Fast relaxation, boson peak, and anharmonicity in Li₂O-B₂O₃ glasses

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Glasses with a composition $x \text{Li}_2 \text{O} \cdot (1-x) \text{B}_2 \text{O}_3$ were investigated by low-frequency Raman scattering in the composition range x=0-0.28. The evolution of the quasielastic line, the boson peak, the Debye frequency, and some other glass parameters with the composition was analyzed. The frequency of the boson peak ω_b shifts with changing x by a factor of 3 and the width of the quasielastic spectrum at room temperature is always equal to $\sim 0.24\omega_b$. The Grüneisen parameter of the glasses is estimated on the basis of the light scattering data for the boson peak frequency within the frames of the anharmonic model of the fast relaxation and using the sound velocity data—for the Debye frequency. The anharmonic properties are compared with the fragility of these glassformers; it is shown that the fragility increases with anharmonicity. It is shown also that the width of the glass transition region correlates with the anharmonic properties. © 2000 American Institute of Physics. [S0021-9606(00)70939-1]

I. INTRODUCTION

Low-frequency Raman scattering spectra in glasses exhibit two characteristic features: the boson peak in the THz region and the quasielastic line with a characteristic width on the order of a few hundreds GHz.¹ The former arises due to excess vibrations with a characteristic length on the intermediate range scale and the latter, due to fast relaxation processes. There is still no commonly accepted theory of these glass features. For the boson peak there are some phenomenological models²⁻⁹ and computer simulation results.^{10,11} Recently, a theory which predicts the boson peak was proposed.¹² Fast relaxation was attributed to the thermally excited transitions in the double well potentials,^{13,14} to the anharmonicity of vibrations¹⁵ or to fluctuations of the dynamical free volume holes.¹⁶ Recently, a significant progress was achieved in understanding the fast relaxation in glasses after applying tandem Fabry-Perot technique for measuring the broadband (1-600 GHz) relaxation spectra.¹⁷⁻²¹ In particular, it was shown that in glassy silica the fast relaxation process is caused by thermally activated jumps in the double well potentials at least up to the room temperature,²¹ while in boron oxide glass the anharmonicity of the vibrations may be the dominant contribution.²⁰ In the latter case the quasielastic light scattering (QELS) apparently has a vibrational nature, i.e., it is a relaxational part of the vibrational response function. It means that the QELS spectrum should have some connection with the most prominent part of the acoustical spectrum-the boson peak. However, no systematic study of such connection is known.

Lithium borate glasses provide very good opportunity to check the correlation of some glass properties with the boson

^{a)}On leave from Institute of Automation and Electrometry, Russian Academy of Sciences, Novosibirsk, 630090, Russia. peak and QELS and between both of them because the frequency of the boson peak changes very strongly with Li_2O fraction; the shift of the boson peak by a factor of 3 was reported²² for lithium borate glasses. Fragility of these glassformers also demonstrates a strong dependence on the lithium fraction.²³ It was shown that there is a correlation between the QELS, boson peak, and fragility in various glassformers²⁴ which is still unexplained. The purpose of this paper is to analyze the behavior of the boson peak and QELS in a series of the lithium borate glasses of different composition and to check the possible correlations between them and other properties of the glasses.

II. EXPERIMENT

A. Samples

A series of lithium borate glasses of the composition xLi₂O·(1-x)B₂O₃ was prepared at regular intervals of 0.02 mole fraction through the composition range $0 \le x \le 0.28$. An

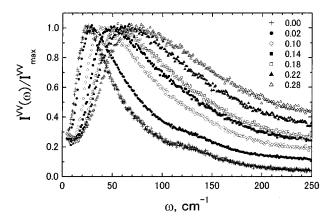


FIG. 1. The boson peak in lithium borate glasses of various composition x.

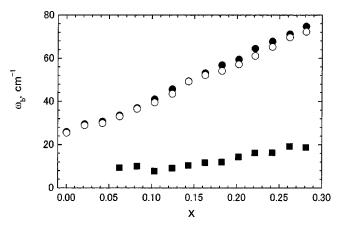


FIG. 2. Boson peak frequency in VV (solid circles) and VH (open circles) scattering. Solid squares, the width of the QELS τ^{-1} .

attempt was made to prepare bulk glasses with higher x, up to x=0.36, but all of their respective melts crystallized quickly after casting. Analytical reagent grade LiOH·H₂O (of purity greater than 99.0%, the main residue is lithium carbonate) and H₃BO₃ (of purity greater than 99.7%) were used as the starting materials. The details of the sample preparation are described elsewhere.²⁵

B. Measurements

The room temperature Raman scattering was measured using an Ar ion laser with the power 50 mW, an additive triple-grating spectrometer Joben Yvon, and a photon counting system. The samples we immersed in silicone oil with a quartz optical cell of 15 mm \times 15 mm \times 20 mm to prevent them from absorbing moisture from the air. Both polarized (VV) and depolarized spectra (VH) are measured in a backscattering geometry in the frequency range 5–300 cm⁻¹.

III. RESULTS AND DISCUSSION

A. Boson peak

In Fig. 1 the polarized (VV) spectra of the boson peak are shown for a series of composition *x*. The amplitude is normalized to unity at the maximum of the boson peak. For *x* from 0 to 0.28 the frequency of the maximum of the boson peak, ω_b , increases by a factor ~3, from 25 to 79 cm⁻¹ (Fig. 2). However, the spectral shape of the boson peak remains the same at all *x*; in Fig. 3 the combined plot obtained by scaling the frequency on the frequency of the maximum ω_b for each composition is shown. All curves coincide well except a small shoulder near $\omega/\omega_b ~3.7$ which gradually decreases with increasing *x*.

It is known that increase of the mole fraction of Li_2O in lithium borate glasses leads to a significant increase of sound velocities and density.²⁵ Since the boson peak is in acoustical region of the vibrational spectrum, one can suspect that its shift with the change of the composition is a trivial one and arises merely due to the change of the average elastic constants and density. In this case it should shift just like, e.g., the Debye frequency ω_D . We compared the dependence of ω_b and ω_D on x. The Debye frequency

$$\omega_D = v_D (6\pi n)^{1/3} \tag{1}$$

[where $v_D = (2/v_l^3 + 1/v_l^3)^{-1/3}$ is the Debye velocity] we estimated using the data on the transversal (v_l) and longitudinal (v_l) sound velocities and particle density *n* from Ref. 25. For the reader's convenience we plot these data in Fig. 4. In Fig. 5 the dependence of ω_b and ω_D on *x* is shown. To make the comparison clearer we normalized the frequencies by their values at x=0. Figure 5 shows that the relative shift of the boson peak is faster than that of the Debye frequency (except a small interval x<0.05): both depend on *x* linearly but the slope is equal to 6.9 for $\omega_b(x)/\omega_b(0)$ and 4.3 for $\omega_D(x)/\omega_D(0)$. In particular, at x=0.28 the shift of the nor-

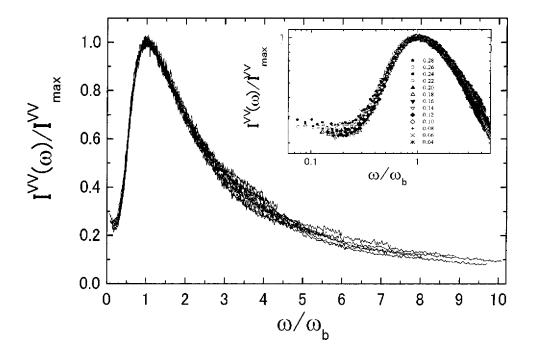


FIG. 3. The scaled boson peak spectra for all compositions *x*. Inset: $\log \omega$ plot of a part of the boson peak.

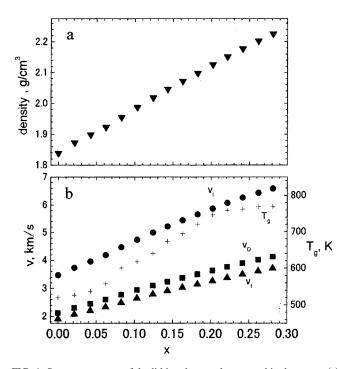


FIG. 4. Some parameters of the lithium borate glasses used in the paper: (a) density; (b) longitudinal (solid circles), transversal (solid triangle) sound velocities and glass transition temperature (cross). The data are from Ref. 25. Solid squares, Debye velocity.

malized Debye frequency is by 30% lower than that of the boson peak. This means that the evolution of the boson peak is different from one determined by scaling of the elastic constants and density; the shape of the full acoustical spectrum changes and mirrows more rapid evolution of the boson peak in comparison with the higher frequency acoustical vibrations. We note that at x < 0.05 some other properties of the lithium borate glasses behave differently from those at x > 0.05 (see, e.g., Figs. 6, 9, and 11).

Sound velocity and frequency define a characteristic length, $L \sim v/\omega$. For the Debye frequency this length is of the order of the minimum possible acoustic wavelength determined by the interatomic distance. For the boson peak this length, which is a few times longer, should correspond to some structure features which are responsible for the appear-

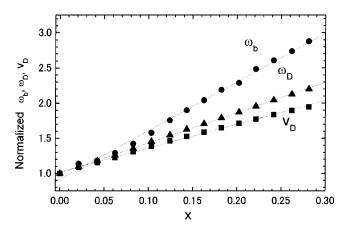


FIG. 5. Boson peak frequency (solid circles), Debye frequency (solid triangles), and Debye velocity (solid squares) normalized to 1 at x=0.

ance of these excess acoustical vibrations. This may be the size of the typical structure nanoheterogeneities,²⁻⁴ or correlation length of the structure,⁵⁻⁷ or a size of the soft-potential atomic configurations.⁸ In silica glass the boson peak vibrations were identified with a specific motion of the a few connected SiO₄ tetrahedra.⁹⁻¹¹ In any case, for the boson peak *L* corresponds to a medium-range scale important for characterization of structure correlations in glasses. In Fig. 6 a characteristic length defined as

$$L_b = v_D / \omega_b \tag{2}$$

is shown as a function of composition for the lithium borate glasses. The length L_b decreases from 2.7 to 1.9 nm, i.e., by the factor 1.42, in the interval x = 0 - 0.28. However, the decrease of L_b itself does not necessary means the decrease of the extent of the medium-range correlations in the glass, because the density of the samples increases with x. To characterize the behavior of the structure correlations one needs to compare the decrease of L_b with the behavior of a length which characterizes the average interparticle distance. The latter can be defined as $a(x) = (\rho(x)/M(x))^{-1/3}$, where $M(x) = xM_{\text{Li}_2\text{O}} + (1-x)M_{\text{B}_2\text{O}_3}, \qquad M_{\text{Li}_2\text{O}} = 29.8M_{\text{au}},$ and $M_{\rm B_2O_3}$ = 69.62 $M_{\rm au}$ are the respective molar masses, $M_{\rm au}$ is the atomic mass unit. The dependence of a on x is shown in Fig. 6 (inset b); it is a perfect linearly decreasing function. On the inset a in this figure the ratio L_b/a is shown. This ratio decreases more or less monotonically from 6.8 at x =0 to 5.2 at x = 0.28. Thus, L_b decreases not only in absolute value (by 42%), but also in units of interatomic distance (by 30%). This means that increase of the Li₂O concentration leads to decrease of the medium-range structure correlations; at concentration z% the decrease of the characteristic length is roughly also z% at least in the interval 5%-28%.

B. Quasielastic scattering

At lowest frequencies of the measurements the quasielastic light scattering dominates the spectra, Fig. 1. QELS is seen in the frequency interval from $0.06\omega_b$ up to $\sim 0.3\omega_b$ (e.g., 4–25 cm⁻¹ for x=0.28). To enhance the QELS part of the spectra, they are shown in the insert of Fig. 1 in the log ω coordinate. Similar to the boson peak, QELS roughly scales on a universal curve for different compositions.

QELS arises due to a fast relaxation process with a relaxation time τ on the order of a picosecond.¹ It was investigated in many glasses by means of the light and neutron scattering (see, e.g., Ref. 26 for a small review). Recently, broad band light scattering measurements by tandem Fabry-Perot interferometer¹⁷⁻²⁰ revealed a broad (1-100 GHz) lowfrequency wing of the fast relaxation in glasses described by a power law with an exponent $\alpha < 1$. However, in the frequency region close to the boson peak the QELS spectrum normally was successfully fitted by a Lorentzian.^{15,16,27-30} In the present paper we fitted the fast relaxation spectrum also by a Lorentzian which was defined by the two parameters: the width τ^{-1} and the strength of the relaxation process, δ^2 , which is equal to the ratio of the integrals over the fitted QELS spectrum and that over the boson peak. The vibration spectrum we fitted by the power law $I \propto \omega^a$ extrapolated from

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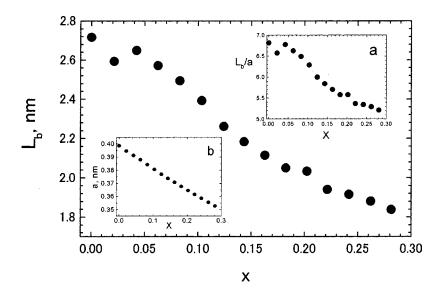


FIG. 6. Correlation length defined by Eq. (2). Insets: (a) the ratio of the correlation length L_b to the average interatomic distance a; (b) the dependence of a on the composition.

the interval around $0.5\omega_b$. The fits coincide with the experimental curves within the experimental error. The master curve in Fig. 3 gives a=2 with the slopes of the individual curves scattered around this value in $\sim 10\%$ interval. Within the same error we found $\delta^2 \approx 0.09$ in all samples except those with $x \le 0.04$; at these low concentrations no reliable estimation of the QELS parameters was available due to the finite low-frequency resolution of the experiment of about 5 cm^{-1} . The dependence of the parameters τ^{-1} and ω_{b} on the composition is shown in Fig. 2. As it is expected already from the fact that QELS for different compositions scales on the master curve together with the boson peak (Fig. 3), the width of QELS also scales with the boson peak frequency. Figure 7 shows that in rough approximation there is a linear correlation between τ^{-1} and the boson peak frequency. In Fig. 8 the ratio τ^{-1}/ω_b is plotted as a function of Li₂O fraction; this figure shows that roughly $\tau^{-1} \approx 0.24 \omega_h$.

These findings support the anharmonic model of the fast relaxation.^{15,31} Within the frame of this model the anharmonicity of the lattice leads to the fast vibrational relaxation. Qualitatively, this relaxation process occurs due to the cou-

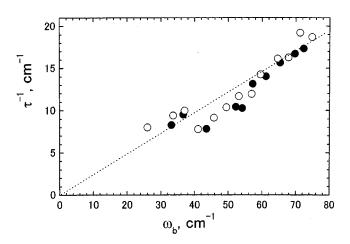


FIG. 7. Correlation between the inverse fast relaxation time τ^{-1} (the width of the QELS line) and the boson peak frequency for a sequence of the lithium borate glasses of various composition. Solid circles, VV; open circles, VH polarization.

pling of a vibration mode to the fluctuations of vibration occupation number $n(\omega)$ (or to temperature at high enough *T*). Since $n(\omega)$ is determined by the squared amplitude of the vibrations, such coupling is provided in first approximation just by the third order anharmonic coefficient, or Grüneisen parameter γ . Such relaxation process gives rise to a self-energy term $M_{\Omega}(\omega)$ in the vibration susceptibility $\chi_{\Omega}(\omega)$; in a single time approximation it can be written as

$$M_{\Omega}(\omega) = \frac{\delta^2 \Omega^2}{1 - i\omega\tau}.$$
(3)

Here Ω is the frequency of the vibration and the parameter δ characterizes the strength of the relaxation process. The model¹⁵ predicts that

$$\delta^2 = \frac{\gamma^2 T}{M v_t^2},\tag{4}$$

where *M* is the molar mass. Finally, in the same single-time approximation the reduced QELS intensity, $I_r^{\text{QELS}} = I^{\text{QELS}}/\omega(n(\omega)+1)$, is equal to

$$I_r^{\text{QELS}}(\omega) \approx \frac{2}{\pi} \frac{\delta^2 \tau}{1 + \omega^2 \tau^2} \int I_r^{\text{vib}}(\Omega) d\Omega, \qquad (5)$$

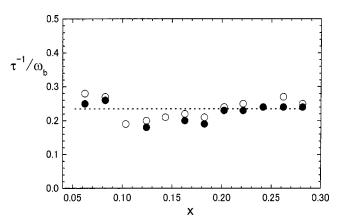


FIG. 8. The ratio of the width of QELS line to the boson peak frequency for data from Fig. 4. Solid circles, VV; open circles, VH polarization.

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where $I_r^{\text{vib}}(\Omega)$ is the reduced intensity of the light scattering by vibrations in the limit $\delta = 0$. $I_r^{\text{vib}}(\Omega)$ essentially represents the boson peak, so $I_r^{\text{QELS}}(\omega)$ is proportional to the integral over the boson peak. Integrating over all ω , Eq. (5) has

$$\delta^2 = \frac{\int I_r^{\text{QELS}}(\omega) d\Omega}{\int I_r^{\text{vib}}(\Omega) d\Omega},\tag{6}$$

so, according to Eq. (4) the integral over the QELS spectrum gives information about the Grüneisen parameter.

The anharmonic model makes prediction that the width of the QELS spectrum correlates with the frequency of the boson peak, namely,

$$\tau^{-1} \approx \omega_b / 3. \tag{7}$$

This relation is given by a simplified estimation, thus the coefficient between τ^{-1} and ω_b may be somewhat different from 1/3. There was a phenomenological observation^{15,31} that in various glasses at $T = T_g$ this relation holds surprisingly good. However, up to now there was no data on the relation between τ^{-1} and ω_b in a sequence of glasses with similar chemical composition in a broad range of ω_b values. The lithium borate glasses give a rare case of the very strong shift of the boson peak as a function of a one smooth variable, composition x, making it possible to check the prediction of Eq. (7). As we have seen above, our data holds this relation with the coefficient 0.24 instead of 1/3. Although 1/3is only a rough estimate, one should note that the fast relaxation time, as it is known from the analysis of the light scattering data in other glasses (e.g., Refs. 15, 16, and 27-30) normally weakly decreases when temperature increases; since we analyzed the room temperature data, one can expect at the glass transition τ^{-1} will be a little bit higher. The data of Ref. 28 in supercooled B_2O_3 indeed gives for τ^{-1} the value $\omega_b/3$ at $T = T_g . ^{15}$

C. Anharmonicity

 $\gamma_D = 1/3 - k_D / \varkappa$

Using the value $\delta^2 \approx 0.1$ obtained by fitting the QELS spectrum as described above, one can estimate the Grüneisen parameter γ for the boson peak within the anharmonic model of the fast relaxation Eq. (4). Taking the v_t and T_g data for our set of the lithium borate glasses from Ref. 25 (Fig. 4), we obtained γ shown in Fig. 9(a). Figure 9(a) shows that estimated from QELS data Grüneisen parameter increases from 4 to 6 in the interval x=0.06-0.28. This means that the anharmonicity of the boson peak vibrations in lithium borate glasses increases with an increase of the lithium oxide fraction.

For lithium borate glasses the temperature variations of the sound velocity and of the density are known.³² Thus, one can determine the Grüneisen parameter $\gamma_D = \partial \ln \omega_D / \partial \ln V$ for the Debye frequency defined by Eq. (1). If \varkappa is a volume thermal expansion coefficient, $\varkappa = \partial \ln V / \partial T$ and k_l , k_t , and k_D are the thermal coefficients for the longitudinal, transversal, and Debye velocities, respectively (e.g., $k_t = \partial \ln v_t / \partial T$), then

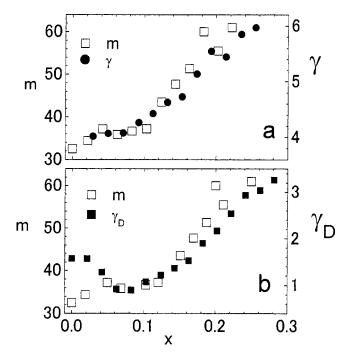


FIG. 9. Correlation between the Grüneisen parameters and fragility for glasses of different composition. (a) Grüneisen parameter for the Debye frequency γ_D (solid squares) and fragility *m* (open squares); (b) Grüneisen parameter for the boson peak γ (solid circles) and fragility. The data for fragility are taken from Ref. 23.

where

$$k_D = \frac{v_D^3}{3} \left(\frac{2k_t}{v_t^3} + \frac{k_l}{v_l^3} \right).$$
(9)

The parameter γ_D is shown in Fig. 9(b). Since the high-frequency vibrations near the end of the acoustical spectrum relax weaker than the boson peak vibrations, γ_D is less than the respective parameter γ for the vibrations in the region of the boson peak. However, the general behavior of γ_D and γ is similar except the region of low-concentration of Li₂O, where we have no data for γ .

An important property of glassformers is the so-called fragility which shows how quickly the viscosity η , or α -relaxation time, changes in the glass transition region if temperature is scaled by T_g .³³ The nature of the fragility is still not fully understood. In Ref. 24 it was noted that the intensity of the QELS in glassformers, normalized to that of the boson peak at $T = T_g$, correlates with the fragility. The anharmonic model of the fast relaxation predicts that the intensity of QELS in a glass is determined by the cubic anharmonic coefficient, so in Ref. 15 it was claimed that the fragility increases together with anharmonicity. This was checked and confirmed in Ref. 15 for a few different glassformers. Here we have a possibility to check the correlation for a sequence of glassformers of similar composition. The latter makes unimportant such factors as different types of chemical bonding and structure which may influence the comparison in the case of different materials.

The fragility is quantified by the parameter m defined as

$$m = \frac{d \ln \eta}{dT_g/T}$$
 at $T \to T_g$. (10)

(8)

We took the data for the fragility of lithium borate glasses for some x in the literature.²³ In Fig. 9 the dependence of the parameter m on the composition is compared with that of the Grüneisen parameters. One can see that indeed there is a correlation of the fragility with the Grüneisen parameters; the higher anharmonicity corresponds to higher fragility.

In Ref. 34 a correlation between the fragility and the correlation length was discussed. It was found that the in different materials the correlation length has a tendency to decrease with increasing fragility of the glassformer. Such comparison of course has more sense in a series of samples of a similar chemical composition. We note that indeed, in the case of lithium borate glasses fragility increases while the correlation length decreases with increasing the Li₂O fraction (Figs. 6 and 9), except the region x < 0.08 where the behavior is more complicated. This corresponds to the ideas that fragility is higher in more structurally (but not chemically) homogeneous glasses and decreases with increase of the structure heterogeneity on the nanometer scale.³⁵

The structural basis for changing of the fragility in lithium borate glasses is not clear. The increase of the fragility in lithium borate glasses with x is in agreement with the model of Vilgis³⁶ in which the degree of fragility is connected to the fluctuations of the coordination number in the system, Δz . Network glasses with fixed covalent bonding have $\Delta z = 0$, and, respectively, they are strong glassformers. Fragile systems always have nonzero Δz . In pure boron oxide the boron atom has a fixed valence equal to 3. Possible small fluctuations of the number of covalent bonds are connected only to the structure defects. In the case of lithium borate glasses some boron atoms have valence 4 in the structural units $Li^+BO_4^-$. Thus, the number of the covalent bonds of boron atoms fluctuates between 3 and 4. A NMR study of Jellison et al.³⁷ showed that the fraction of 4-coordinated boron atoms, p_4 , for $0 \le x \le 0.28$ is determined by x as follows: $p_4 = x/(1-x)$. The average number of valence bonds per boron atom is $\overline{z} = 4p_4 + 3(1-p_4) = 3 + x/(1-x)$. It is easy to find the squared fluctuation of z, $\overline{\Delta z^2} = \overline{z^2} - \overline{z^2} = p_4(1)$ $-p_4$). One has $\overline{\Delta z^2} = x(1-2x)/(1-x)^2 \approx x + O(x^3)$. Thus, neglecting the third power term of the small parameter x, we see that the correlation between the fragility m and Δz^2 means that there should be also the same correlation between m and x in Fig. 9. Thus, at least in the case of the lithium borate glasses fragility indeed correlates with the degree of fluctuations of the boron coordination number.

Differential thermal analysis (DTA) of the lithium borate glasses in the vicinity of the glass transition²⁵ reveals the onset temperature T_g , the peak temperature T_p , and the end temperature T_e in terms of the phenomenological theory of the glass transition.³⁸ The former temperature is often referred to as the glass transition temperature. Figure 10 shows schematically a part of the DTA heating curve in the vicinity of the glass transition with the definition of the three temperatures. In DTA, the temperature difference is measured between the sample to be studied and the standard sample. So, in Fig. 10, ΔT is the difference of temperatures between the sample to be studied and the standard sample. The details of the measurements are described in Ref.

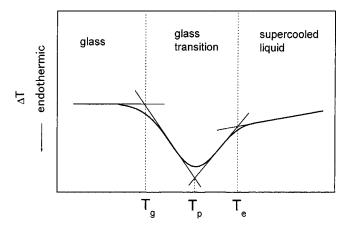
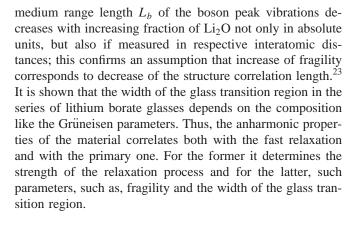


FIG. 10. Schematic differential thermal analysis curve of the glass transition.

25. Here we compare the width of the glass transition region which we define as $\Delta T_g = T_e - T_g$ with the anharmonic properties and fragility. We are motivated by the results of the analysis of the glass transition in terms of the mode coupling theory.³⁹ The theory predicts the so-called critical temperature T_c of the ideal glass transition; it was found that T_c $>T_g$. Phenomenological analysis has shown that the relative interval $(T_c - T_g)/T_g$ is larger in strong glassformers and lesser in the fragile.^{26,35,40} Here we check if such correlation exist also between the width of the glass transition, fragility, and anharmonicity. In Fig. 11, both $T_g/\Delta T_g$ and the Grüneisen parameter for the Debye frequency are shown as a function of the composition. The correlation of the relative width of the glass transition with the anharmonic parameter is very good even at low x < 0.1. The parameters γ and mcorrelates with $T_g/\Delta T_g$ as good as γ_D ; we show only the latter since it was determined in a broader interval of x. These results show that at least in a series of lithium borate glasses with different composition the normalized width of the glass transition strongly correlates with the fragility and anharmonicity of the samples; this stresses the importance of the anharmonic effects for the theory of the glass transition.

IV. CONCLUSION

Due to a strong dependence of the boson peak frequency on the composition the lithium borate glasses give an excellent opportunity to investigate correlations between the lowfrequency Raman spectra (quasielastic scattering and the boson peak), anharmonicity, and fragility. We fitted the quasielastic line by a single relaxation time function and using this data and the anharmonic model of the fast relaxation¹⁵ found the Grüneisen parameter for the boson peak vibrations. We estimated also the Grüneisen parameter for the Debye frequency and found out that both parameters increase when Li₂O fraction increases. Comparison of the anharmonicity parameters with the fragility in the samples of the same composition showed that they strongly correlate; higher anharmonicity corresponds to higher fragility. In a good agreement with the predictions of the anharmonic model¹⁵ the width of the quasielastic line is proportional to the boson peak frequency. We found a characteristic-



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