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Computability in basic quantum mechanics. (English) Zbl 06917933

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This paper constructs an admissible representation for the Hilbert lattice $\mathcal{L}$ [*P. Pták* and *S. Pulmannová*, Orthomodular structures as quantum logics. Transl. from the Slovak. Dordrecht etc.: Kluwer Academic Publishers; Bratislava: Veda (1991; Zbl 0743.03039)], based upon which admissible representations St and Obs for the set of quantum states and that of quantum observables are presented. The construction is fulfilled by making use of the fairly abstract methods of topological domain theory. These data types come endowed with the notion of computability in the sense of Weihrauch's Type Two Effectivity [loc. cit.], in which a map between admissible representations is computable iff it is realized by an element of $\mathcal{K}_{2,eff}$, namely, a computable element of the Baire space also known as a total recursive function. The resulting category $\boldsymbol{AdmRep}_{eff}$ is a full reflective subcategory of what is called Kleene-Vesley topos $\mathcal{KV}$, which is the effective part of $\boldsymbol{RT}(\mathcal{K}_2)$ as described in [*J. van Oosten*, Realizability. An introduction to its categorical side. Amsterdam: Elsevier (2008; Zbl 1225.03002)]. The following effective version of von Neumann's spectral theorem is established.

Theorem. The spectral theorem for self-adjoint operators holds in $\boldsymbol{RT}(\mathcal{K}_2)$ and $\mathcal{KV}$.

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MSC:

18C50 Categorical semantics of formal languages
03D78 Computation over the reals, computable analysis
81P10 Logical foundations of quantum mechanics; quantum logic (quantum-theoretic aspects)

Full Text: DOI arXiv

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