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Slenderness. Volume 1. Abelian categories. (English) Zbl 06947511
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The notion of slenderness evolved from intrinsic and interesting observations regarding homomorphisms from infinite products of the integers into the group of integers. Slenderness was once defined in the category of torsion-free Abelian groups as countable coordinatewise slenderness, though, curiously enough, it turned out that torsion-freeness comes from slenderness [*R. Dimitrić*, Glas. Mat., III. Ser. 21(41), 327–329 (1986; Zbl 0618.20038)]. The first proof of the slenderness of $\mathbb{Z}$ was given by *E. Specker* [Port. Math. 9, 131–140 (1950; Zbl 0041.36314)]. The notion of slenderness was extended to general Abelian categories by the author in the first half of the decade of the 1980s. Nowadays, slenderness is both a theory and a program, and this volume is dedicated to presenting the ideas and relevant results of the theory and outlining the main aims of the program. Slenderness is a theory because it encompasses general results from seemingly disparate areas of algebra, topology, set theory and geometry. Slenderness is a program that has as one of its goals a classification and characterization of slender objects in general, and in specific categories in particular. The volume consists of six chapters together with an introductory chapter (Chapter 0) to fix terminology and summarize the main results in the text and an appendix introducing the reader to a minimum of set theory, in particular to the non-measurable cardinals. A synopsis of the volume goes as follows.

An imitation of the passage from polynomials to infinite power series in analysis is made use of in algebra through completions of algebraic objects. The construction is significant in the development of the theory of slenderness, and Chapter 1 gives a sufficiently detailed treatment of it. The chapter addresses the question of the existence of linear topologies on objects, subject to various properties such as non-discreteness, the Hausdorff property and metrizability. Infinite products of objects play a central role in the theory of slenderness, and some of their topologies are introduced in this chapter. The final topics in this chapter are meant to point the inquisitive reader in the direction of future study in the subject of completions and metrizability.

Chapter 2 is concerned with inverse limits, discussing the Mittag-Leffler condition, the surjectivity property and the flabbiness conditions.

Chapter 3 sets the stage for more detailed study of slender modules and slender rings in subsequent chapters, introducing several equivalent definitions of slenderness. The notion of slenderness reaches out diverse areas such as set theory (Boolean algebras and non-measurable cardinals) and topology (via product and linear topologies). This chapter relies largely on the author's [Commun. Algebra 11, 1685–1700 (1983; Zbl 0578.13010); Lect. Notes Math. 1006, 375–383 (1983; Zbl 0517.18013); Slenderness in Abelian categories. PhD Thesis, Tulane University, New Orleans (1983)].

The author's interest in pure injectivity stems from the fact that many filtered quotients of infinite products have this property, one example being objects of the form

$$\prod A_n / \coprod A_n$$

that feature prominently in the general characterization of slenderness. Chapter 4 addresses only results to be useful in later discussions, a number of proofs being omitted in favor of references to relevant sources.

Chapters 5 and 6 deal with slenderness of rings and modules, and in particular rings of functions, following aesthetic principles in presenting beautiful results with beautiful proofs. Chapter 6 draws most of the results from Heinlein's [Vollreflexive Ringe und schlanke Moduln. Dissertation der Naturwissenschaftlichen Fakultät der Friedrich-Alexander-Universität Erlangen-Nürnberg, Elrangen (1971)].

The next volume is devoted to generalizations and dualizations of the theory of slenderness.

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MSC:

18-02	Research exposition (monographs, survey articles) pertaining to category theory	Cited in **1** Review
18Exx	Categorical algebra	
03C65	Models of other mathematical theories	

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