Coupling of the trace of the energy-momentum tensor to two photons

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We derive a theorem on the coupling of the trace of the energy-momentum tensor to two photons.

The purpose of this paper is to derive a theorem on the coupling of the trace of the energy-momentum tensor to two photons.[1]

Theorem. Consider the vertex

$$\langle 0|\theta_{\mu\mu}(0)|\gamma(\epsilon_1,k_1),\gamma(\epsilon_2,k_2)\rangle = (\epsilon_1\cdot\epsilon_2\,k_1\cdot k_2-\epsilon_1\cdot k_2\,\epsilon_2\cdot k_1)F(q^2)\,. \quad (1)$$

Here $\theta_{\mu\mu}$ is the trace of the energy-momentum tensor, $\theta_{\mu\nu}$, and $q=k_1+k_2$. Then

$$F(0)=0\,. \quad (2)$$

Proof. The most general form of the vertex

$$\langle 0|\theta_{\mu\nu}(0)|\gamma(\epsilon_1,k_1),\gamma(\epsilon_2,k_2)\rangle$$

is given by

$$[\tfrac{1}{2}(F^1_{\mu\rho}F^2_{\nu\rho}+F^1_{\nu\rho}F^2_{\mu\rho})-\tfrac{1}{4}\delta_{\mu\nu}F^1_{\lambda\rho}F^2_{\lambda\rho}]A(q^2) + F^1_{\lambda\rho}F^2_{\lambda\rho}(k_1-k_2)_\mu(k_1-k_2)_\nu B(q^2) + \tfrac{1}{2}(F^1_{\mu\alpha}F^2_{\nu\beta}+F^1_{\nu\alpha}F^2_{\mu\beta})(k_1+k_2)_\alpha(k_1+k_2)_\beta C(q^2)\,. \quad (3)$$

Here $F^i_{\alpha\beta}=(k_i)_\alpha(\epsilon_i)_\beta-(k_i)_\beta(\epsilon_i)_\alpha$ $(i=1,2)$. To derive Eq. (3), we have used the gauge invariance and the conservation law of the energy-momentum tensor.

From Eq. (3) we obtain

$$\langle 0|\theta_{\mu\mu}(0)|\gamma(\epsilon_1,k_1),\gamma(\epsilon_2,k_2)\rangle = (\epsilon_1\cdot\epsilon_2\,k_1\cdot k_2-\epsilon_1\cdot k_2\,\epsilon_2\cdot k_1)\,q^2[-2B(q^2)+\tfrac{1}{2}C(q^2)]\,. \quad (4)$$

Hence

$$F(q^2)=q^2[-2B(q^2)+\tfrac{1}{2}C(q^2)]\,. \quad (5)$$

Since $B(q^2)$ and $C(q^2)$ are free from the pole at $q^2=0$, we get

$$F(0)=0. \quad \text{Q.E.D.} \quad (6)$$

Next we check the theorem in a simple example. We calculate $F(0)$ in QED up to the order e^2. The trace of the energy-momentum tensor is given by

$$\theta_{\mu\mu}=m\bar\psi\psi+(\tfrac{1}{4}n-1)Z_3(\partial_\lambda A_\rho-\partial_\rho A_\lambda)^2 \quad (7)$$

or

$$\theta_{\mu\mu}=m\bar\psi\psi+\sum_i M_i\bar\psi_i\psi_i\,, \quad (8)$$

depending on the methods of the regularization. Equation (7) corresponds to the n-dimensional regularization,[2] while Eq. (8) corresponds to the Pauli-Villars regularization.[3] In Eq. (7) n denotes the dimension of the space-time and in Eq. (8) ψ_i and M_i represent the fields and masses of the auxiliary fields. We may consider the fields in Eqs. (7) and (8) as the renormalized fields when we calculate the matrix element up to the order e^2.

To obtain the matrix element of the energy-momentum tensor, we should keep, *throughout the calculation*, the finiteness of the matrix element and the conservation law of the energy-momentum tensor. To keep the finiteness we introduce the auxiliary fields (the regulators) in the Pauli-Villars method, while we continue the dimension of the space-time from four to an arbitrary number n in the n-dimensional regularization. Simultaneously we should include the energy-momentum tensor of the auxiliary fields in the former, while we take the energy-momentum tensor in an arbitrary n-dimension space-time in the latter. We take the limit $M_i\to\infty$ or $n\to 4$ at the final step. Then the energy-momentum is conserved at each step of the calculation. Without modifying the energy-momentum tensor as above, the energy-momentum is not conserved. Thus, the second terms in Eqs. (7) and (8) are important. The contributions from the second terms in Eqs. (7) and (8) cancel, at $q^2=0$, those from the first term, $m\bar\psi\psi$.

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[1]For related work, see R. J. Crewther, Phys. Rev. Lett. 28, 1421 (1972); M. S. Chanowitz and J. Ellis, Phys. Lett. 40B, 397 (1972); Phys. Rev. D 7, 2490 (1973).

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