Zoethout, Jetze

Internal partial combinatory algebras and their slices. (English) Zbl 07290838

Theory Appl. Categ. 35, 1907-1952 (2020).

A *partial combinatory algebra* (PCA) is an abstract model of computation of computation generalizing the classical notion of computability on the set of natural numbers, These models are to be studied from a standpoint of category theory. Every PCA A gives rise to a category $\mathsf{Asm}(A)$ of assembles, which may be put down as the category of all data types to be implemented in A, Furthermore, the ex/reg completion of $\mathsf{Asm}(A)$ is always a topos, called the *realizability topos* of A and denoted by $\mathsf{RT}(A)$, in which the internal logic is governed by computability in A. The fundamental theorem of topos theory claims that the slice category $\mathsf{RT}(A)/I$ is always a topos, but it is not a realizability topos in general. This paper is concerned with the question whether there is a natual class of categories containing all categories of the form $\mathsf{Asm}(A)$ and being closed under slicing. The author finds out a solution in Stekelenburg's generalization of PCAs [*W. P. Stekelenburg*, "Realizability categories", Preprint, arXiv:1301.2134], where a PCA is not a set but an object in a given regular category.

A synopsis of the paper consisting of siz sections goes as follows. §2 recalls the relevant notions from [loc. cit.]. §3 makes PCAs the objects of a 2-category. §4 investigates the interaction of the 2-category with the construction of Asm. §5 presents an explicit description of the slice of a category of assembles, making use of it to calculate a number of examples of slices. §6 addresses the notion of computational density [*P. Hofstra* and *J. van Oosten*, Math. Proc. Camb. Philos. Soc. 134, No. 3, 445–463 (2003; Zbl 1046.03038)] within the setting of this paper.

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MSC:

03G30 Categorical logic, topoi
68Q09 Other nonclassical models of computation
18B25 Topoi

Keywords:

partial combinatory algebra; assemblies; toposes; slicing

Full Text: Link

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