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Equidistribution of orbits of isometries on compact Riemannian manifolds. (English)

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The question of the uniform distribution of orbits of semigroups of isometries on Riemannian manifolds dates back to H. Weyl, whose theorem [*H. Weyl*, Rend. Circ. Mat. Palermo 30, 377–407 (1910; JFM 41.0528.02)] on the equidistribution on $\mathbb{R}/\mathbb{Z}$ of the arithmetic progressions $\{n\theta\}_{n\in\mathbb{N}}$ for θ irrational is to be seen as the equidistribution of the orbits of the semigroup generated by the rotation of angle $2\pi\theta$ radians on the circumference $S_1=\mathbb{R}/\mathbb{Z}$. *V. I. Arnol'd* and *A. L. Krylov* [Sov. Math., Dokl. 4, 1–5 (1963; Zbl 0237.34008); translation from Dokl. Akad. Nauk SSSR 148, 9–12 (1963)] made use of a different notion of equidistribution some decades later to study the orbits on a sphere S_2 of the semigroup generated by two rotations A,B, assuring that, providing there exists a point $x\in S_2$ whose orbit is dense, then this orbit is equidistributed. Very recently *I. Stewart* [Extr. Math. 34, No. 1, 99–122 (2019; Zbl 07121589)] has observed that this theorem still obtains with respect to the orbits of the semigroup generated by a finite number of reflections of $\mathbb{R}^3$.

This paper considers the action of a finitely generated semigroup S of isometries on a compact Riemannian manifold, establishing

Theorem. If there is a dense orbit, then every orbit is equidistributed in the sense that, for any point $x\in X$ and for any system of generators $S=(a_0=1,a_1,\dots,a_r)$, we have

$$\lim_{n\to\infty}\frac{1}{(r+1)^n}\sum_{i_1,\dots,i_n=0}^{r} f(a_{i_n}\dots a_{i_1}x)=\frac{1}{m(X)}\int_X f\,dm$$

for any f in $C(X)$, where m is the measure induced by a volume form.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

58C35 Integration on manifolds; measures on manifolds
28C99 Set functions and measures on spaces with additional structure

Keywords:

equidistribution; isometries; Riemannian manifold

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