

H Dibaryon in Lattice QCD

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The mass of a doubly strange dibaryon called the *H* particle is calculated in lattice QCD on a $16^3 \times 48$ lattice with a renormalization-group-improved gauge action and Wilson's quark action. We find that the *H* is below the $\Lambda\Lambda$ threshold for strong decay. Furthermore, the case where the *H* is slightly below the *NN* threshold for weak decay is consistent with our numerical results and is not in conflict with the stability of nuclei.

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Jaffe¹ predicted, on the basis of a bag-model calculation, the existence of a flavor-singlet, spin-zero, six-quark (two up, two down, and two strange) state called the *H* particle with binding energy 80 MeV relative to the threshold for strong decay to $\Lambda\Lambda$. The basic idea of Jaffe is that the binding of the *H* particle is mainly caused by the hyperfine splitting

$$H_{\text{hfs}} = - \sum_{i>j} \frac{(\lambda_i \cdot \lambda_j)(\sigma_i \cdot \sigma_j)}{m_i m_j}, \quad (1)$$

where λ_i and σ_i are Gell-Mann and Pauli matrices. In the flavor-SU(3)-symmetry case ($m_u = m_d = m_s$), H_{hfs} is proportional to

$$\Delta = - \sum_{i>j} (\lambda_i \cdot \lambda_j)(\sigma_i \cdot \sigma_j), \quad (2)$$

which has the eigenvalues -24 , 8 , and 48 for the **490**, **189**, and **1** representations of color-spin SU(6), respectively. The *H* particle, which belongs to the **490**, is a bound state due to this negative number, -24 , in the bag-model calculation by Jaffe. Subsequent calculations² of the mass of the *H* particle in bag models as well as in chiral models have yielded values from 1.03 to 2.3 GeV, compared with the $\Lambda\Lambda$ threshold at 2.23 GeV. It is not known how accurate quantitatively we may expect the results are in these phenomenological models.

On the other hand, lattice QCD³ provides us with a way to calculate numerically the mass of the *H* particle from first principles. Mackenzie and Thacker⁴ calculated the mass of the *H* particle on a $6^2 \times 12 \times 18$ lattice and concluded that the *H* is above the $\Lambda\Lambda$ threshold. However, we think that the lattice size is too small to investigate the mass of the *H* particle for the reason which we will describe below. We report here the result of the calculation of the mass of the *H* particle on a $16^3 \times 48$ lattice in the quenched (valence) approximation which neglects dynamical quark loops.

In lattice QCD, the mass of the *H* particle can be determined from the exponential decay of the propagator of the *H* particle in the Euclidean time. The propagator of the *H* particle can be constructed by a sum of tensor products of quark propagators. The method of the construction of the *H* propagator is almost identical with that of Ref. 4, except for one point in our obtaining an explicit wave function of the *H* particle. We will describe this point elsewhere, because it is a technical point.

We use the identical quark propagators which have been used by Itoh, Iwasaki, and Yoshié⁵ to calculate the known hadrons: We have taken the Wilson quark action⁶ and a renormalization-group-improved gauge action which is given by

$$S = \frac{1}{g^2} \left[c_0 \sum \text{tr}(\text{simple plaquette}) + c_1 \sum \text{tr}(1 \times 2 \text{ rectangular loop}) \right], \quad (3)$$

with $c_1 = -0.331$ and $c_0 = 1 - 8c_1$. (We have reported several items of evidence⁷ that it is better to use a renormalization-group-improved action for obtaining the continuum limit of a lattice theory.) We have chosen $\beta = 6/g^2 = 2.4$ which is expected in the scaling region.⁷ The inverse of the lattice spacing is 1810 MeV. This value roughly corresponds to that at $\beta \approx 6.0$ with the standard one-plaquette action.

The lattice size is $16^3 \times 48$. We have calculated quark propagators at five hopping parameters 0.14, 0.145, 0.15, 0.1525, and 0.154 (which correspond to pions of mass around 1900, 1530, 1100, 860, and 700 MeV, respectively) on fifteen gauge configurations, separated by 100

sweeps after a thermalization of 1000 sweeps with a pseudo heat-bath method. We use periodic boundary conditions for both gauge fields and quark fields. In addition to the five hopping parameters we also calculate in this work quark propagators at $K=0.1245$ and 0.155 which correspond to pions of mass around 3100 and 560 MeV, respectively. The $K=0.154$ and $K=0.1245$ correspond to the strange and charm quarks, respectively, because the masses of the vector meson at these hopping parameters are roughly equal to the physical m_ϕ and $m_{J/\psi}$. For the *u* and *d* quarks we have to extrapolate physical quantities up to $K_c = 0.1569(2)$.

We calculate the propagators of π , ρ , N , Δ , Σ , Λ , and H for fifteen combinations of the hopping parameters for three flavors (K_1, K_2, K_3): $K_1=K_2=K_3=0.1245$, 0.14, 0.145, 0.15, 0.1525, and 0.154; $K_3=0.154$ with $K_1=K_2=0.14$, 0.145, 0.15, 0.1525, and 0.155; $K_1=K_2=0.154$ with $K_3=0.14$, 0.145, 0.15, and 0.1525. For each combination of hopping parameters, we have 30 propagators for the H and 60 propagators for Λ , because we calculate the propagators of both particle and antiparticle, and of each spin state. We take the average of these propagators. In addition to the particles given above, we also calculate the two-point functions of $\Lambda\Lambda$ and NN as well as the propagators of the H_1 and H_{189} particles which are members of the 1 and 189 representations, respectively.

If $m_H < 2m_\Lambda$, $G_H(t) \sim \exp(-m_H t)$ for large t . Here $G_H(t)$ is the propagator of the H with momentum-zero state. On the other hand, if $m_H > 2m_\Lambda$, $G_H(t) \sim \exp(-2m_\Lambda t)$ for large t . Therefore, if $m_H < 2m_\Lambda$, $G_H(t)/[G_\Lambda(t)]^2 \sim \exp[(2m_\Lambda - m_H)t]$ for large t , while $G_H(t)/[G_\Lambda(t)]^2 \sim \text{const}$ for large t when $m_H > 2m_\Lambda$.

We show in Fig. 1 the results of the ratio $G_H(t)/[G_\Lambda(t)]^2$ for the case $K_1=K_2=0.154$ and $K_3=0.1525$, and for the case $K_1=K_2=K_3=0.1245$. When the quark

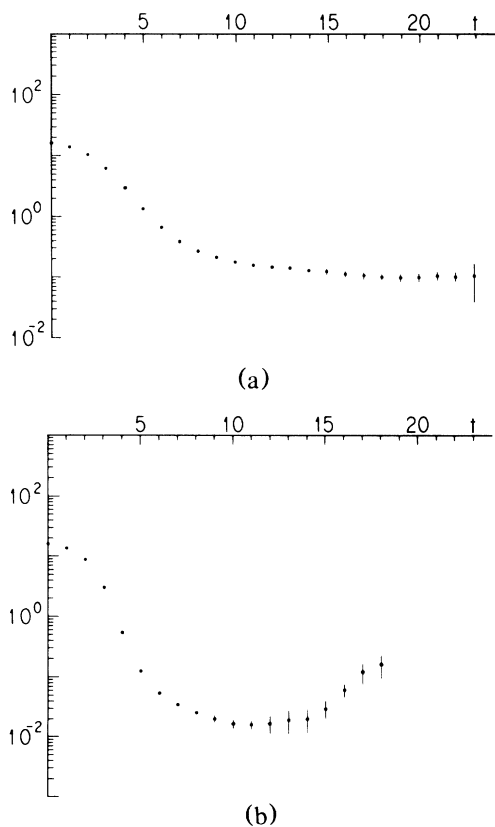


FIG. 1. The ratio of the H propagator to the square of the Λ propagator vs the Euclidean time; (a) $K_1=K_2=K_3=0.1245$; (b) $K_1=K_2=0.154$, $K_3=0.1525$.

mass is heavy ($K=0.1245$), the errors of the propagators are small up to large t ($t \leq 24$). In this case the ratio G_H/G_Λ^2 tends to a constant toward large t ($t \geq 12$). This implies $m_H \geq 2m_\Lambda$. On the other hand, as the quark mass becomes lighter, the errors of the propagators become large toward large t . When the statistical error is larger than 100% at some t_c , we disregard the data for $t \geq t_c$, because we think such data are statistically meaningless. Thus in the case $K_1=K_2=0.154$, $K_3=0.1525$ we obtain the Λ propagator up to $t=19$ and the H propagator up to $t=18$. Similar behavior of the N propagator has been observed in Ref. 5. We see a rise in the ratio G_H/G_Λ^2 from $t=12$ up to $t=18$. This implies $m_H < 2m_\Lambda$.

Investigation of the autocorrelation of the Λ and H propagators shows that there is no long-time correlation which exceeds 100 sweeps. Thus we may take it that all the data on fifteen configurations are statistically independent. We have estimated the error for the ratio $G_H(t)/G_\Lambda^2(t)$ taking into account the correlation between G_H and G_Λ on each configuration, because there is a positive correlation between G_H and G_Λ . Although the estimated error for the ratio G_H/G_Λ^2 in Fig. 1 is rather large for the region where the ratio rises, we believe from the above analyses that the signal of a rise is statistically meaningful.

We show in Fig. 2 the result of G_{H_1}/G_Λ^2 for the case $K_1=K_2=0.154$, $K_3=0.1525$. In this case the H_1 propagator decays rapidly and we obtain the H_1 propagator only up to $t=13$. Note the difference concerning the behavior of the propagator between the H_{490} and H_1 states. We interpret this as indicating that the H_1 state is a scattering state and therefore it spreads over wide space at large t . Consequently the fluctuation of the propagator at large t becomes large.

In Fig. 3 we display m_H and $2m_\Lambda$ for the fifteen combinations. The m_Λ is determined by a two-mass fit to the

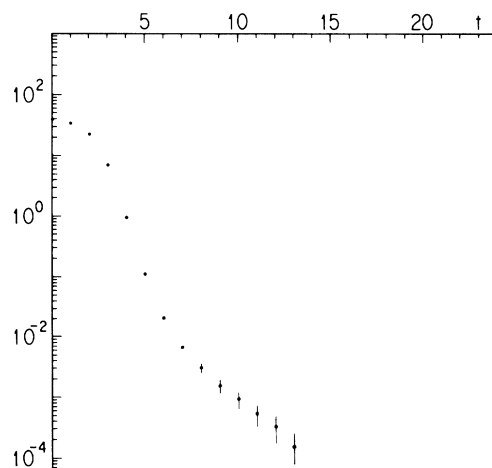


FIG. 2. The same as Fig. 1(b) except for the H_1 propagator.

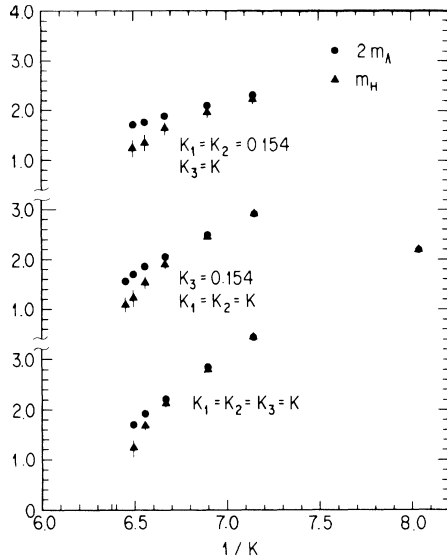


FIG. 3. The m_H and $2m_\Lambda$ for fifteen combinations of hopping parameter.

propagator for $6 \leq t \leq 24$ or t_{\max} where $t_{\max} = t_c - 1$ with the mass difference 500 MeV fixed, as for the nucleon in Ref. 5. [Note that the mass difference between $\Lambda(1600)$ and $\Lambda(1115)$ is about 500 MeV.] The justification of this procedure is described in Ref. 5. On the other hand, the m_H is determined by a one-mass fit to the propagator for $13 \leq t \leq 24$ or t_{\max} , because we have no information for excited states. (Only for the case $K_1 = K_2 = 0.155$, $K_3 = 0.154$ where the data for the H is noisy, we fit the data for $13 \leq t \leq 17$ including noisy data at $t = 14$.) It should be noted that a one-mass fit gives, in general, a larger estimated value for m_H compared with the real value. The results are displayed versus K for the following three cases: case a, $K_1 = K_2 = K_3 = K$; case b, $K_1 = K_2 = 0.154$, $K_3 = K$; and case c, $K_1 = K_2 = K$, $K_3 = 0.154$. We see that for all of the three cases, as K increases m_H clearly becomes less than twice m_Λ and the difference $2m_\Lambda - m_H$ increases.

In order to obtain the physical m_H and m_Λ , we have to extrapolate the results to the point $K_1 = K_2 = 0.1569(2)$ and $K_3 = 0.154$. There are several ways to do this. One of them is to fit the masses for case c, $K_1 = K_2 = K$ and $K_3 = 0.154$, by a quadratic function of $1/K$. We obtain $m_\Lambda = 1190(100)$ MeV and $m_H = 1450(250)$ MeV. Another way we have tried is to fit the data of all three cases a, b, and c (except for the case $K_1 = K_2 = K_3 = 0.1245$; this point is too far from the physical point) by a quadratic function of two variables $x = 1/K_1 = 1/K_2$ and $y = 1/K_3$. In this case we obtain $m_\Lambda = 1210(95)$ MeV and $m_H = 1710(140)$ MeV. Thus the value for m_Λ does not change so much according to the way of fit and is consistent with the physical value within 10% error. On the other hand, the m_H does depend on the way of fit. Combining two fits we estimate that $m_H \approx 1555\text{--}1850$

MeV: A clear lower bound for m_H is 1555 MeV in order that hypernuclei must not strongly decay by the process $N + \Lambda \rightarrow H + K$. In general, the extrapolation procedure introduces systematic errors. The best way is, of course, to do the calculation at $K = 0.1569$ for the u and d quarks. However, because of the critical slowdown it is very time consuming to do the calculation at small quark mass. For the case of the known hadrons, we have obtained results⁵ which agree with the physical values with an error of at most 10%–15% with the extrapolation procedure. Therefore we expect a similar result for the H .

Let us clarify the reason for the discrepancy between our result and the result in Ref. 4. The rise G_H/G_Λ^2 in Fig. 1 is seen above $t = 12$. However, the propagator of the H was obtained only up to $t = 8$ in Ref. 4, because the linear extension in the temporal direction is 18. Consequently there was no chance to see the rise in G_H/G_Λ^2 . We also think that the linear extension in the spatial directions, 6 in lattice units, is too small, because it is about a half of the electromagnetic diameter of the proton (although in Ref. 4 the lattice spacing was estimated to be 0.9 GeV^{-1} , it has turned out later to be 1.5 GeV^{-1}). Of course it is an open question whether one can obtain a similar result to ours with the one-plaquette action around $\beta \approx 6.0$ on a $16^3 \times 48$ lattice, because the onset of scaling depends on the form of action.⁷

Of course we think our lattice size is not large enough to suppress completely finite-size effects. It is difficult to estimate how large finite-size effects are without doing a similar calculation on a larger lattice. However, we think that finite-size effects are not so large, for the following two reasons: (i) As mentioned above, the behavior of the H propagator is similar to that for the Λ propagator and is better than those for the H_1 propagator and for the NN propagator. We interpret this as meaning that the size of the H is comparable to that of Λ , and that the H_1 state and the NN state spread over wider space. (ii) We have calculated the Λ and H propagators using antiperiodic boundary conditions for quark fields in the cases $K_1 = K_2 = K_3 = 0.154$. Although the propagator of the H_{490} is obtained only up to $t = 15$ because of large fluctuations, it agrees with that with periodic boundary conditions up to $t = 15$ within almost 1 standard deviation. A rise in G_H/G_Λ^2 is seen for $12 \leq t \leq 15$, although the rise has been seen for $12 \leq t \leq 18$ for the periodic case. Thus we conclude that finite-size effects are not large. Clearly the best test of finite-lattice-size effects is to do a similar calculation on a larger lattice.

In order to do the calculation on a large lattice such as $16^3 \times 48$, we have to use the quenched approximation. However, if one recalls the success of valence-quark models one may expect that the quenched approximation will give reasonable values for hadron masses with about 10% errors.

From all the above analyses we conclude that the m_H is below the $\Lambda\Lambda$ threshold and, furthermore, if we take

literally the estimate for m_H , the m_H is below the NN threshold (1880 MeV). However, when we consider possible errors such as those due to finite-size effects, we cannot exclude the possibility that $m_H > 2m_N$.

Physically there is a significant difference between the case $m_H < 2m_N$ and the case $m_H > 2m_N$. If $m_H < 2m_N$, nuclei can decay by second-order weak interactions $p+p \rightarrow H+e^++\nu+e^++\nu$. We would like to point out that the most stringent lower limit for this decay lifetime comes from the proton-decay experiment,⁸ because water contains the oxygen nucleus. We roughly estimate that the lower limit of the lifetime from the experiment is $\tau \geq 10^{30}$ yr. Therefore the crucial issue is whether the estimated m_H is consistent with this limit. The decay rate crucially depends on the Q value. If we assume that the Q value is 100 MeV and that the radii of Λ and H are the same and the quark distributions in Λ and H are Gaussian, we roughly obtain $\tau \approx 10^{20}$ yr. If quarks in Λ and H are strongly correlated (it is very likely that three quarks in baryons are cigar shaped, for example), the decay is in general more suppressed and there is a possibility that $\tau \geq 10^{30}$ yr. If the Q value is of order of 10 MeV, $\tau \geq 10^{30}$ yr. Therefore we estimate $m_H \geq 1800$ MeV from the constraint of the stability of the nucleus. Combining this estimate with our numerical results, we conclude that the case where the m_H is slightly below the threshold NN , e.g., 1850 MeV, is not in conflict with experiment (see also Fitch⁹) and is consistent with our numerical results. In this case the H is completely stable. This has significant implications for the cosmology such as a possible origin of the dark matter. How many H were created in the early Universe and how many survive? These important problems are out of the scope of the present paper. We would like to discuss these points and present more details of our results in a future publication.

Let us finally discuss the nature of the H particle. The dependence of the binding energy on the quark mass displayed in Fig. 3 is roughly in accord with the basic idea of Jaffe that the binding of the H particle is mainly caused by the hyperfine splitting: The mass in Eq. (1) is the constituent quark masses and the consistent quark mass for each hopping parameter can be determined, for example, by use of the phenomenological mass formula quoted in Ref. 5. Thus the nature of the H particle on a

lattice is essentially identical to that of the H particle introduced by Jaffe except for one point: The coefficient of the hyperfine splitting in the bag model is common to mesons, baryons, and the H particle, because it is a perturbative one. On the other hand, on a lattice the coefficient for the mesons (π and ρ) is about 3 times that for the baryons (N and Δ), as in accord with experiment (see Ref. 5), because the term is a nonperturbative one. On a lattice the coefficient for the H particle is equal to or is slightly larger than that for the mesons. This is the reason why we have obtained a larger value for the binding energy of the H than that in the bag model by Jaffe.

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