluated at -0.50 for $i = f$ and -0.60 for $i = l$, while the slope of the supply function, $\partial \ln q_i^S/\partial \ln p_i$, is evaluated at 0.80 for $i = f$ and 0.68 for $i = l$. These relatively steep slopes tend to make the internal prices unstable but the quantities demanded and supplied stable. The shifts of the demand and supply functions, $\partial \ln q_i^D/\partial \ln p_k$ and $\partial \ln q_i^S/\partial \ln p_k$, are somewhat large for changes in the price of cash crop but are not for those in the other prices. Thus, the elasticity $(\partial \ln p_i^*/\partial \ln p_k)_1$ of the internal prices is of moderate size for changes in the price of cash crop but is not for those in the other prices as shown in Table 3.1.

Next, these results are used to examine the response $(\partial \ln p_i^*/\partial \ln p_k)_2$ of the internal prices in the case of two missing markets. If we substitute the demand and supply functions of food and labor into the equations in (3.7) for $i = f$ and $i = l$, and differentiate both sides of them with respect to p_k, then we obtain

$$(\partial \ln p_i^*/\partial \ln p_k)_2 = \{D_{ij}D_{jk} - D_{jj}D_{ik}\}/|D| \quad (i, j \in NT;\ k \in T), \tag{3.9}$$

where D is a 2×2 matrix of D_{ij} ($i, j = f, l$). To explain this response of the internal price in an intuitive way, it is helpful to consider an interactive process between the internal markets for food and labor. A rise in the market price p_k has its initial effects $(\partial \ln p_f^*/\partial \ln p_k)_1$ and $(\partial \ln p_l^*/\partial \ln p_k)_1$ on the internal prices of food and labor, respectively, just as in the preceding analysis of one missing market. Due to cross price effects between the two commodities, such a rise in the internal price of commodity j shifts the demand and supply functions of the other commodity i to change its internal price.

[4] The price elasticities of labor supply are evaluated using the relation $\partial \ln q_l^S/\partial \ln p_l = -(\partial \ln c_l/\partial \ln p_l)(p_l c_l/Y)/(p_l x_l/Y)$, where $p_l c_l/Y = 0.383$ and $p_l x_l/Y = 0.255$ can be obtained from Table 1 of de Janvry et al.

Recalling equation (3.8), this change in the internal price p_i^* may be expressed as $(-D_{ij}/D_{ii})\times(\partial \ln p_j^*/\partial \ln p_k)_1$. Let $\alpha \equiv -D_{fl}/D_{ff}$ and $\beta \equiv -D_{lf}/D_{ll}$, which are evaluated at 0.32 and 0.55 respectively for the values in Table 3.2.

Repeating this interaction between the two internal markets gives an expression for the response of the internal prices in the case of two missing markets.

$$(\partial \ln p_f^*/\partial \ln p_k)_2 = (\partial \ln p_f^*/\partial \ln p_k)_1 + \alpha_1(\partial \ln p_l^*/\partial \ln p_k)_1 + \alpha_2(\beta_1(\partial \ln p_f^*/\partial \ln p_k)_1) + \ldots,$$

$$(\partial \ln p_l^*/\partial \ln p_k)_2 = (\partial \ln p_l^*/\partial \ln p_k)_1 + \beta_1(\partial \ln p_f^*/\partial \ln p_k)_1 + \beta_2(\alpha_1(\partial \ln p_l^*/\partial \ln p_k)_1) + \ldots, \quad (3.10)$$

where $\alpha_m = \alpha$ and $\beta_m = \beta$ $(m = 1, 2, \ldots)$. For example, the initial rise $(\partial \ln p_f^*/\partial \ln p_c)_1 = 0.57$ in the internal price of food for changes in cash crop price p_c raises the internal price p_l^* of labor by $\beta_1(\partial \ln p_f^*/\partial \ln p_c)_1 = 0.55\times0.57 = 0.31$ in the internal market for labor, which in turn increases the internal price p_f^* of food by $\alpha_2(\beta_1(\partial \ln p_f^*/\partial \ln p_c)_1) = 0.32\times0.31 = 0.10$ in the internal market for food, and so on. Since $\alpha_m = \alpha$ and $\beta_m = \beta$ for all m, the equations in (3.10) are rearranged as

$$(\partial \ln p_f^*/\partial \ln p_k)_2 = \sigma(\partial \ln p_f^*/\partial \ln p_k)_1 + \alpha\sigma(\partial \ln p_l^*/\partial \ln p_k)_1,$$

$$(\partial \ln p_l^*/\partial \ln p_k)_2 = \sigma(\partial \ln p_l^*/\partial \ln p_k)_1 + \beta\sigma(\partial \ln p_f^*/\partial \ln p_k)_1, \quad (3.11)$$

where $\sigma = \sum_{t=0}^{\infty}(\alpha\beta)^t$ and $\alpha\beta > 0$. The infinite series σ converges if $\alpha\beta < 1$, or equivalently, if $|D| = D_{ff}D_{ll} - D_{fl}D_{lf} > 0$, which should be satisfied for well-behaved production and utility functions. Hence, σ converges to $1/(1-\alpha\beta)$ and the equations in (3.11) are rewritten as

$$(\partial \ln p_i^*/\partial \ln p_k)_2 = (D_{ii}D_{jj}/|D|)(\partial \ln p_i^*/\partial \ln p_k)_1 + (-D_{ij}D_{jj}/|D|)(\partial \ln p_j^*/\partial \ln p_k)_1, \quad (3.12)$$

which proves to give another expression for the elasticity of the internal price in (3.9).

The multipliers α and β in the interactive process are positive and the two initial effects of changes in the price of cash crop are also positive under our assumptions. Hence, starting from the initial effects, the internal prices p_i^* continue to rise by gradually diminishing amounts and their responses eventually converge to the magnitudes shown on the right hand side of equation (3.9) or (3.12). Similar arguments apply to changes in the other market prices, since the two initial effects $(\partial \ln p_f^*/\partial \ln p_k)_1$ and $(\partial \ln p_l^*/\partial \ln p_k)_1$ have the same sign for each of the cases k = v and k = m. The intuitive interpretation above illustrates how the relation $|(\partial \ln p_i^*/\partial \ln p_k)_1| <$

$|(\partial \ln p_i^*/\partial \ln p_k)_2|$, or the intensified internal instability, occurs in the simulation study by de Janvry et al.

Based on the interpretation above, we can summarize which assumptions produce the intensified internal instability of the household assumed by de Janvry et al. For one thing, the two initial effects, $(\partial \ln p_f^*/\partial \ln p_k)_1$ and $(\partial \ln p_l^*/\partial \ln p_k)_1$, in equation (3.12) have the same sign for each of the three cases k = c, v, m, which results from plausible assumptions on the household's price response.[5] For example, since the difference D_{ii} is negative in equation (3.8), the positive initial effects of changes in the price of cash crop depend on the signs of the shifts $\partial \ln q_i^D/\partial \ln p_c$ and $\partial \ln q_i^S/\partial \ln p_c$ of the demand and supply functions shown in Table 3.2. These shifts reflect plausible assumptions that food and leisure are normal goods, the demand for labor increases with the price of cash crop, and that the two crops are competitive. For another thing, the differences D_{fl} and D_{lf} of the cross price effects between food and labor are positive. These positive differences reflect the substitutability between food and leisure as well as the naturally expected behavior of producers, that is, their demand for labor increases with the price of food, while their supply of food decreases with the price of labor.

Finally, we can also summarize the reasons why the internal instability is marked for changes in the price of cash crop but is not for those in the other prices. For one thing, the initial effects associated with the price of cash crop are of moderate size, while those associated with the other prices are very small. The latter reflects the assumptions made by de Janvry et al. that the demand and supply functions of food and labor hardly shift in response to changes in the prices of other variable inputs and manufactured goods as shown in Table 3.2. For another thing, the coefficients $D_{ii}D_{jj}/|D|$ and $-D_{ij}D_{ji}/|D|$ of the initial effects in equation (3.12) are evaluated at 1.21 and 0.39 for i = f, j = l, and at 1.21 and 0.67 for i = l, j = f, respectively. The magnitude of these coefficients depends on that of the multipliers α and β in the interactive process introduced above, which in turn depends crucially on that of the cross price effects $\partial \ln q_i^D/\partial \ln p_j$ and

[5] Conversely, if the two initial effects have opposite signs as found by de Janvry et al. for the case of an increased food productivity, the associated internal instability cannot be observed, given the conditions $D_{fl} > 0$ and $D_{lf} > 0$.

$\partial \ln q_i^S/\partial \ln p_j$ (i, j $\in$ NT; i $\neq$ j). This is a natural consequence because no cross price effects between food and labor imply that $D_{fl} = D_{lf} = 0$ hence $\alpha = \beta = 0$. In this case, no interactions occur between the two internal markets, so that the elasticity of the internal price in equation (3.9) or (3.12) is reduced to the one in equation (3.8). The cross price effects between food and labor are not elastic as shown in Table 3.2. However, they are large enough to expand the initial effects $(\partial \ln p_i^*/\partial \ln p_k)_1$ and $(\partial \ln p_j^*/\partial \ln p_k)_1$ into $1.21\times(\partial \ln p_i^*/\partial \ln p_k)_1$ and $0.39\times(\partial \ln p_j^*/\partial \ln p_k)_1$ for i = f, j = l, and into $1.21\times(\partial \ln p_i^*/\partial \ln p_k)_1$ and $0.67\times(\partial \ln p_j^*/\partial \ln p_k)_1$ for i = l, j = f, respectively, through the interactions of the two internal markets. Thus, the moderate initial effects of changes in the price of cash crop are expanded markedly, while the very small initial effects of changes in the other prices are not.

3.4 Concluding Remarks

This chapter offered a two-step interpretation for the response of internal prices in an agricultural household model with missing markets for both food and labor. It is found that the response of their internal prices depends exclusively on the sign and magnitude of the initial effects of changes in exogenous variables and on those of the cross price effects between food and labor.

The initial effects are identified with the response of the internal prices in the case of one missing market, so that they are determined by the slopes and shifts of the demand and supply functions in the individual internal markets. Under the assumptions made by de Janvry et al., both the demand and supply functions are relatively steep for food and labor, so that their internal prices tend to be unstable. In addition, changes in the price of cash crop cause relatively large shifts of the demand and supply functions, while those of the other market prices do not, which makes a large difference between the associated initial effects.

On the other hand, the magnitude of the cross price effects between food and labor determines how much the initial effects are expanded by the interactions of the two internal markets. Under the assumptions made by de Janvry et al., the cross price effects are not so elastic but are large enough to expand markedly the initial effects of changes in the price of cash crop. It is this expansion of the initial effects that describes the mechanism producing the intensified internal instability of agricultural households.

Table 3.1. Elasticities of the Internal Prices under Three Types of Market Failures

Number of Missing Markets	n = 2	n = 1 (Labor)	n = 1 (Food)
Internal Price of Food p_f			
$\partial \ln p_f / \partial \ln p_c$	0.87	0.00	0.57
$\partial \ln p_f / \partial \ln p_v$	–0.06	0.00	–0.03
$\partial \ln p_f / \partial \ln p_m$	0.19	0.00	0.13
Internal Price of Labor p_l			
$\partial \ln p_l / \partial \ln p_c$	0.92	0.45	0.00
$\partial \ln p_l / \partial \ln p_v$	–0.11	–0.07	0.00
$\partial \ln p_l / \partial \ln p_m$	0.17	0.07	0.00

Note: "n = 1 (Labor)" represents the case of one missing market for labor, while "n = 1 (Food)" represents a similar case for food. In the case "n = 2", both of the two markets are missing.

Table 3.2. Shifts and Slopes of the Demand and Supply Functions of Food and Labor

	p_c	p_v	p_m	p_f	p_l
Food					
q_f^D (= c_f)	0.21	–0.07	0.17	–0.50 *	0.20 *
q_f^S (= x_f)	–0.54	–0.04	0.00	0.80	–0.22
Labor					
q_l^D (= x_l)	0.17	0.05	0.00	0.37	–0.60
q_l^S (= $T_l - c_l$)	–0.40	0.14	–0.09	–0.34 *	0.68 *

Note: The values in this table are the elasticities of quantities q_i^D and q_i^S (i = f, l) with respect to price p_j (j = c, v, m, f, l) with the other prices being fixed, where q_i^D and q_i^S denote the demand and supply of commodity i, respectively. The values of the elasticities with an asterisk do not include income effects.