Vainerman, Leonid; **Vallin, Jean-Michel**

Yetter-Drinfel'd algebras and coideals of weak Hopf C^*-algebras. (English) Zbl 07308188
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Continuing the study of coactions of weak Hopf C^*-algebras on C^*-algebras and their applications [*L. Vainerman* and *J.-M. Vallin*, Methods Funct. Anal. Topol. 23, No. 1, 76–107 (2017; Zbl 1389.18008); J. Algebra 550, 333–357 (2020; Zbl 1446.16037)], this paper aims to characterize braided-commutative Yetter-Drinfel'd C^*-algebras over weak Hopf C^*-algebras in categorical terms, investigating quotient type coideal subalgebra of a given weak Hopf C^*-algebra $\mathfrak{E}$ and coideal subalgebras invariant with respect to the adjoint action of $\mathfrak{E}$ with an explicit description of quotient type coideal subalgebras of the weak Hopf C^*-algebras associated with Tannak-Yamagami categories as an example.

A synopsis of the paper consisting of seven sections goes as follows. §2 through §4 present basic definitions and facts for the comprehension of the main results of the paper. §2 describes three C^*-multitensor categories associated with any weak Hopf C^*-algebra, while §3 explains how to reconstruct a weak Hopf C^*-algebra if one of these categories is given. §4 is concerned wtih coactions. The following result (Theorem 4.3) is presented as a slight generalization of the main result in [*L. Vainerman* and *J.-M. Vallin*, Methods Funct. Anal. Topol. 23, No. 1, 76–107 (2017; Zbl 1389.18008)].

Theorem. Given a weak Hopf C^*-algebra $\mathfrak{E}$, the following categories are equivalent:

1. The category of unital $\mathfrak{E}$-C^*-algebras with unital $\mathfrak{E}$-equivariant-homomorphisms as morphisms.
2. The category of pairs $(\mathcal{M}, M)$, where $\mathcal{M}$ is a left module C^*-category with trivial associativities over $UCorep(\mathfrak{E})$ and M is a generator in $\mathcal{M}$, with equivalence classes of unitary module functors respecting the prescribed generators as morphisms.

§5 investigates an importantspecial class of $\mathfrak{E}$-C^*-algebras called braided-commutative Yetter-Drinfel'd C^*-algebras, characterizing the corresponding C^*-module categories:

Theorem. Given a weak Hopf C^*-algebra $\mathfrak{E}$, the following categories are equivalent:

1. Category $YD_{brc}(\mathfrak{E})$ of unital braided-commutative Yetter-Drinfel'd C^*-algebras with unital $\mathfrak{E}$- and $\widehat{\mathfrak{E}}$-equivariant *-homomorphisms as morphisms.
2. Category $Tens(UCorep(\mathfrak{E}))$ of pairs $(\mathcal{C}, \mathcal{E})$, where $\mathcal{C}$ is a C^*-multitensor category whose associativities reduce to the changing of brackets and $\mathcal{E} : UCorep(\mathfrak{E}) \to \mathcal{C}$ is a unitary tensor fucntor such that $\mathcal{C}$ is generated by the images of $\mathcal{E}$, with morphisms $(\mathcal{C}, \mathcal{E}) \to (\mathcal{C}', \mathcal{E}')$ equivalence classes of pairs $(\mathcal{F}, \eta)$, where $\mathcal{F} : \mathcal{C} \to \mathcal{C}'$ is a unitary tensor functor and $\eta : \mathcal{F}\mathcal{E} \to \mathcal{E}'$ is a natural unitary monoidal functor isomorphism.

Moreover, given a morphism $[(\mathcal{F}, \eta)] : (\mathcal{C}, \mathcal{E}) \to (\mathcal{C}', \mathcal{E}')$, the corresponding homomorphism of braided-commutative Yetter-Drinfel'd C^*-algebras is injective (surjective) iff $\mathcal{F}$ is faithful (full).

§6 investigates, as an application of the above theorem, coideal C^*-subalgebras belonging to the category: quotient type and invariant with respect to the adjoint action of a weak Hopf C^*-algebra and the relationship between them. It is established, as a generalization of Takeuchi's corresponding result in [*M. Takeuchi*, Commun. Algebra 22, No. 7, 2503–2523 (1994; Zbl 0801.16041)], that

Theorem. Any quotient type coideal C^*-subalgebra is invariant. Conversely, for any invariant coideal C^*-subalgebra I of $\mathfrak{E}$, there exists a unique, up to isomorphism, quantum subgroupoid (i.e., a weak Hopf algebra $\mathfrak{H}$ equipped with an epimorphism $\pi : \mathfrak{E} \to \mathfrak{H}$) such that I is isomorphic as a $\mathfrak{E}$-C^*-algebra to the quotient type coideal C^*-subalgebra $I(\mathfrak{H}\backslash\mathfrak{E})$.

§7 describes invariant and quotient type coideal C^*-subalgebras of weak Hopf algebras constructed using the Tambara-Yamagami categories [*D. Tambara* and *S. Yamagami*, J. Algebra 209, No. 2, 692–707 (1998; Zbl 0923.46052)] whose simple objects are elements of a finite abelian group G and one separable element m.

Reviewer: Hirokazu Nishimura (Tsukuba)

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16T05 Hopf algebras and their applications
46L05 General theory of C^*-algebras

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