Baez, John C.; **Master, Jade**
Open Petri nets. (English) Zbl 07283037
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The principal objective in this paper consisting of seven sections together with an appendix is to describe two forms of semantics for *open Petri nets*. The first is an *operational* semantics. Let Petri be the category of Petri nets and Petri net morphisms, Set the category of sets and mappings and CMC the category of commutative monoidal categories and strict monoidal functors. There is a canonical functor $L:\mathsf{Set}\to\mathsf{Petri}$ as well as a canonical functor $L':\mathsf{Set}\to\mathsf{CMC}$. A symmetric monoidal double category $\mathbb{O}\boldsymbol{pen}\,(\mathsf{Petri})$ is constructed in Theorem 13 of §3, where objects are sets and horozontal 1-cells from a set X to a set Y are open Petri nets, namely, cospans in Petri of the form

$$\begin{array}{ccccc} & & P & & \\ & \overset{i}{\nearrow} & & \overset{o}{\nwarrow} & \\ LX & & & & LY \end{array}$$

with P a Petri net as well as i and o Petri net morphisms, while a symmetric monoidal double category $\mathbb{O}\boldsymbol{pen}\,(\mathsf{CMC})$ is constructed in Theorem 16 of §4, where objects are sets and horozontal 1-cells from a set X to a set Y are open commutative monoidal categories, namely, cospans in CMC of the form

$$\begin{array}{ccccc} & & C & & \\ & \overset{i}{\nearrow} & & \overset{o}{\nwarrow} & \\ L'X & & & & L'Y \end{array}$$

with C a commutative monoidal category as well as i and o strict monoidal functors. It is shown in Theorem 17 of §4 that this semantics gives a map from $\mathbb{O}\boldsymbol{pen}\,(\mathsf{Petri})$ to $\mathbb{O}\boldsymbol{pen}\,(\mathsf{CMC})$, sending any Petri net P to the free symmetric monoidal category FP generated freely by the places and transitions of P while acting on open Petri nets in a compositional way.

The second is a *reachability* semantics, giving a map from $\mathbb{O}\boldsymbol{pen}\,(\mathsf{Petri})$ to the double category $\mathbb{R}\boldsymbol{el}$ of relations. The double category $\mathbb{R}\boldsymbol{el}$ of relations is addressed in §5, where

- objects are sets,
- vertical 1-morphisms are functions $f:X\to Y$,
- horizontal 1-cells from X to Y are relations $R\subseteq X\times Y$,
- 2-morphisms are squares

$$\begin{array}{ccc} X_1 & \overset{R\subseteq X_1\times Y_1}{\rightarrow} & Y_1 \\ f\ \downarrow & & \downarrow\ g \\ X_2 & \overset{S\subseteq X_2\times Y_2}{\rightarrow} & Y_2 \end{array}$$

  with $(f\times g)\,R\subseteq S$.

It is shown in Theorem 23 of §6 that there is a lax double functor $\blacksquare:\mathbb{O}\boldsymbol{pen}\,(\mathsf{Petri})\to\mathbb{R}\boldsymbol{el}$. Appendix is a brief review of double categories.

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MSC:

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68Q55 Semantics in the theory of computing

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