

Using a conservative value of 1.3° for the upper limit phase shift and a circulating flux of 152 Gcm^2 , we evaluate from Eq. (10) the upper limit for γ_2 to be

$$\gamma_2 < 4.9 \times 10^{-12}.$$

Thus within experimental uncertainty, no measurable AB effect has been found for neutrons.

This conclusion suggests more than the mere absence of a dynamic charge on the neutron. More generally, the experiment was capable of detecting the existence of any nonstandard coupling to the electromagnetic field resulting in an AB effect, even though the particle involved was uncharged.

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Instanton Contributions in Two-Dimensional Nonlinear O(3) σ Model

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The correlation length in the two-dimensional nonlinear O(3) σ model is calculated with contributions of instantons included. It is given by $\xi = 0.0125a \exp[2\pi/f(a)]f(a)/2\pi$ where $f(a)$ is the coupling constant defined in lattice regularization scheme and a is the lattice spacing. This number remarkably coincides with the result of Monte Carlo simulations by Shenker and Tobochnik.

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The O(3) nonlinear σ model in two dimensions bears many similarities with a non-Abelian gauge model in four dimensions: Both possess asymptotic freedom, n -instanton solutions, and no intrinsic scale parameters. An O(3)-invariant regularization of the former gives the O(3) Heisenberg spin model, while a gauge-invariant regularization of the latter gives a non-Abelian lattice gauge model. In the O(3) Heisenberg model

no phase transition would occur at any finite temperature. This corresponds to the fact that in the lattice gauge theory a confining phase would survive even when $g \ll 1$. It is believed that these kinds of low-temperature (weak coupling) behavior are due to nonperturbative effects. Numerical calculations in the lattice-regularization scheme include automatically all such effects. Indeed, recent Monte Carlo simulations per-

formed by Creutz¹ and Wilson² for a four-dimensional (4d) non-Abelian lattice gauge model as well as by Shenker and Tobochnik³ for the two-dimensional (2d) Heisenberg model confirm these conjectures for low-temperature behavior. In both cases a sharp crossover takes place from strong to weak coupling. In the weak-coupling region the renormalization of the coupling constant is consistent with the perturbative renormalization group.

Now the problems are whether it is in principle possible to take account of these nonperturbative effects by continuum theories and, if possible, how they can be taken account of. The answer will be that at least in the 2d nonlinear O(3) σ model, in the weak-coupling region, nonperturbative effects can be exactly taken account of by instanton configurations (not by instanton-anti-instanton configurations).

The nonlinear O(3) σ model is defined by the Lagrangian

$$L = \frac{1}{2f} \sum_{i=1}^3 (\partial_\mu \sigma_i)(\partial_\mu \sigma_i); \quad \sum \sigma_i^2 = 1. \quad (1)$$

$$Z \propto \sum_{q=0}^{\infty} \frac{1}{(q!)^2} z^{2q} \int \prod_{i,j=1}^q d^2 a_i d^2 b_j \exp(-2 \sum_{i,j}^q \ln|a_i - b_j| + 2 \sum_{i \neq j} \ln|a_i - a_j| + 2 \sum_{i \neq j} \ln|b_i - b_j|), \quad (3)$$

where

$$z \propto \mu \exp[-2\pi/f(\mu)]. \quad (4)$$

They concluded that the system is infrared finite as the Coulomb system in 2d is infrared finite⁷ and that the system generates a mass dynamically.

Since in this Letter we would like to compare the continuum theory and the lattice theory quantitatively, we have to be careful with any numerical factor and dependence of the coupling constant on regularization method. In the Pauli-Villars regularization the fugacity z is given by⁸

$$z = \mu \exp\left(-\frac{2\pi}{f(\mu)}\right) \frac{8}{f} e^{-1}. \quad (5)$$

The Coulomb system in 2d is equivalent to the sine-Gordon system in 2d.⁷ To rewrite Eq. (3) in terms of the sine-Gordon field φ , we use the fact that the inversion of the 2d d'Alembertian is given by

$$G(r) = (2\pi)^{-1} \ln(r + 1/\Lambda). \quad (6)$$

Then we obtain⁹

$$Z \propto \int D\varphi \exp\left[\int \left(\frac{1}{2} \partial_\mu \varphi \partial_\mu \varphi + 2z' \cos \alpha \varphi\right) d^2 x\right], \quad (7)$$

The partition function is given by

$$Z = \int D\sigma \exp(-\int L d^2 x). \quad (2)$$

When $f \ll 1$, we may apply the steepest-descent method to evaluate the path integral. Solutions of the equation of motion are known as instantons.⁴ It should be noted that instanton-anti-instanton configurations are not exact solutions. The contributions of instantons are evaluated in very interesting papers by Fateev, Frolov, and Schwarz,⁵ and Berg and Lüscher,⁶ including one-loop quantum corrections around instantons. Their results may be summarized as follows: (1) Ultraviolet divergences of one-loop corrections around the solutions are consistent with the perturbative renormalization group. (2) The N -instanton state is able to be described in terms of $2N$ "particles" in 2d. Each of these particles is a half of an instanton and will be called "half-instanton" here. (3) The interaction between the "half-instantons" is logarithmic. That is, the N -instanton state is equivalent to the Coulomb gas in 2d.

The partition function they obtained is given by

where

$$z' = z\Lambda \quad (8)$$

and

$$\alpha^2 = 4\pi. \quad (9)$$

Note that in this functional-integral approach tadpole-type diagrams are included. To relate this approach to the operator formalism, we have to sum up tadpole-type diagrams. Doing this we obtain⁹

$$L = -\frac{1}{2} \partial_\mu \varphi \partial_\mu \varphi + 2z'' N_m(\cos \alpha \varphi), \quad (10)$$

where

$$z'' = \frac{1}{2} e^\gamma (m/\Lambda) z' = \frac{1}{2} e^\gamma m z. \quad (11)$$

Here N_m denotes the normal-ordering operation defined by the mass m . By adjusting as

$$2z'' = m^2/\alpha^2, \quad (12)$$

we obtain

$$m = 4\pi e^\gamma z. \quad (13)$$

As Coleman¹⁰ and Mandelstam¹¹ showed, the sine-Gordon model in 2d is equivalent to the massive Thirring model in 2d, which is described by

the Lagrangian

$$L = i\bar{\psi}\gamma_{\mu}\partial_{\mu}\psi + M\bar{\psi}\psi - \frac{g}{2}\bar{\psi}\gamma_{\mu}\psi\bar{\psi}\gamma_{\mu}\psi. \quad (14)$$

When $\alpha^2 = 4\pi$,

$$g = 0 \quad (15)$$

and

$$M = \frac{1}{2}e^{-\gamma}m. \quad (16)$$

Here $\gamma = 0.577\ 215\ 665\dots$ is Euler's constant.

Now the two-point function of the nonlinear O(3) σ model may be written in terms of the massive Thirring field.⁵ It turns out that the correlation length is exactly given by the inverse of the mass of the Thirring field, since the Thirring field is free. Therefore we obtain

$$\xi = \frac{1}{M} = \frac{e}{8} e^{2\pi f(\mu)} \frac{f(\mu)}{2\pi} \frac{1}{\mu}. \quad (17)$$

To compare with the result of Shenker and Tobochnik,³ we have to rewrite Eq. (17) in terms of the coupling constant defined in lattice regularization, $f(a)$. The transformation factor was obtained by Parisi.¹² Thus we obtain

$$\xi = [\exp(1 - \pi/2)/32\sqrt{2}] e^{2\pi f(a)} \frac{f(a)}{2\pi} a, \quad (18)$$

where a is the lattice spacing. The numerical coefficient is given by

$$C = \exp(1 - \pi/2)/32\sqrt{2} = 0.0125. \quad (19)$$

This number remarkably coincides with the numerical value 0.01 obtained by Monte Carlo simulations.³

We turn to discussions of implications of our results. Instantons give a full account of the correlation length from deep in the weak-coupling

region ($f \ll 1$) up to the crossover point ($f \sim 1/1.3$). This means that the following conjectures may be misleading: (i) The dilute instanton picture will be relevant and (ii) instantons will become important only when the coupling constant becomes large up to around the crossover point. We conjecture that in 4d non-Abelian gauge theories also, instantons play the same role as in the 2d nonlinear σ model. This view has been taken by the present author in Refs. 9 and 13.

Note added.—The cluster property of the vacuum has been carefully analyzed. It has been concluded that the vacuum of the 2d nonlinear O(3) σ model is twofold degenerate and that a new kind of topological symmetry breaking occurs. The details will be published elsewhere.¹⁴

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