Consistency Condition for Propagators

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It is shown that there is a strict condition for the propagators of higher-spin particles. We require the consistency of the propagator when a particle propagates by means of self-energy-type interactions. We require a kind of unitarity. Then this requirement turns out to be a strict condition in the case of higher-spin particles. Its nontriviality is due to the fact that free fields of higher-spin particles contain redundant components, which are suppressed by means of subsidiary conditions, not satisfied by the currents coupled with the fields. The usual propagators do not satisfy our requirement, while those which have been introduced in connection with the $O(4)$ symmetry satisfy it. Also, a difficulty of the scalar-tensor theory of the graviton is pointed out.

In a previous paper[1] we argued that the propagators used usually for higher-spin particles are not correct. The term which is proportional to $(s-\mu^2)$ is essential when we discuss the $O(4)$ symmetry of the amplitudes at zero invariant mass. In this paper we want to introduce a consistency condition for propagators and to clarify the reason why the usual propagators of higher-spin particles are not correct. Also, we discuss the propagator of the graviton and point out a difficulty of the scalar-tensor theory of the graviton.

It is well known[2,3] that quantum field theories describing higher-spin particles, in general, are inconsistent in the presence of interactions. We require here at least the consistency of the propagator when a particle propagates under self-energy-type interactions. We formulate this requirement as follows:

$$d'=d'\pi d+d\,, \tag{1}$$

where d', d, and π are the dressed propagator, the free propagator, and the proper self-energy part, respectively. Equation (1) is trivial in the case of spin-0 particles and is used to determine the renormalized mass from the bare mass. This equality is, however, a strict condition, as shown below in the case of higher-spin particles; hence we want to call Eq. (1) a consistency condition for the propagators. This is nontrivial because of the following fact[4]: Free fields of higher-spin particles contain redundant components that are suppressed by means of subsidiary conditions, which the currents coupled with the fields do not satisfy.

Then we will show how Eq. (1) acts as a consistency condition, taking the case of the spin-2 massive particle as an example. Before entering into the details, we introduce the projection operators

$$P_{2,\mu;\nu}=\tfrac{1}{2}(\bar{\delta}_{\mu_1\nu_1}\bar{\delta}_{\mu_2\nu_2}+\bar{\delta}_{\mu_1\nu_2}\bar{\delta}_{\mu_2\nu_1})-\tfrac{1}{3}\bar{\delta}_{\mu_1\mu_2}\bar{\delta}_{\nu_1\nu_2}\,, \tag{2a}$$

$$P_{1,\mu;\nu}=(1/2k^2)(k_{\mu_1}k_{\nu_1}\bar{\delta}_{\mu_2\nu_2}+\bar{\delta}_{\mu_1\nu_1}k_{\mu_2\nu_2}+k_{\mu_1}k_{\nu_2}\bar{\delta}_{\mu_2\nu_1}+\bar{\delta}_{\mu_1\nu_2}k_{\mu_2}k_{\nu_1})\,, \tag{2b}$$

$$P_{0aa,\mu;\nu}=\tfrac{1}{12}(\delta_{\mu_1\mu_2}-4k_{\mu_1}k_{\mu_2}/k^2)\times(\delta_{\nu_1\nu_2}-4k_{\nu_1}k_{\nu_2}/k^2)\,, \tag{2c}$$

$$P_{0ab;\mu,\nu}=\tfrac{1}{12}\sqrt{3}(\delta_{\mu_1\mu_2}-4k_{\mu_1}k_{\mu_2}/k^2)\delta_{\nu_1\nu_2}\,, \tag{2d}$$

$$P_{0ba,\mu;\nu}=\tfrac{1}{12}\sqrt{3}\delta_{\mu_1\mu_2}(\delta_{\nu_1\nu_2}-4k_{\nu_1}k_{\nu_2}/k^2)\,, \tag{2e}$$

$$P_{0bb,\mu;\nu}=\tfrac{1}{4}\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2} \tag{2f}$$

for the convenience of calculations. Here

$$\bar{\delta}_{\mu\nu}=\delta_{\mu\nu}-k_\mu k_\nu/k^2\,. \tag{2g}$$

We can decompose the propagators and the self-energy part in Eq. (1) by means of these projection operators. Then Eq. (1) must be satisfied separately by the coefficients of the projection operators. The coefficients of P_2 and P_1 can easily be determined by means of Eq. (1) while, in general, the coefficients of the P_0's do not exist.[5] To satisfy Eq. (1) d_{0aa}, d_{0ab}, and d_{0bb} (coefficients of P_{0ba}, P_{0ab}, and P_{0bb} of the propagators) must satisfy the relation $d_{0ab}{}^2=d_{0aa}d_{0bb}$. The requirement that the propagator for a massive particle is free of zero-energy poles permits only the case in which the coefficients of P_{0ba}, P_{0ab}, and P_{0bb} are zero.

Now the usual propagator can be decomposed by means of the projection operators as

$$\frac{1}{s-\mu^2}\left\{P_2+\frac{(s-\mu^2)}{\mu^2}\left[-P_1+\frac{\frac{1}{2}s-\mu^2}{\mu^2}P_{0aa}-\frac{s}{2\sqrt{3}\mu^2}(P_{0ab}+P_{0ba})+\frac{\mu^2+\frac{1}{2}s}{3\mu^2}P_{0bb}\right]\right\}, \tag{3}$$

while the propagator proposed in Ref. 1 can be decomposed as

$$\frac{1}{s-\mu^2}\left[P_2+\frac{(s-\mu^2)}{\mu^2}\left(-P_1+\frac{\frac{1}{2}s-\mu^2}{\mu^2}P_{0aa}\right)\right]. \tag{4}$$

Thus, *Eq. (3) does not satisfy Eq. (1), while Eq. (4) does.*

The propagator (4) can not be derived directly from a Lagrangian, but can be derived only by means of a

[1] Y. Iwasaki, Phys. Rev. **173**, 1608 (1968).
[2] K. Johnson and E. C. G. Sudarshan, Ann. Phys. (N.Y.) **13**, 126 (1961).
[3] L. M. Nath, Nucl. Phys. **68**, 660 (1965).
[4] This point is due to a discussion with Dr. J. Arafune.
[5] In general, the free propagator depends on the self-energy part. This is absurd.

limiting process.[6] From this fact we may suppose that a consistent theory describing higher-spin particles can be obtained only by a limiting process if one takes the Lagrangian approach.

The case of massless particles must be discussed separately, because terms such as $1/\mu^2$ do not exist in this case. Another important point in this case is the fact that the current is conserved, which is the necessary condition for Eq. (1) to hold.

After calculations similar to those in the case of massive particles, we find two solutions of Eq. (1) in the case of the spin-2 particle:

$$(1/s)[\tfrac{1}{2}(\delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}+\delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1})-\tfrac{1}{2}\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2}], \tag{5}$$

$$(1/s)[\tfrac{1}{2}(\delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}+\delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1})-\tfrac{1}{3}\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2}]. \tag{6}$$

Solution (5) corresponds to the propagator of the linearized Einstein theory and contains only helicity-(± 2) states as intermediate states.

On the other hand, the propagator of the scalar-tensor theory[7] of the graviton also contains the helicity-0 state as an intermediate state and can be written, using one parameter α, as

$$(1/s)\{[\tfrac{1}{2}(\delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}+\delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1})-\tfrac{1}{2}\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2}] +\alpha\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2}\}, \tag{7}$$

where the limit of α determined by the deflection of light near the sun is $\alpha<0.1$. Hence solution (6) can be rejected as the propagator of the graviton. No scalar-tensor theory,[8] except Eq. (5), can satisfy Eq. (1). Thus, *the only solution which satisfies both Eq. (1) and the experimental facts is solution (5).*

We want to comment here that there is a gap between propagators of massless particles and those of massive particles in the case of higher-spin particles, and that the propagator of massless particles does not correspond to an irreducible representation of the $O(4)$ symmetry.

A full account of the properties of propagators of higher-spin particles, including self-energy parts, will be published elsewhere.[9]

[6] See footnote 9 of Ref. 1.

[7] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961); W. Thirring, Acta Phys. Austriaca Suppl. **5**, 391 (1968); R. H. Sexl, Fortschr. Physik **15**, 269 (1967).

[8] Even if the interaction between the spin-0 part and the source and that between the spin-2 part and the source are introduced independently, the parts of spin 0 and spin 2 are coupled when the sources are the same.

[9] Y. Iwasaki, Progr. Theoret. Phys. (Kyoto) **44**, No. 5 (1970).

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General Treatment of the Multiple Factorizations in the Dual Resonance Models; the N-Reggeon Amplitudes*

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The author presents a compact way of carrying out repeated factorizations on the dual amplitude. Prescriptions are given for writing down the multiply-factorized tree amplitudes. As an application of the prescriptions, the N-Reggeon amplitude is derived.

I. INTRODUCTION

In this paper, we use the fact that the projective transformation of cross ratio is invariant under changing of the projective frames (duality) to generalize the multiple-factorization technique developed in a previous paper.[1] We find a neat and compact way of carrying out the mutliple factorizations on the dual amplitude. As a consequence of this, we obtain a set of prescriptions which enables us to write down directly the multiply-factorized tree amplitudes by simply examining the corresponding tree diagrams. Applying the prescriptions in a particular case, we obtain the formula for the N-Reggeon amplitudes.

In Sec. II we explicitly carry out the third and the fourth factorizations and prove the factorization of the quadruply-factorized tree into two triply-factorized trees. We thus discuss the application of the quadruply-

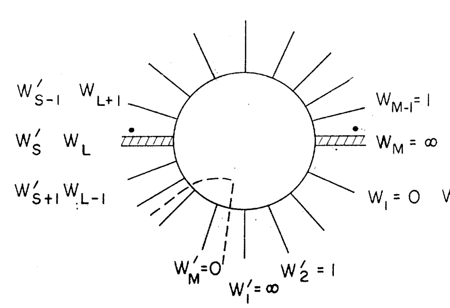

FIG. 1. Doubly-factorized tree. w_i refers to the frame defined by $w_M=\infty$, $w_1=0$, $w_{M-1}=1$, and w_i' refers to a new frame $w_M'=0$, $w_1'=\infty$, $w_2'=1$ for the third factorization.

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[1] L. P. Yu, Phys. Rev. D **2**, 1010 (1970).