Comparison of different factor solutions in factorial structure of motor ability

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最近の運動能力の因子構造の研究には，一つの相関行列に，いくつかの異る因子解法にもとずく因子解を求め，得られた因子パターン行列を比較検討し，因子の妥当性，客観性を確めつつ因子構造についての結論を導こうとする方法論がとられるようになってきている。これは，因子分析に必要な面倒な計算が電算機によって容易に行なえるようになってからの傾向である(1970年以降)。しかし，因子解法には，相互に直接導かれるカノニカル因子解法と主因子解法，及び前述の2解法とは異るが相互に直接導かれ得る主成分分析法とイメツヂ因子解法，さらに，これらは全く異る最尤度因子解法，アルファ因子解法の4種に大きく分類することが出来る。これら4つの異った解法，又は発想の異る6つの因子解法相互の因子解法それ自身についての比較検討については，理論的関連についてはPsychometricsの領域において検討されてきてはいるが，これらすべてを同時に比較検討してはいない。かつ，運動能力データを用いて，具体的適合性についての検討は皆無である。我が国では主成分分析法が比較的多く用いられてきている。因子分析的方法の弱点の一つは，異る解法による解は必ずしも同一ではなく，すなわち，異る解法による因子の不変性の保証のない所にある。この意味から，いかなる因子解法にもとずく因子解がどのような実際的特徴をもつものであるか，また，運動能力の因子構造の検討には，いかなる因子解法による解が適当であるのか，及び，我が国でこれまで多く採用されて来た主成分分析法，主因子解法は適当な解を与えるものといえるのかについて，10項目の運動能力テストよりなる相関行列に主成分分析法，主因子解法，カノニカル因子解法，イメッヂ因子解法，最尤度因子解法，アルファ因子解法の6解法を適用し，得られた因子解にNormal Varimax基準による直交解，及び独立群化法（Independent cluster method）による斜交解，計12因子解を導き，これらを相互に比較検討し，各因子解の特徴を検討した。

I. Introduction

Since factor analysis procedures were introduced in piysical educatdon research early 1930's, many studies have applied some factor analytic procedures to have found many useful findings. Since large scaled electronic computer could be used widely in research, various factor solutions that had been developed mainly in psychomerics research area have been utilized widely in various research fields. Most of factor solutions or factoring procedures were developed before 1960's ; e. g., Centroid factor (Thurston, L., 1931)*[19], Principal factor (Hotelling, H., 1933)*[8], Bi-factor (Holzinger, H., 1937),*[9] Multiple-group factor (Horst, P., 1937*[11], Guttman, L., 1944*[3], Holzinger, K., 1944*[10], Thurston, L., 1934*[20]), Maximum likelihood factor (Lawley, D. N., 1940*[14]), Image factor (Guttman, L., 1953)*[4], Alpha factor (Kaiser, H. F., and Caffrey, J., 1965)*[13], Alpha-max factor (Bentler, P, M., 1986)*[1], Minres factor (Harman, H. H. and Jones, W. H., 1966)*[5], and so on. Among them, Cenoroid factor, Bi-factor and Multiple group factor solutions became out of date already, because they were devised with some arbitrary assumption and mainly for manual use. While other factor solutions are based upon some exact mathematical back ground. However they had never been utilized for any correlation matrices of large order until the large scaled electronic computer could be used, because they need computation of eigen values and their associated eigen vectors of given matrix and also matrix inversion that are almost impossible to be accomplished manually.

Since middle of 1960's in Japan and beginning of 1960's in U. S. A., when the large scaled electronic computer could be used widely for general user with FORTRAN instead of assembly or machine language, these factor solutions based upon strict mathematical foundation began to be used in place of classical factor solutions, such as Centroid factor soution or Bi-factor solution and so on, and then Centroid factor, Bi-fator, and Multiple-group factor solutions have been out of date unless they are used as the initial approximation to final solution.

In the field of health, physical education and sport science, many factor analytic studies have been reported by many researchers since 1930*[15]. Most of them which have been reported in Japan since 1965 utilized the principal component or principal factor solutions, and only a few studies utilized other solutions ; e. g., hierarchical factor solution derived from multiple-group factor solution (Matsuura, Y., 1969)*[15], Maximum likelihood factor solution (Matsuura, Y., 1981).*[18] It has been well known that the same factor solution would not always be resulted in even from same correlation matrix with different factor solution applied. Thus it is worth recognizing the characteristics of each factor solution and discussing on which factor solution is more appropriate to any given data ; particularly physical fitness and/or motor ability data, because any studies of this type have never been done in physical education research field.

In this study six direct factor solutions ; Principal component, Principal factor, Canonical factor, Alpha factor, and Maximum likelihood factor solutions, were applied, and multiple orthogonal and oblique factor solutions were derived from these six direct factor solutions. Thus twelve derived factor solutions ; six orthogonal solutions and six oblique solutions, were obtained from a correlation matrix of motor ability measures, and they were compared.

II. Basic Concept of Six Factor Solutions and their Relationships

Let R, C_I^2, U_I^2, C_0^2, and U_0^2 be a correlation matrix with unity in diagonal, a diagonal matrix with communalities estimated by squared multiple correlation ; SMC, a diagonal matrix with uniquenss ; $(I-C_I^2)$, a diagonal matrix with final communality estimates, and a diagonal matrix with final uniqueness estimates ; $(I-C_0^2)$, respectively.

Suppose that $U_I^{-1}\ R_I^{-1}\ U_I^{-1}$ be decomposed into $Q_1\ (e_{1i})\ Q_1'$, where (e_{1i}) is a diagonal matrix with eigen values and Q_1 is the associted eigen vectors ; that is,

$$U_I^{-1} R U_I^{-1} = Q_1\ (e_{1i})\ Q_1' \tag{1}$$

Guttman's Image variance-covariance matrix can be expressed as

$$G = R + U_I^2 R^{-1} U_I^2 - 2U_I^2 \tag{2} \quad {}^{*4,16}$$

per-and post-multiply U_I^{-1} to (2), then

$$U_I^{-1} G U_I^{-1} = U_I^{-1} R U_I^{-1} + U_I R^{-1} U_I - 2I$$

This may be simplified further to the following ;

$$U_1{}^1 G U_I^{-1} = Q_1 \,\mathrm{diag}\left(\frac{(e_{1i}-1)^2}{e_{1i}}\right) Q_1' \tag{3}$$

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$$G = U_1 Q_1 \operatorname{diag}\left(\frac{(e_{1i}-1)^2}{e_{1i}}\right) Q_1' U_1 \tag{4}$$

Then Image factor solution is given by

$$A = U_1 Q_1 \operatorname{diag}\left(\frac{(e_{1i}-1)^2}{e_{1i}}\right)^{\frac{1}{2}} \qquad (5)^{*6,16}$$

From $U_1^{-1}RU_1^{-1}=Q_1\ (e_{1i})\ Q_1'$, Pre-and post-multiply U_1,

$$R = U_1 Q_1 (e_{1i}) Q_1 U_1 \ , \qquad \text{so}$$

$$A = U Q (e_{1i})^{\frac{1}{2}} \qquad (6)^{*6,16}$$

gives principal component solution.

Furthermore, $(R-U_1^2)$ is a correlation matrix with communalities estimated by SMC in diagonal. Then

$$\begin{aligned} U_1^{-1}(R-U_1^2)U_1^{-1} &= U_1^{-1} R U_1^{-1} - I \\ &= Q_1 \operatorname{diag}(e_{1i}) Q_1' - \operatorname{diag}(1) \\ &= Q_1 \operatorname{diag}(e_{1i}-1) Q_1' \end{aligned}$$

Thus,

$$(R-U_1^2)=U_1Q_1\operatorname{diag}\ (e_{1i}-1)\ Q_1'U_1, \qquad \text{so}$$

Principal factor solution with communalities estimated by SMC is given by

$$A = U_1 Q_1 \operatorname{diag}(e_{1i}-1)^{\frac{1}{2}} \tag{7}$$

Principal factor solution with communalities estimated by SMC is given by
Therefore, principal component solution, Image factor solution, and principal factor solution with communalities estimated by SMC are related directly. On the other hand, Canonical factor solution is based upon the following concept. The canonical correlation between the observed variables and factors (derived variables) are equal to eigen values of characteristic equation, $|AA'-\lambda R|=0$, where A stands for factor pattern matmix and R for correlation matrix.*[16] Then let U_0^2 be a uniqueness estimate and R-U_0^2 be an approximation for AA′, because AA′ is a reproduced correlation matrix. The above equation is expressed as

$$|(R-U_0^2)-\lambda R| = 0 \tag{8}$$

This may be simplified further to the following ;

$$|U_0^{-1}(R-U_0^2)U_0^{-1}-\mu I| = 0, \qquad \text{where } \mu=\frac{\lambda}{1-\lambda} \tag{9}$$

Then, suppose that $U_0^{-1}(R-U_0^2)U_0^{-1}$be decomposed into $Q_2\ (e_{2i})\ Q_2'$, where (e_{2i}) is a diagonal matrix of eigen values and Q_2 the associated eigen vector matrix.
Then, Canonical factor solution is given by

$$A = U_2 Q_2 (e_{2i})^{\frac{1}{2}} \qquad (10)^{*16}$$

Alpha factor solution is based upon such concept that sample common factors are to be determined so as to show maximum correlation with the population common factors.
Then, Kaiser and Caffrey utilized coefficient of generalizability ; coefficient Alpha, developed by Cronbach (1951).[*2]
Coefficient Alpha is expressed in the following ;

$$\alpha = \frac{n}{n-1}\left(1 - \frac{w' C_0^2 w}{w'(R-U_0^2)w}\right) \qquad (11)$$

where w'stands for weight vector. Maximizing α is equivalent to maximizing

$$\lambda = \frac{W' (R-U_0^2) W}{W'C_0{}^2W},$$

and then the problem leads to solving the following characteristic equation ;

$$| C_0^{-1} (R-U_0^2) C_{01}^{-} - \lambda I | = 0 \qquad (12)$$

Then, suppose that C_0^{-1} $(R-U_0^2)$ C_0^{-1}can be decomposed into Q_3 (e_{3i}) Q_3', so Alpha factor solution is given by

$$A = C_0 Q_3 (e_{3i})^{\frac{1}{2}} \qquad (13)^{*13,16}$$

The Concept of Maximum likelihood factor solution developed by Lawley (1944) starts with defining a correlation matrix R as a sample estimate of population correlation matrix P. According to the basic concept of factor analysis,

$$P = \bar{A}\bar{A}' + \bar{U}^2, \qquad (14)$$

where $\bar{A}$ and $\bar{U}^2$ stand for population pattern matrix and population uniqueness matrix with uniqueness in diagonal. And suppose R may be expressed $AA'+U^2$, where A and U^2 are sample estimates of $\bar{A}$ and $\bar{U}^2$, respectively.

$$R = AA' + U^2 \qquad (15)$$

Then, suppose that U^2 may be estimated by U_0^2, and then let $U_0^{-1}(R-U_0^2)$ be decomposed into Q_4 (e_{4i}) Q_4', so Maximum likelihood factor solution is given by

$$A = Q_4 (e_{4i})^{\frac{1}{2}} \qquad (16)^{*16}$$

However, U_0^{-1} $(R-U_0^2)$ is not symmetric. This leads to eigen value problem of unsymmetric matrix, Practically, Maximum likelihood factor solution can be accomplished by some iteration procedures.[*'14'16] Furthermore, some variation of these six models have been devised ; e. g., Alphamax,[*13] Minres,[*5] Multi-step,[*17] and so on, but they are fundamentaly based upon either of these six factor solutions, and either of them is utilized on the process to accomplish their final solutions.

III. Method

In order to make comparison between these twelve solutions simple, ten motor ability variables were chosen ; that is, four physique measures : stature, body weight, chest girth, and sitting height, four fundamental motor skill : 50m dash, running broad jump, softball throw for distance and vertical jump, two muscular strength measures : back strength and grip strength (mean of rigth and left grip). Data of these measures were extracted from 2184 elementary school boys of 5th and 6th grades. Then a correlation matrix of order 10 was computed with the data of 5th and 6th grades pooled. Six factor solutions were applied to this correlation matrix, and six multiple orthogonal factor solutions were derived with Normal Varimax criterion procedure, and also six multiple oblique factor solutions with Independent Cluster procedure. For communality and uniqueness estimation, such iteration technique as repeating principal factor analysis was used. For maximum likelihood factor solution, Hemmerle algorithm*[7] was ustlized, becausie this algorithm seems to converge faster than any other ones and few computation trouble may happen on the interation process. All the computer programs were devemoped by author himself and FACOM M200 computer of University of Tsukuba Science Information Processing Center was used.

IV. Results and Discussion

Table 1 shows the correlation matrix, and table 2 to 7 show six multiple orthogonal factor solutions resulted. Numbers of factors extracted are two for principal component, principal factor and Maximum likelihood factor solutions, and five for Canonical factor, Image factor, and Alpha factor solutions. In Image factor solution, however, three of five factors extraced showed very small amount of contribution ; e. g., .08639, .0094, and .00227, so they could not be interpreted, so Image factor solution produced two robust factors. Principal component, principal factor, Maximum likelihood factor, and Image factor solutions that producded only two robust factors show very similar factor pattern. On the first factor showed stature, body weight, chest girth and sitting height considerably high loadings, and back strength, girp strength and softball throw for distance showed significant but low loadings. Therefore, this factor was named physique factor. On the second factor showed 50m dash and running broad jump considerably high loadings ; higher than. 70, and softball throw and vertical jump moderate ones, and back and grip strength significant but low ones. This is the case for all the factor solutions which produced two robust factors. Thus this factor was named as fundamental motor skill factor. It is one of common features in these four solutions that back strength and grip strength measures showed low but significant loadings on both factors, and also that softball throw showed low loading on physique factor. Principal component solution showed the highest degree of contribution ; 60.75%, and principal factor solution next ; 54.24%, Maximum likelitood factor solution the third ; 52.73%, and Image factor solution the least ; 44.45%. However, as mentioned precedingly, there was found no discrepancy in the variables that loaded significantly on both factors.

In Canonical factor and Alpha factor solutions which resulted in five factors, three factors were robust but other two factors were not. The first factor extracted in both solutions was loaded highly by stature and sitting height, moderately by body weight, and low but significantly by chest girth and grip strength. This factor showed high congruence with the first facotor ; physique factor extracted in other four solutions which produced two robust factors. Thus the first factor, although it showed high congruence with physique factor in other four solutions, was reasonably interpreted as body linearity factor. The second factor showed high congruence with the second factor of other four solutions ; fundamental motor skill factor. The third factor was loaded highly by chest girth and moderately by body weight, so this was named as body bulk factor. The fourth factor was loaded moderately by vertical jump, and low but significantly by grip strength, so it was interpreted tentati-

Table 1, Correlation matrix

1	1.0000	0.6918	0.5529	0.7450	0.3139	0.3403	0.3837	0.1888	0.3665	0.4552
2	0.6918	1.0000	0.7603	0.6632	0.3025	0.2885	0.3418	0.1863	0.3862	0.4289
3	0.5529	0.7603	1.0000	0.5488	0.4582	0.4387	0.3854	0.2285	0.3779	0.4554
4	0.7450	0.6632	0.5488	1.0000	0.2929	0.3264	0.3885	0.1744	0.3524	0.4116
5	0.3139	0.3025	0.4582	0.2929	1.0000	0.6280	0.4886	0.4523	0.3477	0.3125
6	0.3403	0.2885	0.4387	0.3264	0.6280	1.0000	0.4571	0.4585	0.3204	0.3227
7	0.3837	0.3418	0.3854	0.3885	0.4886	0.4571	1.0000	0.3502	0.3763	0.3606
8	0.1888	0.1863	0.2285	0.1744	0.4523	0.4585	0.3502	1.0000	0.2137	0.3366
9	0.3665	0.3862	0.3779	0.3524	0.3477	0.3204	0.3763	0.2137	1.0000	0.3248
10	0.4552	0.4289	0.4554	0.4116	0.3125	0.3227	0.3606	0.3366	0.3248	1.0000

Note ; Variable No. is as follows: 1. Stature, 2. Body weight, 3. Chest girth, 4. Sitting height, 5. 50m dash, 6. Running broad jump, 7. Softball throw for distance, 8. Vertical jump, 9. Back strength, 10. Grip strength

Table 2, Principal Component

	FACTOR	I	II
1	STATURE	0.846	0.166
2	BODY WEIGHT	0.913	0.127
3	CHEST GIRTH	0.756	0.363
4	SITTING HEIGHT	0.828	0.151
5	50M DASH	0.197	0.838
6	RUNNING BROAD J.	0.208	0.817
7	SOFTBALL THROW	0.317	0.617
8	VERTICAL JUMP	0.062	0.680
9	BACK STRENGTH	0.389	0.386
10	GRIP STRENGTH	0.480	0.376
	AMOUNT OF CONT.	3.372	2.702

Table 3, Principal Factor

	FACTOR	I	II
1	STASURE	0.831	0.177
2	BODY WEIGHT	0.841	0.158
3	CHEST GIRTH	0.729	0.347
4	SITTING HEIGHT	0.790	0.171
5	50M DASH	0.218	0.758
6	RUNNING BROAD J.	0.233	0.725
7	SOFTBALL THROW	0.339	0.543
8	VERTICAL JUMP	0.068	0.656
9	BACK STRENGTH	0.392	0.352
10	GRIP STRENGTH	0.462	0.357
	AMOUNT OF CONT.	3.142	2.282

Table 4, Maximum Likelihood Factor

	FACTOR	I	II
1	STATURE	0.793	0.197
2	BODY WEIBHT	0.846	0.154
3	CHEST GIRTH	0.711	0.355
4	SITTING HEIGHT	0.774	0.183
5	50M DASH	0.211	0.774
6	RUNNING BROAD J.	0.223	0.745
7	SOFTBALL THROW	0.328	0.539
8	VERTICAL JUMP	0.099	0.573
9	BACK STRENGTH	0.383	0.344
10	GRIP STRENGTH	0.461	0.331
	AMOUNT OF CONT.	3.050	2.223

Table 5, Canonical Factor

	FACTOR	I	II	III	IV	V
1	STATURE	0.832	0.172	0.202	0.095	0.171
2	BODY WEIGHT	0.606	0.105	0.593	0.093	0.184
3	CHEST GIRTH	0.386	0.309	0.728	0.079	0.177
4	SITTING HEIGHT	0.764	0.167	0.228	0.066	0.187
5	50M DASH	0.112	0.739	0.183	0.178	0.195
6	RUNNING BROAD J.	0.175	0.710	0.141	0.218	0.119
7	SOFTBALL THROW	0.256	0.447	0.091	0.184	0.384
8	VERTICAL JUMP	0.050	0.416	0.035	0.617	0.091
9	BACK STRENGTH	0.244	0.243	0.177	0.090	0.458
10	GRIP STRENGTH	0.342	0.170	0.247	0.346	0.272
	AMOUNT OF CONT.	2.079	1.677	1.131	0.649	0.621

Table 6, Image Factor

	FACTOR	I	II	III	IV	V
1	STATURE	0.736	0.236	0.086	0.010	0.002
2	BODY WEIGHT	0.781	0.207	0.130	0.003	0.001
3	CHEST GIRTH	0.661	0.356	0.215	0.004	0.0
4	SITTING HEIGHT	0.720	0.223	0.087	0.018	0.002
5	50M DASH	0.229	0.637	0.067	0.028	0.005
6	RUNNING BROAD J.	0.238	0.624	0.030	0.030	0.004
7	SOFTBALL THROW	0.319	0.495	0.041	0.001	0.028
8	VERTICAL JUMP	0.105	0.507	0.010	0.040	0.008
9	BACK STRENGTH	0.361	0.333	0.010	0.015	0.042
10	GRIP STRENGTH	0.434	0.338	0.003	0.071	0.008
	AMOUNT OF CONT.	2.647	1.798	0.085	0.009	0.003

Table 7, Alpha Factor

	FACTOR	I	II	III	IV	V
1	STATURE	0.831	0.167	0.204	0.104	0.170
2	BODY WEIGHT	0.607	0.104	0.578	0.090	0.199
3	CHEST GIRTH	0.395	0.306	0.733	0.087	0.171
4	SITTING HEDGHT	0.769	0.175	0.217	0.063	0.179
5	5OM DASH	0.111	0.745	0.181	0.169	0.191
6	RUNNING BROAD J.	0.175	0.707	0.140	0.218	0.118
7	SOFTBALL THROW	0.262	0.450	0.086	0.187	0.374
8	VERTICAL JUMP	0.050	0.423	0.037	0.612	0.083
9	BACK STRENGTH	0.244	0.250	0.176	0.085	0.457
10	GRIP STRENGTH	0.341	0.170	0.236	0.351	0.280
	AMOUNT OF CONT.	2.097	1.691	1.108	0.647	0.615

vely as leg explosive strength factor, but the fifth factor was loaded significantly lut low by back strength and sofotball throw for distance, but it seemed to be rather hard to interprete this factor as back strength. Thus, in Canonical and Alpha factor solutions, body linearity, body bulk, fundamental motor skill, and leg explosive strength factors were extracted and interpreted. Therefore it can be concluded that Canoical factor and Alpha factor solutions are likely to produce more robust factors than principal component, Principal factor and Image factor solutions. Jackson, A. S. (1972)*[12] showed Canocial factor and Image factor solutions produced more factors than principal component bnd Alpha factor solutions at his factor analytical study of muscular strength. In this study, however, Image factor solution produced five factors but the loadings of three factors were all negligible. Table 8 to 13 show six oblique factor pattern matrices and factor structure matrices and factor correlation matrices. In the oblique solutions derived from principal cmponent, Principal factor and Maximum likelihood factor solutions, the simple structure of pattern matrix seemed to be accomplished more perfectly than in the orthogonal solutions, But muscular strength measures showed low but significant loadings only on the first factor which was named "physique" factor in principal factor and Maximum likelihood factor solutions, and showed low but significant loadings on both factors in

Table 8, Oblique Solution derived from Principal Component Solution

		PATTERN		STRUCTURE	
	FACTOR	I	II	I	II
1	STATURE	0.864	-0.005	0.862	0.369
2	BODY WEIGHT	0.947	-0.062	0.920	0.347
3	CHEST GIRTH	0.713	0.230	0.813	0.538
4	SITTING HEIGHT	0.849	-0.017	0.842	0.350
5	50M DASH	-0.017	0.868	0.358	0.861
6	RUNNING BROAD J.	0.001	0.843	0.365	0.843
7	SOFTBALL THROW	0.173	0.602	0.433	0.676
8	VERTICAL JUMP	-0.119	0.726	0.195	0.675
9	BACK STRENGTH	0.314	0.334	0.458	0.470
10	GRIP STRENGTH	0.410	0.304	0.541	0.481
	CORRELATION	1.000	0.432		

Table 9, Oblique Solution derived from Principal Factor Solution

		PATTERN		STRUCTURE	
	FACTOR	I	II	I	II
1	STATURE	0.890	-0.074	0.847	0.453
2	BODY WEIGHT	0.909	-0.099	0.852	0.421
3	CHEST GIRTH	0.705	0.159	0.797	0.563
4	SITTING HEIGHT	0.845	-0.067	0.807	0.417
5	50M DASH	-0.046	0.814	0.420	0.788
6	RUNNING BROAD J.	-0.016	0.770	0.425	0.761
7	SOFTBALL THROW	0.178	0.552	0.476	0.624
8	VERTICAL JUMP	-0.180	0.746	0.247	0.643
9	BACK STRENGTH	0.314	0.279	0.474	0.459
10	GRIP STRENGTH	0.393	0.261	0.543	0.486
	CORRELATION	1.000	0.572		

Table 10, Oblique Solution derived from Maximum Likelihood Factor Solution

	FACTOR	PATTERN I	PATTERN II	STRUCTURE I	STRUCTURE II
1	STFTURE	.0.825	-0.015	0.817	0.369
2	BODY WEIGHT	0.920	-0.086	0.875	0.347
3	CHEST GIRTH	0.678	0.190	0.778	0.538
.4	SITTING HEIGHT	0.808	-0.025	0.795	0.350
5	50M DASH	-0.027	0.817	0.403	0.861
6	RUNNING BROAD J.	-0.003	0.779	0.407	0.843
7	SOFTBALL THROW	0.185	0.513	0.455	0.676
8	VERTICAL JUMP	-0.084	0.621	0.243	0.675
9	BACK STRENGTH	0.313	0.275	0.458	0.470
10	GRIP STRENGTH	0.406	0.267	0.541	0.481
	CORRELATION	1.000	0.526		

Table 11, Oblique Solution derived from Canonical Factor Solution

PATTERN

	FACTOR	I	II	III	IV	V
1	STATURE	0.893	0.034	-0.039	0.038	0.011
2	BODY WEIGHT	0.177	-0.120	0.759	0.046	0.001
3	CHEST GIRTH	-0.236	0.148	1.029	-0.043	0.014
4	SITTING HEIGHT	0.769	0.028	0.034	-0.001	0.051
5	50M DASH	-0.038	0.751	0.039	-0.025	0.106
6	RUNNING BROAD J.	0.112	0.746	-0.045	0.045	0.019
7	SOFTBALL THROW	0.152	0.247	-0.128	0.072	0.425
8	VERTICAL JUMP	-0.010	0.185	-0.126	0.700	0.057
9	BACK STRENGTH	0.008	-0.040	0.084	0.003	0.569
10	GRIP STRENGTH	0.113	-0.160	0.197	0.385	0.206

STRUCTURE

	FACTOR	I	II	III	IV	V
1	STATURE	0.893	0.353	0.721	0.374	0.550
2	BODY WEIGHT	0.768	0.340	0.868	0.365	0.536
3	CHEST GIRHT	0.621	0.524	0.882	0.419	0.575
4	SITTING HEIGHT	0.835	0.340	0.700	0.341	0.538
5	50M DASH	0.302	0.810	0.425	0.555	0.594
6	RUNNING BROAD J.	0.336	0.781	0.412	0.567	0.539
7	SOFTBALL THHROW	0.405	0.569	0.415	0.484	0.638
8	VERTICAL JUMP	0.171	0.563	0.226	0.734	0.381
9	BACK STRMENGTH	0.394	0.384	0.433	0.342	0.602
10	GRIP STRENGTH	0.476	0.375	0.511	0.521	0.511

CORRELATION

	FACTOR	I	II	III	IV	V
1	I	1.000	0.342	0.807	0.362	0.582
2	II	0.0	1.000	0.481	0.686	0.665
3	III	0.0	0.0	1.000	0.441	0.634
3	IV	0.0	0.0	0.0	1.000	0.573
5	V	0.0	0.0	0.0	0.0	1.000

principal component solution. However the structure matrices of these three solutions showed all variables correlated significantly with both factors, although it was natural shat physique measures showed especially high correlations with physique factor and fundamental motor skill measures also high correlatinos with fundamental motor skill factor, The factor correlation was the lowest in the solution derived from principal component ; .43226, and the ones from Maximum likelihood factor and principal factor solutions, .52638, and .57234, respectively.

In the oblique solutions derived from Canonical factor and Alpha factor solutions, the features that were found in the orthogonal solutions derived from them tended to be more accentuated ; that is, the first factor was loaded highly only by stature and sitting height, so it was interpreted as body linearity factor, and the third factor loaded highly only by body weight and chest girth, so it could be interpreted as body bulk factor, and running and jumping factor, back strenbth and throwing factor, and leg explosive strength factor were extracted as robust factors. And the simple structure was accomplished more sufficiently than in the orthogonal solutions. In the oblique solution derived from

Table 12, Oblique Solution derived from Alpha Factor Solution

		PATTERN				
	FACTOR	I	II	III	IV	V
1	STATURE	0.892	-0.024	-0.003	0.022	0.030
2	BODY WEIGHT	0.190	-0.009	0.730	0.057	-0.100
3	CHEST GIRTI	-0.205	-0.081	1.028	0.002	0.190
4	SITTING HEIGHT	0.795	0.024	0.036	0.043	0.055
5	50M DASH	-0.042	0.123	0.054	0.058	0.757
6	RUNNING BROAD J.	0.119	-0.042	-0.014	0.042	0.741
7	SOFTBAAL THROW	0.090	0.510	-0.135	0.047	0.197
8	VERTICAL JUMP	-0.068	-0.165	-0.078	0.800	0.152
9	BACK STRENGTH	-0.088	0.721	0.055	0.043	-0.093
10	GRIP STRENGTH	0.040	0.224	0.186	0.441	-0.215

		STRUCTURE				
	FACTOR	I	II	III	IV	V
1	STATURE	0.894	0.633	0.720	0.461	0.332
2	BODY WEIGHT	0.764	0.613	0.861	0.425	0.315
3	CHEST GIRTH	0.627	0.640	0.886	0.482	0.502
4	SITTING HEIGHT	0.838	0.612	0.694	0.421	0.327
5	50M DASH	0.317	0.630	0.405	0.602	0.811
6	RUNNING BROAD J.	0.352	0.589	0.391	0.614	0.776
7	SOFTBAAL THROW	0.427	0.651	0.407	0.543	0.568
8	VERTICAL JUMP	0.196	0.423	0.206	0.721	0.573
9	BACK STRENGTH	0.406	0.601	0.436	0.397	0.382
10	GRIP STRENGTH	0.488	0.553	0.501	0.555	0.367

		CORRELATION				
	FACTOR	I	II	III	IV	V
1	I	1.000	0.697	0.803	0.488	0.341
2	II	0.0	1.000	0.711	0.728	0.713
3	III	0.0	0.0	1.000	0.488	0.430
4	IV	0.0	0.0	0.0	1.000	0.745
5	V	0.0	0.0	0.0	0.0	1.000

Image factor solution, although the features of factor pattern and structure matrices are very similar to the other two oblique solutions in which five factors were extracted, they were quite different from those of orthogonal pattern matrix derived from Image fator solutions. Despite that only two of five factors were robust in the orthogonal solution, four factors were robust in the oblique solution. In the orthogonal solution there were some factors that were extracted but could not be interpreted after axis rotation, but such implicit factors could appear as robust factor in the oblique solution. In other words, the orthogonal solution tends to discard some information of factorial structure and to result in some clear-cut structure of motor ability. This is one of strong points for orthogonal solution and one of weak points as well ; e. g., it is very good for selection of test items and test construction. The oblique solution is likely to give more information on factorial structure. For instance, physique domain tended to be devided into body lineaity and body bulk even in the orthogonal solutions derived from Canonical factor and Alpha factor solutions, and this was accentuated more definitely in the oblique solutions derived from the same solutions. The correlation between these two factors,

Table 13, Oblique Solution derived from Image Factor Solution

		PATTERN				
	FACTOR	I	II	III	IV	V
1	STATURE	0.0	0.001	0.736	0.035	0.020
2	BODY WEIGHT	0.726	-0.064	0.127	0.010	-0.002
3	CHEST GIRTH	0.887	0.148	-0.194	-0.010	-0.011
4	SITTING HEIGHT	-0.014	0.026	0.747	-0.015	0.029
5	50M DASH	0.092	0.590	-0.061	-0.006	0.080
6	RUNNING BROAD J.	-0.017	0.615	0.091	0.031	-0.016
7	SOFTBALL THROW	-0.146	0.203	0.162	0.049	0.346
8	VERTICAL JUMP	-0.129	0.270	-0.015	0.796	-0.056
9	BACK STRENGTH	0.101	-0.062	-0.028	-0.010	0.491
10	GRIP STRENGTH	0.129	-0.179	0.073	0.437	0.103
		STRUCTURE				
	FACTOR	I	II	III	IV	V
1	STATURE	0.740	0.414	0.777	0.579	0.653
2	BODY WEIGHT	0.815	0.411	0.787	0.576	0.655
3	CHEST GIRTH	0.769	0.530	0.701	0.638	0.690
4	SITTING HEIGHT	0.720	0.397	0.759	0.558	0.633
5	50M DASH	0.427	0.678	0.377	0.651	0.631
6	RUNNING BROAD J.	0.424	0.666	0.387	0.646	0.627
7	SOFTBALL THROW	0.444	0.556	0.439	0.584	0.585
8	VERTICAL JUMP	0.256	0.512	0.232	0.789	0.455
9	BACK STRENGTH	0.442	0.414	0.430	0.469	0.489
10	GRIP STRENGTH	0.509	0.434	0.503	0.519	0.538
		CORRELATION				
	FACTOR	I	II	III	IV	V
1	I	1.000	0.560	0.946	0.742	0.831
2	II	0.0	1.000	0.492	0.939	0.897
3	III	0.0	0.0	1.000	0.712	0.813
4	IV	0.0	0.0	0.0	1.000	0.978
5	V	0.0	0.0	0.0	0.0	1.000

however, was considerably high ; e. g., .80673 for the one derived from Canonical factor solution, and . 80307 for the one from Alpha factor solution. Therefore, even if these two factors were extracted as separate factors in two orthogonal solutions and three oblique solutions, it can reasonably be hypothesized that these four physique measures may define one physique factor domain just as a physique factor defdned in four orthogonal factor solutions.

V. Conclusions

In multiple orthogonal solutions, the solutions derived from principal compondnt, principal factor, Maximum likelihood, and Image factor solutions produced very similar factor structure ; that is, physique and fundamental motor skill factors, and Canonical factor and Alpha factor solutions produced five factors of which three were robust and could be positively interpreted. In the latter solutions, physique domains teneded to be devided into two domains ; body linearity and body bulk. Therefore, it may be concluded that Canonical factor and Alpha factor solutions tended to produce more robust factors than other solutions. In the oblique solutions derived from six direct factor solutions, the features observed in the orthogonal solutions were likely to be much accentuated. It should be noted that three implicit factors of Image factor solution in orthogonal solution appeared as explicit and robust factors in oblique solution, and two of them could be really interpreted. Oblique solution has some possibility to make some implicit factors of orthogonal solution explicit in obligue factor space. Oblique solutions derived from Canonical factor, Alpha factor and Image factor solutions, showed that body linearity factor correlated very highly with body bulk factor. This implies that four physique measures may define one factor ; physique factor just as in four orthogonal solutions and three oblique solutions. From the orthogonal solutions derived from Canonical and Alpha factor solutions tended to produce too many factors. In Image factor solutions, however, five factors were extracted but only two of them were robust after orthogonal axis rotation, so Image factor solution seemed to meet the practical situation more than others, but degree of contributdon of two factors was the least in all orthogonal solutions.

In this study only a correlation matrix of small order was analized, so the inferences obtained here should be tested furthermore by applying various correlation matrices of larger order.

The computer time for each solution is rather long except principal component solution, because interative computation processes are included ; particularly Maximum likelihood factor solution takes the longest computer time. For practical use of factor analysis, although more studies of this type should be worked out, principal component solution or principal factor solution can give some sufficient results. Of course, this recommendation depends upon what factor analysis is used for. For test construction, reconstruction of rescaled space at least, these two solutions are considered good enough and appropriate.

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