Chatterjee, Saikat; **Koushik, Praphulla**
On two notions of a gerbe over a stack. (English) ｜Zbl 07267887｜
Bull. Sci. Math. 163, Article ID 102886, 30 p. (2020).

This paper is concerned with the relationship between two different notions of a *differentiable gerbe over a differentiable stack*. One is a morphism of stacks abiding by some additional requirements [*K. Behrend* and *P. Xu*, J. Symplectic Geom. 9, No. 3, 285–341 (2011; Zbl 1227.14007); https://www.uni-due.de/~hm0002/stacks.pdf; https://arxiv.org/pdf/math/0605694.pdf]. The other is a Morita equivalence class of a Lie groupoid extension [*C. Laurent-Gengoux* et al., Adv. Math. 220, No. 5, 1357–1427 (2009; Zbl 1177.22001)]. On the one hand, given a Lie groupoid $\mathcal{G}$, the category of principal $\mathcal{G}$-bundles, denoted by $B\mathcal{G}$, is a differentiable stack [loc. cit.]. On the other hand, given a differentiable stack $\mathcal{D}$, there exists a Lie groupoid $\mathcal{H}$ such that $\mathcal{D}$ is isomorphic to $B\mathcal{H}$ [*E. Lerman*, Enseign. Math. (2) 56, No. 3–4, 315–363 (2010; Zbl 1221.14003)]. The central idea in this paper is the correspondence between Lie groupoids and differentiable stacks.

A synopsis of the paper consisting of five sections goes as follows. §2 aims to introduce the differentiable gerbe over a stack. §3 aims to introduce the classifying stack of a Lie groupoid and the notion of $\mathcal{G}$-$\mathcal{H}$ bibundle for Lie groupoids $\mathcal{G}$ and $\mathcal{H}$. §4 addresses the correspondence between the two definitions, exploring the possibility of finding a Morita equivalence class of a Lie groupoid extension from a gerbe over a stack $F : \mathcal{D} \to \mathcal{C}$. §5 considers Morita equivalence class of a Lie groupoid extension, recovering a morphism of stacks obedient to the required properties of a gerbe over a stack on the lines of [loc. cit.].

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MSC:

14A20	Generalizations (algebraic spaces, stacks)
22A22	Topological groupoids (including differentiable and Lie groupoids)
18D30	Fibered categories
18F99	Categories in geometry and topology
53C08	Differential geometric aspects of gerbes and differential characters

Keywords:

differentiable gerbes; differentiable stacks; Lie groupoids; Morita equivalence

Full Text: DOI

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