発育発達現量値曲線に当てはめられた多項式を用いた身体的発育発達に関する研究

— 6歳から18歳 —

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A study on physical growth and development through investigating the polynomial fitted their distance curves

— In the term over 6 and 18 years old —

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従来から，発育曲線に１種または複数種の関数を当てはめ，この関数の数学的特性を考察することによって，発育の特性を検討した研究は1930年代から今日まで，多くはないが報告されて来た。しかし，一つの高次多項式を当てはめて，その一次，二次微分曲線及び三次微分商の考察から発育発達特性を検討したものはない。むしろ，多項式は当てはめの精度からみれば，ロジスチック関数等より低いと批判されてきた。しかし，数学的には，時系列データを一つの関数であらわすには，多項式は他のどの関数より含まれる母数の多い事から自由性は高いといえる。この事から，本研究では発育発達現量値データに対し当てはめの精度を，最小の推定誤差，残差の無作意性の保証の基準で検討しつつ，身長，体重，背筋力，瞬発筋力（垂直跳，立幅跳）の４身体的属性，５変量について，性別に多項式をあてはめた。年齢を独立変数とする８次から11次の多項式で相対誤差の平均が0.1％以上1.2％以下の精度ですべての発育発達現量値曲線が表される事を確かめた。この決定された多項式の一次，二次及び三次微分をもとめ，現量値，速度，加速度曲線の特性について，二次微分を０とした方程式の６歳から18歳の区間の実根を求め，速度曲線の極大，極小，及び現量値曲線の変曲点の位置を性別，変量別に決定し比較検討した。その結果次の結論が得られた。
１)，６歳から18歳の年齢区間で，男子においては，減速的，加速的，再び減速的発育発達が見られるが，女子では加速的，減速的発育発達がみられた。つまり，女子では６歳以前にすでに減速的発育発達は出現しており，それだけ発育発達の位相の変化は男子より早いと推測された。
２)，男子においては，身長の最大発育速度出現時期は他の属性より６か月ほど早いと推測されたが，女子では体重より２年，背筋力より６か月，垂直跳より３か月，立幅跳より11か月早いと推測され，発育発達の位相の属性間に於けるずれが女子において顕著であると考えられた。
３)，最大発育発達速度出現時期の属性間の差異の幅は，男子では1.07年，女子で3.10年であり，属性による発育発達の位相のずれが女子に於いて大であると推測された。
４)，この事から，男子においては体格の発育，筋力の発達が運動能力の発達に有効に関与しているが，女子では制限的に関与していると推測された。

Key words : Growth and development, Polynomial, Derivative

1. Introduction

Several studies(3,5~8,10,11,14,15,18), but not so many, have attempted to clarify the characteristics of growth and development of physical fitness through fitting a certain mathematical function of t; age or the transformed value of actual age, to its distance and/or velocity curves. Most of these studies tried to fit some mathematical function the growth and developmental distance and/or velocity curves determined by the individual longitudinal data and to evaluate the parameters included in function. In these studies, a certain type of exponential, logistic functions and polynomial has been utilized. Actually most of the growth and developmental distance curves may reach the maximum level, then keep this level in some term, and then be followed by decreasing trends. Thus, in the term over 6 and 18 years old; i. e., the growing and developing term, a certain function having an asymptote is more applicable to the growth and developmental distance curve. Only in the short term, polynomial was attempted to fit them. In the author's former study(14,15), it was concluded that the growth and developmental trend even in the term over 0 and 20 years of age could not be described so precisely with a single logistic function but rather successfully with the compound function of logistic, simple vibration functions and polynomial of 2nd or 3rd order. However, the author found that the deterioration trend of physical fitness in the term over 30 and 59 years old could be described so precisely with the polynomial of higher than 2 or 3(16). Furthermore, even if the distance curve holds some complicated traits, it can be assumed that it may be described mathematically by the polynomial of some higher order. If the polynomial of 10th order could be fitted satisfactorily, the first and second derivatives can be easily determined, and they stand for velocity and acceleration of growth and development at each age. As far as the parameters of polynomial are not needed to be evaluated, polynomial can give us the precise information regarding distance and velocity of growth and development. Thus, this study was designed to test the following three hypotheses;

1) The growth and developmental trend of physical fitness can be described so precisely with the polynomial of some order as to clarify its characteristics,

2) The growth and developmental velocity and acceleration can be evaluated by the first and second derivative curves, and

3) Then, the characteristics of growth and development can be analyzed through investigating the polynomial determined.

2. Methods

The data base of physical fitness was constructed in FACOM 760/20 at University of Tsukuba Science Information Processing Center 1985(13) and has been updated every year with the data of several sources; physical fitness and motor ability survey conducted by Ministry of Education, Science and Culture, Tsukuba Growth and Development Study by the author's laboratory, and several other data sets produced by growth and developmental studies conducted since 1983. The updating procedures are as follows;

1) To test the difference in variance between the original population from which the means included in the data base were computed and new population from which new sample means were determined,

2) If there is no significant difference in variance between both population mentioned in 1), to recompute the mean and standard deviation (SD) with the original mean and SD and new ones.

3) To smooth the revised mean and standard deviation with taking the means and SD's of both sides in age from the revised mean into consideration. Then, in this paper, stature, body weight, back strength, vertical jump and standing long jump were selected to discuss, because these measures are important for evaluating physical fitness, and they have been chosen as the measures of various physical fitness test batteries. Furthermore, with the data of physical fitness and motor ability survey conducted annually by Minis-

try of Education, Science and Culture, the author verified that any significant secular changes could not be found during these five years; 1984 through 1988.(13)

Then, the study was worked out by the following procedures.

1) The polynomials of linear through 15th order were determined for the mean growth and developmental distance data filed in the data base as mentioned previously in the term over 6 and 20 years. Although this study actually intended to investigate the characteristics of growth and development from 6 to 18 years old, two data of 19 and 20 years of age were added and the data of 6 through 20 were used to determine the polynomial to keep the continuity of curves at 18 years of age.

2) The residuals were evaluated in terms of relative error, standard error of estimate (SEE) and their randomness.

3) The polynomial producing the minimum SEE, least or as much less as possible mean relative error and random residuals were chosen as the appropriate one to describe the growth and developmental distance curve.

4) The first and second derivatives of appropriate polynomial determined were evaluated at each age. They determined the velocity and acceleration curves.

5) Put the second derivative zero, and this equation was solved. The real roots stand for points of inflexion of the distance curve. In other words, they show the age for start of acceleration or deceleration in distance curves and local minimum or maximum in velocity curves.

3. Results and Discussion

1) Determination of appropriate order of polynomial to describe the growth and developmental distance curves.

Table 1 shows the standard error of estimate for each order of polynomial in each measure, respectively. The polynomial of 11th order shows the minimum SEE for stature, body weight and

Table 1 Standard error of estimate (SEE) for the order of polynomial; linear through 15th, in the physical fitness measures chosen

Sex	Boy					Girl				
Order	S.	B.W.	B.S.	V.J.	S.L.J.	S.	B.W.	B.S.	V.J.	S.L.J.
1	5.468	3.025	8.116	4.379	8.212	6.268	3.928	6.235	3.967	10.478
2	2.340	2.328	6.800	2.481	4.287	1.497	1.861	2.008	.735	1.297
3	1.234	.841	3.311	.842	1.668	1.495	1.146	1.460	.562	1.288
4	1.084	.815	3.216	.835	1.316	.566	.551	.722	.224	.308
5	.535	.319	1.869	.436	.878	.506	.437	.705	.219	1.268
6	.680	.382	2.059	.385	.863	.467	.440	.687	.225	.282
7	.626	.370	2.078	.323	.754	.401	.445	.660	.231	.274
8	1.334	.860	4.134	.317	.630*	.428	.888	.654	.450	.445
9	.606	.408	2.146	.319	.678	.389	.399	.656	.270	.350
10	.397	.253	1.585	.306*	.698	.378*	.335*	.647*	.213	.258
11	.324*	.176*	1.412*	.322	.680	.478	.396	.719	.201*	.250*
12	.328	.191	1.477	.311	.705	.548	.482	.748	.233	.296
13	.888	.472	2.686	.343	.753	.435	.440	.684	.358	.722
14	.596	.394	1.851	.326	.640	.563	.359	.797	.182	.476
15	.331	.183	1.452	.345	.637	.403	.378	.677	.205	.322

Note; 1) * stands for minimum SEE.
2) S. stands for Stature, B.W. for body weight, B.S. for back strength, V.J. for vertical jump, and S.L.J. for standing long jump.
3) Order stands for order of polynomial.

back strength in boy and for vertical jump and standing long jump in girl, and that of 10th for vertical jump in boy and for stature, body weight and back strength in girl, and that of 8th for standing long jump in boy. Then, Table 2 shows the result of randomness check of residuals produced by fitting the polynomials showing the minimum SEE to the distance curves. All Durbin-Watoson ratios were found not significantly different from 2 and all auto-correlations were also found no significant statistically. This means that the polynomials of such order as produced the minimum SEE can describe the growth and developmental distance curves chosen in this study very satisfactorily.

Table 3 shows the coefficients of each term; a_0, a_1, a_2,......a_n, as an example of appropriate polynomials. The selected polynomials include the terms of considerably high order, so $a_i t^i$ may contribute significantly to the estimated value in the term of higher order of t, even if a_i is very small. Therefore, all calculations were worked out with double precision scheme in computer.

Let investigate the mean relative errors presented in Table 2. The largest relative errors were found in back strength for both sexes, but they were 1.182% in boy and 1.076% in girl. Their maximum errors were 3.122% at 12 years old for boy and 4.691% at 7 years old for girl. However, in other measures, the mean relative errors were much less than 1.0% in both sexes, so it can be concluded that the selected polynomials could describe the growth or developmental distance curves so precisely enough as to investigate the characteristics of growth and developmental trend of physical fitness elements chosen in this study.

As an example showing goodness of fit, Table 4 shows the actual and estimated distance values and residuals in stature as a case of high precision and back strength as that of relatively low precision, although the mean relative error is only a little larger than 1.0%. Table 4 shows that the large residuals happened at age 12 and 14 in back strength but even such large residuals were found less than 4% as relative error. In stature all absolute residuals were less than 0.8cm, so their relative error were found less than 0.6%. Thus, it can be safely concluded that these polynomials could describe the growth and developmental distance

Table 2 The appropriate order of polynomial to describe the growth and developmental distance curve of the chosen measure and several statistics of its precision

	Measure	Order	SEE	DW	AC	M.R.E(%)
	Stature	11	.324	2.221	-.219	.173±.131
B	Body weight	11	.176	2.023	-.255	.343±.312
o	Back strength	11	1.412	2.045	-.023	1.182±1.021
y	Vertical jump	10	.306	2.047	-.070	.683±.684
	Standing L.J.	8	.630	2.101	-.054	.284±.194
G	Stature	10	.378	1.754	.105	.236±.148
i	Body weight	10	.335	1.943	-.026	.639±.566
r	Back strength	10	.647	2.028	-.064	1.076±1.250
l	Vertical jump	11	.201	2.116	-.066	.485±.346
	Standing L.J.	11	.250	2.276	-.138	.115±.089

Note; 1) Order stands for the order of polynomial.
2) SEE stands for standard error of estimate,
DW for Durbin–Watoson ratio,
AC for auto–correlation, and
M.R.E for mean relative error.

Table 3 Coefficients of terms in the polynomial selected to describe the growth distance curve of stature

Sex	Boy	Girl
Order	11	10
a_0	.1059754028D03	.1123919525D03
a_1	.1060841179D02	.9767074585D00
a_2	-.1991113663D01	.1439817429D01
a_3	.2840036750D00	-.1265690923D00
a_4	-.1404949650D-01	.3957867622D-02
a_5	.6526678335D-03	-.6716854405D-03
a_6	-.5637117290D-04	.4904011439D-04
a_7	-.1037316906D-05	-.2073412361D-05
a_8	-.2336364560D-06	.3876225492D-06
a_9	.4201939419D-07	-.2812148736D-07
a_{10}	.2857092340D-10	.5648193024D-09
a_{11}	-.6987341261D-10	

Note; $Y=\sum_{i=0}^{n} a_i t^i$,
where n=11 for boy and n=10 for girl and t stands for (age − 5) ; transformed age.
nDm in the table denotes $n x 10^m$.

curves very precisely.

2) The characteristics of growth and development of the physical fitness elements chosen in this study.

Fig. 1 to Fig.10 show the growth or developmental distance, velocity (first derivative) and acceleration (second derivative) curves of stature, body weight, back strength, vertical jump and standing long jump in boy and girl, respectively. As already investigated in Table 2 , it could be recognized that all curves show high degree of goodness of fit and these polynomials can be used so satisfactorily enough to investigate the characteristics of growth and development of these physical fitness elements.

For the first place, in order to investigate the mathematical characteristics of distance curve, such equation as the second derivative being put zero was solved, because these real roots show the inflexion points of distance curve and local minimum or maximum of first derivative curve. Table

Table 4 The actual and estimated values, and residuals; stature and back strength of boy

	Stature; cm			Back strength; kg		
Age	Y	$\hat{Y}$	R	Y	$\hat{Y}$	R
6	115.00	114.86	.14	32.00	31.87	.13
7	121.00	121.29	-.29	40.00	40.09	-.09
8	126.60	126.53	.07	45.00	45.94	-.94
9	131.70	131.55	.15	53.00	51.88	1.12
10	137.40	137.02	.38	61.00	59.80	1.20
11	142.50	143.24	-.74	70.00	70.73	-.73
12	150.00	150.05	-.05	82.00	84.56	-2.56
13	157.20	156.86	.34	100.00	99.91	.09
14	163.00	162.78	.22	118.00	114.44	3.56
15	167.00	166.93	.07	125.00	125.61	-.61
16	168.30	168.88	-.58	130.00	131.89	-1.89
17	169.20	169.08	.12	134.00	134.07	-.07
18	169.30	168.92	.38	137.00	135.57	1.43
	Residual mean=.02			Residual mean=.05		

Note; Y stands for actual value,
$\hat{Y}$ for the estimated value, and
R for the residuals; $(Y-\hat{Y})$.

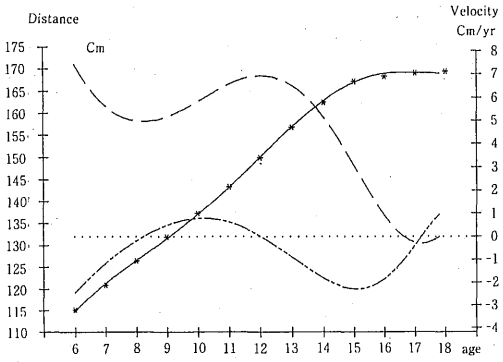

Fig. 1 Distance, velocity, and acceleration curves of stature: Boy

——— distance, — — — velocity, ---- acceleration

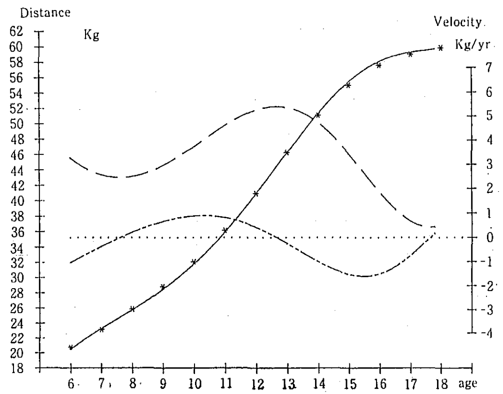

Fig. 2 Distance, velocity, and acceleration curves of body weight: Boy

——— distance, — — — velocity, ---- acceleration

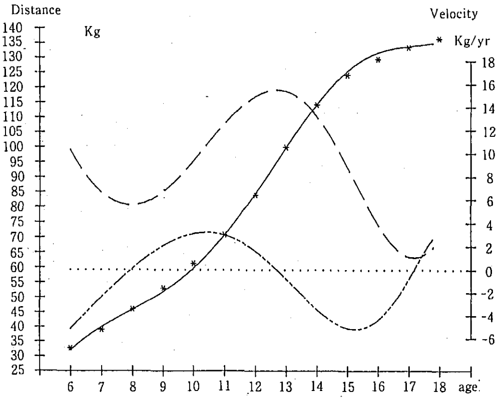

Fig. 3 Distance, velocity, and acceleration curves of back strength: Boy

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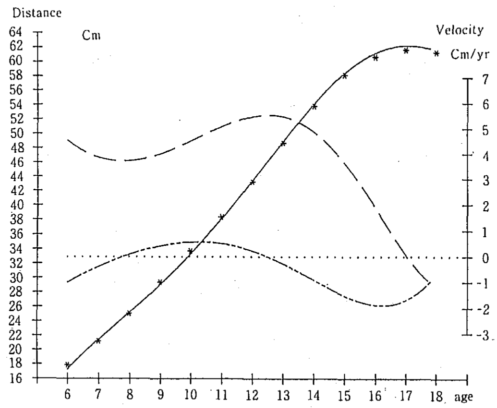

Fig. 4 Distance, velocity, and acceleration curves of vertical jump: Boy

——— distance, — — — velocity, ---- acceleration

5 shows the real roots of the equation in each measure for both sexes, respectively. The real roots were converted to the chronological age and are presented in Table 5.

In boy, all measures have the one root which shows the local minimum in velocity curve, and another which shows the local maximum in it. In stature, body weight and back strength, one more real root was found near between 17 and 18 years old. The former two roots suggested the followings. Up to the age corresponding first root, the second derivative was negative, so the velocity decreased. This means the growth and development would continue with decreasing velocity. Then the growth and development spurt would start and continue up to the age corresponding to the second root and the peak growth and developmental velocity would appear at this age. Thereafter, the growth and development would be decelerated. Then the velocity becomes very small after 17 years old. Therefore, even if the second derivative exist and no matter whether it is positive or negative, it can be inferred that the growth and developmental distance is about to reach its plateau. Therefore, it can be concluded that the growth and development may continue deceleratedly, then acceleratedly and then deceleratedly again in all physical fitness elements chosen here in the term over 6 and 18 years old in boy. Then, at 17 or 18 years old, they may be reaching to their plateau. And the peak velocity may appear fastest at 12.03

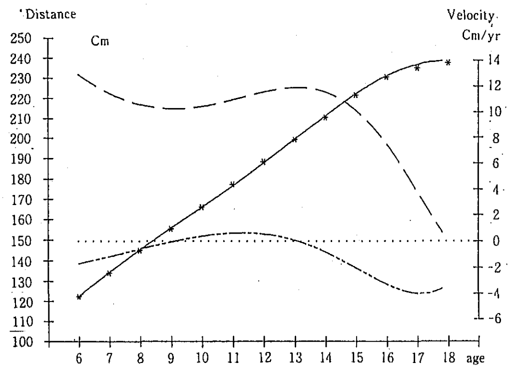

Fig. 5 Distance, velocity, and acceleration curves of stand. long jump: Boy

—— distance, – – – velocity, ---- acceleration

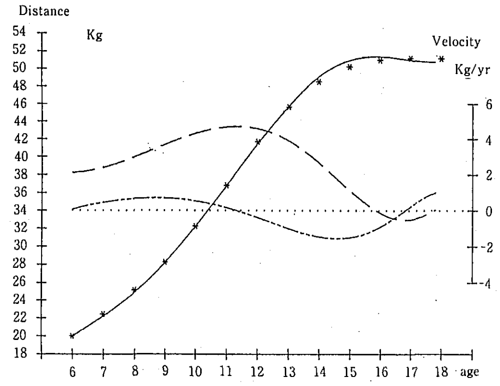

Fig. 7 Distance, velocity, and acceleration curves of body weight: Girl

—— distance, – – – velocity, ---- acceleration

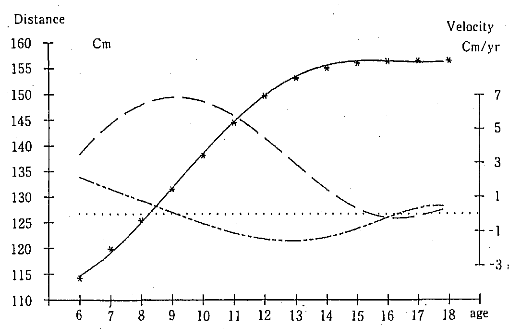

Fig. 6 Distance, velocity, and acceleration curves of stature: Girl

—— distance, – – – velocity, ---- acceleration

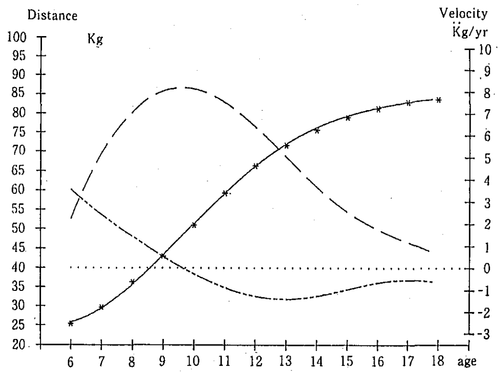

Fig. 8 Distance, velocity, and acceleration curves of back strength: Girl

—— distance, – – – velocity, ---- acceleration

Table 5 The ages corresponding to the real roots of equations that the second derivatives were put zero ; unit is years old

Measure	Boy			Girl	
Stature	8.33,	12.03,	17.21	9.13,	16.31
Body weight	7.62,	12.66,	17.65	11.32,	16.80
Back strength	7.92,	12.70,	17.16	9.63	
Vertical jump	7.77,	12.50		8.88	
Standing L.Jump	9.12,	13.10		8.22	

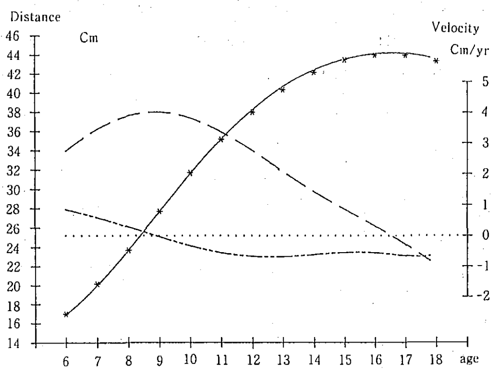

Fig. 9 Distance, velocity, and acceleration curves of vertical jump: Girl

——— distance, — — — velocity, ----- acceleration

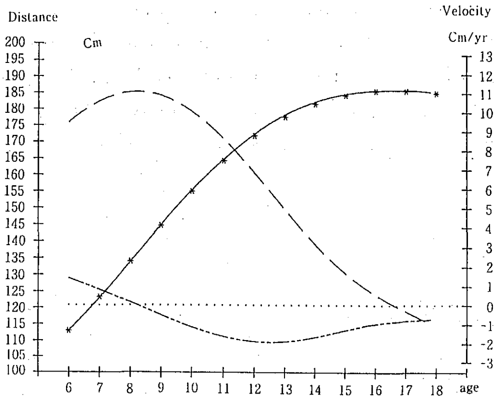

Fig. 10 Distance, velocity, and acceleration curves of stand. long jump: Girl

——— distance, — — — velocity, ----- acceleration

years old in stature and intermediate at 12.50 to 12.70 years old in body weight, back strength and vertical jump, and then latest at 13.10 in standing long jump. Tanner[21,22] suggested that the peak height velocity may appear about 6 months earlier than peak weight velocity. This result was comparable to Tanner's study. The peak age of stature was youngest in all measures but the onset of growth spurt was found a little later than others except standing long jump, so the duration of growth spurt may be shorter in stature than in other measures. Actually the duration of growth and developmental spurt was 4.6 to 5.04 years in body weight, back strength and vertical jump, and 4 years even in standing long jump, but 3.7 years in stature.

In girl, the ages corresponding to the first real roots stand for the age which peak growth or developmental velocity appears. This suggests that the growth and development are already accelerated at 6 years old in all measures. Therefore, it may be assumed that the local minimum of velocity may be found before 6 years old in girl. This shows such understanding widely recognized as the girl matures faster in the growth and development of physical fitness than the boy. In stature and body weight, the second real roots were found between 16 and 17 years old, but their velocities were very small and likely to be about to change a little, so this may suggest that their growth may reach plateau. The age of peak weight velocity was oldest but that of stature was 9.13 years old, so peak growth velocity may appear about 2 years earlier in stature than in body weight. Such difference was only 0.63 years; 7.6 months, in boy. Furthermore, the age of peak developmental velocity in other measures were younger than that of body weight and, even compared with that of stature, they were 3 months younger in vertical jump and 11 months in standing long jump. This is one of quite different characteristics in girl from boy.

Tanner[21] reported that the peak in strength gain is about 14 months after the height peak and 9 after the weight peak. However, in this study, the appearance age of peak velocity in strength is comparable to that of body weight in boy, but it is about 20 months earlier than that of body weight and 6 months later than that of stature in girl. Although any definite reason could not be found, it may be inferred that the sex differences of endocrinological mechanism might partially cause such sex differences as mentioned previously. The excretion of oestrogen in girls rises very sharply and begins to be cyclic at about 10 years old and oestrogen cycles are established at about the time the height spurt begins.[21] This may influence favorably to deceleration in the development of muscular strength in girl, but in boy, the excretion of androgens is activated before peak height velocity appears and the difference in excretion amount between both sexes may become large after oestrogen cycles are established in girl. It has been well established that androgens influence the development of muscular strength favorably.[21] Therefore, the development of muscular strength can be accelerated after peak height velocity appears in boy, but not so in girl. It could be concluded that these curves reflect such sex differences sharply.

In terms of velocity change, the developmental trend of muscular strength is very similar to that of body weight in boy and that of jumping skill; i.e., standing long jump, seems to be realized based upon the growth of physique and the development of muscular strength. This is logical and reasonable inference induced from the relationship between peak ages of different measures. In girl, however, the developmental trends of muscular strength and motor skill are rather similar to that of stature but the developmental spurt may appear before the growth spurt begins in physique. In other words, the growth spurt may decelerate the development of motor skill in girl.

4. Conclusions

It has been well understood that the growth and development distance curves can not be described successfully with a single mathematical function. In this study, however a polynomial of some high order was evidenced to describe them so satisfactorily enough as to be useful for analysis of their characteristics. Through investigating the mathematical characteristics of the polynomial fitted to each physical fitness elements chosen in this study, the followings were induced.

1) In the term over 6 and 18 years old; i.e., growing term, most of physical fitness elements grow or develop deceleratedly, then acceleratedly, and then deceleratedly again in boy. In girl, they do acceleratedly and then deceleratedly in this term. In other words, the growth and developmental spurt begins already before 6 years old in girl, but it begins after 6 years in boy.

2) In boy, the age of peak velocity is about 7 months earlier in stature than other measures.

3) In girl, the age of peak velocity is about 2 years earlier in stature than body weight, but 6 months earlier than back strength, 3 months later than vertical jump and 11 months later than standing long jump.

4) The timing difference in the age of peak velocity between different elements of physical fitness is quite different between both sexes; i.e., that of motor skill; standing long jump, is latest in boy but earliest in girl.

5) Through investigating the timing difference in appearance of peak velocity between different elements of physical fitness, it was inferred that the motor skill development is realized favorably based upon the growth of physique and development of muscular strength in boy but the growth spurt may decelerate the development of motor skill in girl.

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