Barter, Daniel; **Bridgeman, Jacob C.**; **Jones, Corey**

Domain walls in topological phases and the Brauer-Picard ring for vec$(\mathbb{Z}/p\mathbb{Z})$**.** (English)

Zbl 07076856
Commun. Math. Phys. 369, No. 3, 1167-1185 (2019).

The principal objective in this paper is to compute the Brauer-Picard ring for $\boldsymbol{Vec}(\mathbb{Z}/p\mathbb{Z})$ with p prime, making use of ladder string diagrams [*S. Morrison* and *K. Walker*, Geom. Topol. 16, No. 3, 1481–1607 (2012; Zbl 1280.57026); *J. Fröhlich* et al., Adv. Math. 199, No. 1, 192–329 (2006; Zbl 1087.18006)]. The string diagram calculus for fusion categories in [*P. Selinger*, "A survey of graphical languages for monoidal categories", Lect. Notes Phys. 813, 289–355 (2011)] allows the authors to interpret fusion categories as disc-like 2-categories, using the ideas in [*S. Morrison* and *K. Walker*, Geom. Topol. 16, No. 3, 1481–1607 (2012; Zbl 1280.57026)] to compute relative tensor products. Physically, this fusion category corresponds to the quantum double model of $\mathbb{Z}/p\mathbb{Z}$, which is significant for both quantum memories and quantum information processing tasks.

Although the authors consider only $\boldsymbol{Vec}(\mathbb{Z}/p\mathbb{Z})$, the framework is not restricted to this class of fusion categories and future work is expected.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18D20 Enriched categories (over closed or monoidal categories)
16D90 Module categories in associative algebras
81P45 Quantum information, communication, networks (quantum-theoretic aspects)
81T45 Topological field theories in quantum mechanics

Software:

MathOverflow

Full Text: DOI

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