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Projective covers of 2-star-permutable categories. (English. French summary) Zbl 07287127
Cah. Topol. Géom. Différ. Catég. 61, No. 4, 402-426 (2020).

This work is preceeded by the following ones.

- The notion of a *multi-pointed category* as a unifying generalization of pointed and non-pointed contexts with a *good theory of ideals* and unified results from the realm of *ideal determined* categories on the one hand and *Barr-exact Goursat* categories on the other was introduced in [*M. Gran* et al., J. Pure Appl. Algebra 216, No. 8–9, 1905–1919 (2012; Zbl 1257.18011)].
- Notions of permutability of equivalence relations in multi-pointed categories were introduced and investigated in connection with certain diagrammatic characterizations known for regular *subtractive* categories and *Goursat* categories in [*M. Gran* et al., Theory Appl. Categ. 27, 80–96 (2012; Zbl 1252.18022)].
- Generalizations of homological lemmas such as the 3×3 *Lemma* and the *Short Five Lemma* were considered in [*M. Gran* et al., Homology Homotopy Appl. 14, No. 2, 1–22 (2012; Zbl 1258.18011)]. In non-pointed contexts the appropriate notion of exact sequence is that of *exact fork*, which is a sequence consisting of a kernel pair and its coequalizer, while in a more general multi-pointed contexts the pertinent notion is that of a *star-exact* sequence unifying the pointed and non-pointed versions and allowing for the aforementioned multi-pointed homological lemmas.
- The notion of 2-*star permutable* category was studied as a common extension of both regular subtractive and regular Mal'tsev categories, and characterizations of these categories via diagrams such as regular pushouts were generalized to a multi-pointed context in [*M. Gran* and *D. Rodelo*, J. Algebra Appl. 13, No. 8, Article ID 1450068, 15 p. (2014; Zbl 1308.18003)].

This paper aims to address a characterization of *projective covers* of regular 2-star permutable multi-pointed categories, unifying and subsuming the known characterizations in the Mal'tsev [*J. Rosický* and *E. M. Vitale*, Homology Homotopy Appl. 3, No. 3, 453–466 (2001; Zbl 0993.18001)] and subtractive [*M. Gran* and *D. Rodelo*, Diagrammes 67–68, 103–115 (2012; Zbl 1331.08003)] settings. The author applies the characterization in terms of star-symmetry to recover the syntactic condition defining *E-subtractive* varieties in the sense of [*A. Ursini*, Appl. Categ. Struct. 21, No. 3, 209–236 (2013; Zbl 1285.08003)].

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MSC:

18C05 Equational categories
18A35 Categories admitting limits (complete categories), functors preserving limits, completions
08B05 Equational logic, Mal'tsev conditions
18G05 Projectives and injectives (category-theoretic aspects)

Keywords:

multi-pointed category; star relation; Mal'tsev category; subtractive category; projective cover; variety of algebras

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